My Adventures in Simplicity and Complexity at UC Davis

Carlos E. Puente

Cover. Thank you Majdi. It is an honor for me to share this seminar.

Page 2. This presentation is dedicated in memoriam to: Jacobo Goldstein, a fellow Colombian who was the best teacher I ever had; Verne Scott, an athletic professor here who led the committee that proposed I should be hired; Miguel Mariño, another pillar of the department and a friend I dearly miss; Jorge Ramírez and Garry Willgoose, two bright classmates from MIT who departed too early; Diego Echeverry, a friend at Universidad de los Andes who also departed too early; Jim Biggar and Don Nielsen, also pillars of LAWR who supported me; Pete Eagleson, a legendary hydrologist from MIT who called me Colonel Buendía; Fabio Castrellón, my first hydrology teacher at Uniandes; Ken Tanji, the hardest worker I ever saw in our department; and Joe Stasulat, also affiliated with LAWR, who gave me sound advice at times of doubts.

Page 3. The talk is also dedicated to Fabian Bormardelli, Samuel Sandoval, Daniele Zaccaria, Laura Foglia, Majdi Abou Njam, Da Yang, Isaya Kisekka, Helen Dalkhe, Matt and Adele Igel, Xiaoli Dong and Nate Chaney.

Page 4. The seminar is divided into a few parts and it starts explaining an idea that guided my career.

Page 5. As hinted from the dedication, I came to UC Davis via Universidad de los Andes in Bogotá, Colombia and MIT, and such happened in 1986 after following fellow Colombians compete at the Tour de France.

Here you see some very bright people: Alejandro Deeb, who was my undergraduate advisor and with whom I wrote my first article; Darío Valencia, a gentleman working in Medellín who became a clear example to follow; Rafael Bras, my advisor and mentor at MIT who changed my life by bringing me to the US; Juan Valdés, a teacher and friend who was a key member of my dissertation committee; Ignacio Rodríguez-Iturbe, my academic grandfather, as he was the advisor of both Rafael and Juan, and another example for me to follow; and Daniele Veneziano, a high octane scientist who taught me and another person a graduate class, which triggered that I would do likewise with a graduate class of mine here at UC Davis.

These folks were giants for me to emulate and I came to Davis 35 years ago ready to triumph…

But, just in case, I had a plan B…

Page 6. Perhaps I could have gone home and do some music, as I am doing here with my dad…

After a difficult beginning, as I found myself working pretty isolated and with rather limited resources, my first crop of students arrived…

Page 7. Here they are, three prosperous professors: Marc Bierkens from the Netherlands and fellow of AGU; Germán Poveda a Colombian and winner of the Nobel Prize with Al Gore, and Enrique Angel Sanint, also Colombian.

At the time, I realized that opportunities for innovation in hydrology existed in the physics literature via topics related to fractal geometry and chaos theory. As such, I immersed myself into such matters, read several papers and attended a few conferences. Little by little I gained some understanding and on a lovely Friday I got an idea…

Page 8. It all started months before when I received an article from Michael Barnsley about fractal interpolating functions. As the ideas were far reaching, I tried to generalize them to obtain surfaces from which to study the evolution of river networks. Although, at the end, the objects I got contained unnatural ridges, this excursion allowed me to appreciate a bit more about simplicity and complexity, for the usage of rather simple sets gave rise to complex looking patterns and, as in chaos theory, without relying on the concept of chance.

After attending an extension course on fractals at UC Berkeley, I read a paper by Charles Meneveau and Katepalli Sreenivasan regarding the nature of turbulence and, all of a sudden, on such a special Friday, it came to me that the multiplicative cascades of nature's disorder could perhaps be joined with fractal functions in order to arrive at a model of hydrologic phenomena.

So that you may appreciate the thought process, I explain next some of the math in some detail.

Page 9. First, in regards to fractal functions. These are geometric objects generated via the iteration of simple maps, as the two shown here.

Although I am not going to read them, there are some key properties that I ought to emphasize about them.

Page 10. The first thing to notice is that they both take a generic point in the plane (x, y) into another point in the plane.

Page 11. As you can see, the first component on both maps does not depend on y and the second simply yields a linear combination of both x and y.

Page 12. d sub 1 and d sub 2 are parameters, whose magnitudes are both less than 1.

Page 13. x lies, in this example, between 0 and 1 and the two maps pass by the three points shown, in the sense that w sub 1 of (0,0) is (0,0), w sub 2 of (1,0) is (1,0), w sub 1 of (1,0) is (0.5,1) and w sub 2 of (0,0) is also (0.5,1).

Page 14. As implied by the simple dynamics of the 1st component, w sub 1 operates to the left of the domain and w sub 2 does so to the right.

Page 15. With this understood, here are the aforementioned three points…

The next step is to set the parameters d sub 1 to 0.5 and d sub 2 to negative 0.5 and perform iterations of the maps using a coin, such that heads means using w sub 1 and tails w sub 2.

Imagine that one starts at the point in the middle and that the first coin toss gives heads…

Page 16. Then, using w sub 1 generates a point to the left as marked by the arrow.

Page 17. If another head happens, the notions generate another point to the left,

Page 18. and if a tail occurs next, such gives another point, now on the rights and so on.

Page 19. This is what happens after 80 iterations or so,

Page 20. and this is what emerges when many more points are added. As seen, one gets a profile of a fractal mountain, like the surfaces I sought, an interesting object that lacks derivatives in many places.

Such a mountain turns out not to depend on chance as it is obtained irrespective of who tosses the coin and also independently of the type of coin used: a fair one or not.

Page 21. It happens that the kind of object obtained, a function that interpolates the three points pictured earlier, depends on the signs of the two parameters d sub 1 and d sub 2. While the case + - corresponds to the mountain just explained, the case - - yields another symmetric mountain and the case + + defines a cloud.

Page 22. To further emphasize the determinism of the obtained objects, here is an alternative construction of the cloud related to the case + +. As seen, such is built via mid-point additions of powers of z on lines joining the original points and those acquired progressively.

The tossing of a fair or a biased coin defines points that always land on the same attractor cloud, or on the same mountain for the other cases.

Page 23. It happens that the objects, shaped as wires, a notation I invented so that I could explain the notions to my dad who was an orthopedic surgeon, fill up increasing amounts of space in the plane as the magnitude z increases beyond 0.5. When z tends to its limit of one, the objects, that are topologically one dimensional, fill all the plane and have fractal dimensions that tend to 2.

With these notions in place, it is time to talk a bit about the other side of the idea, namely multiplicative cascades related to turbulence.

Page 24. Here it is, in motion, what Meneveau and Sreenivasan found while studying fully developed turbulence in the air. When the Reynolds number is sufficiently large, an irreversible cascade of inwardly rotating eddies gets established, one that splits eddies into eddies and moves the energies from place to place until dissipation.

Page 25. What such researchers found via a host of experiments, both in nature and in the laboratory, including atmospheric turbulence, boundary layer turbulence and the one created on the wake of a cylinder, is that data along lines perpendicular to the flows, that is over one dimension, were universally consistent with the cascade above when the parent eddies divide into smaller eddies that carry 70 and 30 percent of the parent’s energy.

Actual records turned out to be a permutation of the shown object filled with thorns, for the largest eddy in nature does not always happen to the left. Invariably however, 1D data exhibited quantized layers, as the ones here, that by being supported by fractal dusts, define a multifractal object.

This profound result is surprising to me to this day, for one may imagine that the power of the air should be an absolute mess, but, instead, such has a clear structure as indicated by the inverted parabola here that, in excellent agreement, draws the spectrum of singularities of the records and that of the simple cascade model.

Page 26. With this in place, we may return to the lovely Friday of my illumination. Bridging the two concepts relied on realizing that the generic encyclopedia of turbulence also appeared while finding a fractal interpolating function using a biased coin that would imply going to the left 70% of the time and to the right 30% of the time. Here dx is the histogram over x of the acquired points on the mountain, which gave credence to the idea thinking that finding histograms over y would produce interesting sets.

Page 27. This is what my collaborator Enrique Ángel, for an angel guided the calculations, found over y, a complex looking histogram that could be interpreted as a transformation of turbulence.

Page 28. As seen, the idea turned out to be good for dy looks like normalized real data, as measurements gathered in hydrology and also in geophysics and beyond.

Page 29. With the passage of time and while trying to explain the idea, I realized that the construct reminded us of the celebrated allegory of the caveman by Plato, for what we see, in an output object dy, is just the shadow produced by the wire when illuminated by the input multifractal dx.

Page 30. And the notions were also rather Platonic in a deeper sense, for although dy appears to be random, it is not: it is an entirely deterministic object emanating from the two deterministic components that parameterize it.

Page 31. These are the ideas, with some extensions, that I developed during my career and that define the so-called fractal-multifractal, FM method…

But, there is a relevant case I ought to show you now and that is what happens as the fractal functions fill-up space,

Page 32. This is what occurs when the mountain case is extended to its limit. Surprisingly, dy now becomes a Gaussian bell and such a result happens to be universal for any non-discrete input dx.

Page 33. This was, no doubt, a very surprising PUENTE, bridge in English, that links, from x to y, two distinct behaviors of nature: dissipation to conduction, or, if you wish, advection to diffusion.

Page 34. In case you are wondering, the sign combination when both d sub 1 and d sub 2 are negative yields in the limit the oscillation between two bells, which triggers the question: and what happens to the cloud?

Page 35. This is the stupendous surprise found with the + + case, when z equals 0.995. Remarkably, a luminous bell concentrated at infinity happens in the limit,

Page 36. and the maximally positive wire filters almost any type of disorder and takes it all above, except for a discrete dx,

Page 37. which begged the questions: what is the transformation? Is it something like LOVE capable of transforming almost anything into beauty?

While pondering this theorem of science that I never thought I would discover, I had an epiphany in which the diagram became a reality in me. I vividly experienced warmth, peace and love as others have claimed, and my life was not the same since.

Page 38. Here there are some friends who accompanied me during those unexpected and beautiful days. Alvaro Aldama, a classmate from MIT and professor at Princeton at the time, who explained to me what was to be born from above; Father Rafael García Herreros, a saint to be, who after my illumination encouraged me to dream; Father Diego Jaramillo, who guided my attempts of communicating the science; and Father Camilo Bernal, a classmate of mine in Mathematics and Engineering at Universidad de los Andes in Bogotá, who has taught about perseverance since.

As time passed, and as I recovered from an emotional breakdown that took me to Colombia for a year and a half, but not to play drums altogether, one through which I learned that overworking was not good for me,

Page 39. some friends came to my rescue. On the one hand, Akin Orhun, whom I am sure some of you remember, who upon learning about my hydrologic research had the vision that I should be trying the ideas with economic data, to which I always said that I did not know enough economics; Daniel Fessler, a friend of Akin and a professor at the UC Davis school of law, who tried to get me a benefactor from a person already helping the Vatican Observatory, and who ultimately helped me get a donation from Pacific Bell (what a perfect name!) that allowed me to hire economist Marta Juliao.

These were very special times, improbable times I ought to say, for as time passed Marta was destined to be my closest collaborator

Page 40. and Akin the godfather of our beloved Cristina.

Page 41. At this time, I would like to show you some results in hydrology and beyond, but not in economics, for such never took place. Some of the research I shall share here was funded early on by NASA, NSF, USDA and others, but such was increasingly difficult to find as time went on.

Page 42. But before I do so, I would like to pay tribute to some other collaborators that have accompanied my career with their wisdom and hard work: Sivakumar Bellie, who teaches at ITT Bombay; Jason Huang, the best programmer I have ever worked with; Andrea Cortis, my man; Mahesh Maskey, the most persevering of them all; Don Jorge Pinzón; Jason Emmons; Martha Cecilia Díaz; René Lobato; José Angulo and Aldo Romero.

Page 43. And also the longest genealogy starting with the pioneer Nelson Obregón, followed by Victor Peñaranda, followed by David Serrano and until Fernando Torres and María Paula Sanabria, altogether two great great grand children in science.

Page 44. Here is an application of the FM approach that was part of my research for several years. On the left, a representation from a wire passing by 5 points, and not three, that starting on a distinct multifractal dx gives rise to a deterministic object dy that preserves statistical and chaotic properties of the data set on the right, which portrays rainfall data measured in Boston every few seconds.

Clearly, to two vertical sets are not identical, but look as cousins of one another and when I showed those

Page 45. to the one and only Benoit Mandelbrot, and upon telling him that the set on the right was rainfall in Boston, he exclaimed that dy was also rainfall, to which I replied: thank you very much. The FM method generates objects that resemble real data, and if the master said so, such was a welcomed reassurance.

In what follows I shall show you some additional results regarding the modeling of rainfall (and later streamflow), specifically data gathered at a daily scale.

Page 46. For this purpose, the iteration of two maps suffices, but subject to some conditions that ensure that the obtained dy's contain holes, as it does not rain every day.

Page 47. This gives rise to representations having nine parameters: x coordinates x sub 1 and x sub 2; y coordinates y sub 1, y sub 2 and y sub 3; the two parameters d sub 1 and d sub 2, as before; p as the bias of the coin guiding the iterations; and phi sub v providing an additional vertical cutoff to rid rain traces.

Page 48. All in all, given a data set, the goal is to explore the nine-dimensional FM space and find the representation that minimizes the mean root square error in accumulated records and such using a particle swarm procedure that, after years of experimentation, proved to be the best.

Page 49. So here we go. This page shows four years of accumulated records in black and their corresponding FM fits, in red, for daily rainfall at Cherry Valley, California, whose low root mean square errors (less than 2%) and maximum errors in accumulated sets show the merit of the geometric approach.

Page 50. Here is a comparison of observed, in blue, versus FM sets, in red, from 1958 to 2008, that further illustrates the goodness of the yearly representations, which leads to rather small errors when sets are considered as a whole.

Page 51. Given the quality of the yearly encodings, it becomes interesting to ask if there are trends in FM parameters. Here you see the best three sets of parameters that do not exhibit clear trends other that variability, including no obvious changes due to climate change.

Page 52. Such a lack of variability may be further studied by finding autocorrelations and phase diagrams of best parameters, which, in the case of interpolating coordinates here, exhibit a clear complexity.

Page 53. The same happens for the other parameters that show clouds of points in their phase diagrams and non-significant correlations, which confirm that FM parameters are as variable as rainfall totals, as seen in the last row below.

We performed this kind of an analysis to a few stations within the state and their variability was their salient feature. There were no obvious trends on FM parameters due to climate change and it was not possible to draw rainfall predictions based on them, a humbling reminder that natural complexity rules precipitation.

Page 54. Although daily rainfall predictions, one year ahead, are not possible, the FM approach may be used to obtain simulations that preserve entropy and histogram information, yielding sets that share the textures of real sets.

Page 55. The FM approach may also be used to disaggregate weekly, bi-weekly and monthly rainfall data into daily rainfall, as shown here fitting the broad records while doing calculations at the fine scale. As seen in red, the obtained downscales preserve the textures in the data.

Page 56. Having shown some results related to rainfall, now I can do likewise with the less variable records of streamflow. This entails using three simple maps,

Page 57. which lead to the nine FM parameters highlighted: the coordinates x sub 1, x sub 2, y sub 1 and y sub 2; the vertical scalings d sub 1, d sub 2 and d sub 3; and the biases of two coins p sub 1 and p sub 2 that together with 1 – p sub 1 – psub 2 guide the iteration of the three maps.

As before, all is done using a particle swarm procedure to search this nine-dimensional space looking for the best FM representation of a given set.

Page 58. This page shows examples of real, in black, and FM representations, in red, for two distinct looking daily sets gathered at the Sacramento River over two years, starting on October 1st. As seen, the accumulated records are very well represented and so are the records.

Page 59. Here is a comparison of observed, in blue, versus FM sets, in red, from 1950 to 2015, that further illustrates the goodness of the yearly representations, with statistical qualifiers that are clearly superior to those found for rainfall sets.

Page 60. As moving averages of data are often used, as in the pandemic, this slide portrays streamflow records averaged over decades and their FM counterparts. As expected, the geometric representations are even better, especially in the Nash-Sutcliffe statistic of the data, NSD.

Given the quality of the FM encodings, for annual and averaged data, it becomes relevant to study, as before, the evolution of FM parameters.

Page 61. Here it is what is found for the annual records, with red lines indicating averages over five years. As seen, there are some trends in some parameters but they do reflect some natural variability.

Page 62. And this is the counterpart for averaged records over decades, which, as seen, exhibits more stable trends on some parameters, leading perhaps to useful information regarding climate change effects.

When all is said and done, the key question is: could the FM method be used to predict?

Page 63. As seen here, for the averaged records the answer appears to be yes. On the left there are forecasts based on simple time series models of FM parameters and on the right based on fitting neural networks to such. As seen, decent predictions, one year ahead, are possible having small errors.

But, could it be done likewise with actual data? Could streamflow be predicted a year ahead?

Page 64. Sadly, but expectedly, the answer to such is yes and no, as seen here employing neural networks of FM parameters. Sometimes there are some decent forecasts, but the ever changing streamflow geometries, reflected in the varying FM parameters, often yield pretty bad forecasts, which once again provides a humbling realization of the irregularity of nature.

Page 65. As found for rainfall, the FM method may be used to simulate daily streamflow, preserving, in this case, the autocorrelation and the histogram of the data. As opposed to stochastic methods that preserve a few points on such attributes, simulations may be done that render all such statistical information.

Page 66. The FM approach may be used to disaggregate streamflow from months to days, as illustrated here for two stations on the Rio Grande-Bravo. What makes this slide, prepared by Laura Garza and

Samuel Sandoval, particularly interesting is that the obtained downscales exhibit all the desired features seen over the year hence providing suitable sets for ecological predictions.

…Not having time to tell you about even better results with temperature records, it is time for me to move on and show you some extensions to higher dimensions.

Page 67. Similar maps having more coordinates, with the first component decoupled from the others, may be used

Page 68. and such that the previous parameters little d sub n become matrices capital d sub n, shown here in polar coordinates defining radial and angular parameters.

Page 69. The iteration of such maps, based on suitable conditions regarding interpolating points, generate wires from x into the plane (y, z) that have fractal dimensions that could be any number between 1 and 3.

Page 70. Here there are some examples of what may be generated without chance. On the left, a convoluted wire gives rise to a complex looking shadow over the (y, z) plane having a hole in the center. On the right, a wire living in four dimensions yields a three-dimensional set, and its two-dimensional sides, as in pollution or rainfall studies.

Page 71. Here there are other examples that concentrate the mass in a few locations and which may perhaps be used in ecological studies. These complex deterministic sets need to be further studied as they may represent geostatistics in the next decades.

Page 72. Here is an application of these ideas on the modeling of groundwater pollution. Following the point injection of chloride at the Borden site, in Canada on day 1, the left patterns show vertically averaged concentrations at three later dates and on the right the corresponding shadows of fitted wires as obtained via the FM approach. As seen, the overall shapes of the plume, moving in the north east direction, are nicely preserved.

Page 73. It happens that the time evolution of the FM parameters for this case do show some clear trends: the coordinates in y and z grow linearly following the center of mass of the plume; the rotations stabilize by pairs as the patterns established a direction of motion; and the radial parameters are all high above a value of 0.85. When such FM trends are extrapolated, such result in faithful predictions of the plume 725 and 800 days after injection, which otherwise would have had elliptical shapes according to stochastic theories of transport.

This application illustrated the “heretical” idea that geometry may sometimes be enough to predict without using the underlying partial differential equations, something that I have tried to argue saying that geometry reflects the physics and chemistry taking place in nature, even if we cannot go and measure the FM parameters of a given set.

And having said this, let’s go to the limit and see what is obtained.

Page 74. This slide shows what happens as the fractal dimension of a wire tends to three: the multifractal set dx, as before, illuminates the bulky object plotted on the right as from x to y and x to z, and it produces as a shadow over the (y,z) plane a bell on the left, in this case a circular one reflecting no correlation.

Page 75. This happens universally for any non-discrete dx when the norms of the matrices D sub n tend to one. Varying the angular parameters and the signs of the radial parameters may define also elliptical bells, and, as before, oscillations between bells.

Page 76. An interesting property that explains, but does not prove, the nature of the universality of the Gaussian result for any non-discrete dx, is the fact that any piece of the space-filling wire, no matter how small, also defines bells as shadows, something very special as a piece of wire essentially clones the whole set.

Page 77. Although there is a proof of the one-dimensional bell result, the higher-dimensional case lacks a demonstration and such triggered an idea.

Page 78. As the limit when the radial parameters tend to 1 is key, the first thing to notice is that the previous pages arise for magnitudes of 0.995 while using, say, one mullion iterations.

Page 79. Then it occurred to us, to myself and a postdoctoral fellow Aaron Klebanoff not portrayed, to plot 2,000 dots at a time to study how circles were formed.

Page 80. Our expectation was that the circular bell would come from the superposition of circles and donuts, as in a game we all played as kids.

Page 81. But Aaron left to become a professor and I could not see much in the calculations that I stopped, as a good friend dissuaded me from doing something not hydrological, as I was up for tenure.

Page 82. Well, I got my tenure and one of the first things I did after was to go back to these limiting ideas and repeat the calculations increasing from the magnitudes of the key parameters from 0.995 to 0.9999, and this is what I discovered.

Page 83. There were no circles or donuts, but exquisite patterns making up the circular bell. The ones on the left have the angular parameters equal to 90 degrees, and the ones on the right 60 degrees.

Page 84. The superposition of ever varying beautiful rosettes give perfect circles and bells,

Page 85. and this result happens only in the space-filling limit that in a remarkable way define hidden order in chance, that is, guided by the coin used in the iterations, which does affect the specific patterns obtained, but not the overall nature of the Gaussian result.

Page 86. How not to exclaim with the Psalmist “How precious to me are your designs, O God”?

Page 87. As the discoveries just materialized, we went on to study what was found inside the 2D bell, and here you find some of its most relevant designs.

Page 88. On the left, and found with my wife who turned from economics to geometry, there are examples of the infinitely many ice crystals growing in nature by diffusion that the bell contains. Such lovely sets make up a geometric example of an aleph, a term coined by Argentine philosopher Jorge Luis Borges, and in a rather delightful way redeemed my attempts, as it all had to do with water, that is, with hydrology!

Page 89. On the right, there is another surprise. While the top image shows what appears in biochemistry books regarding the shadow of B-DNA, that is, a ten-sided rosette reflecting the 36 degree folding of the three-dimensional object, the bottom image portrays a pattern inside the 2D bell that shares the same topological properties in circles and spikes, one that has the additional property of being defined by the precise sequence of iterations given by the binary expansion of pi!

Searching for the proverbial needle on a haystack, together with pioneer Nelson Obregón we found this remarkable result that links a key element of the geometry of life with the most celebrated number in history. Later on, I dared to write a paper about this on a journal called Complexity and argued that perhaps speaking about a blind watchman was not altogether the posture to have.

Page 90. Having learned my lesson regarding the importance of the positive outward spiral in number 9, here you may see the evolution of a pattern guided by the first 100,000 bits of pi. The exotic dynamics and joyous beauty in these sets certainly inspires art projects to be housed at museums, who knows maybe one day here on campus.

Page 91. Well, knowing that all of these, and surely more, happens silently inside a bell, I had to write a book I never dreamt doing. Here you see the artistic cover done by my dear friend Duartecito, also Colombian, who also did the covers of other books and lovely figurines, as you will see.

Page 92. Knowing what I learned, I also had to write a poem that says: *The bell peals intensely/revealing mighty deed,/in shadows, as by magic,/designs of life crossbreed./Its central theme exalts/independence as a feast,/reparation of the broken/in enduring code within./O patterns of pure beauty,/giving harmony to chance,/O quintessential simplicity/ascribing a meaning to π./In fullness of dimension,/while defeating all strife,/dwells by ardent iteration/a reflection of God's art.*

Page 93. I did write two other books and on a parable based on the notion of the hypotenuse you may appreciate Duarte's minimalistic figurines. The idea here is to use nature's multiplicative cascades to speak about our own modes of division, which sadly lead to our own turbulence, as reflected in the concentration of resources and the dissipation of friendships and societies.

By learning about the differences between equilibrium and thorny multifractals, or to say in an alternative way, between their accumulated objects as shown here on the right: a straight ramp with equation $Y = X$ and a well-named devil's staircase, the reader is guided to appreciate that there is a unique solution for peace and harmony that corresponds to the maxim "love one another".

Page 94. As shown here, this piece has received some nice comments from some important people: Nobel prize winner José Saramago who said that my parable was as fascinating as an unknown book by Borges, the same celebrated Argentine I mentioned before; and Templeton prize winner in Science and Spirituality John Polkinghorne who said that the parable conveyed spiritual insights in an attractive way. These opinions have not made my parable a best seller, but such are not too bad for a UC Davis hydrologist, even if such tries to follow a rather famous colleague of water in John the Baptist.

I ought to say that it was not possible for me to have my little book published properly, as several publishing houses told me that such did not fit their lists. But, heeding the wisdom of our own Greg Pasternack, what you see here happened via Amazon, together with versions in Spanish, Italian and Farsi.

Page 95. Continuing with parables, there is another one from science that led to my other improbable book, another undreamt piece of work that I have shared at national and international conferences in the scientific field of complexity and also in the field of science and religion. This specific parable pertains to the fig tree of science shown on the left, an object known as the Feigenbaum tree, in honor of a rather brilliant scientist that established that there is order in the path to chaos.

When this tree, also known as the diagram of bifurcations, emerged in science, some people stopped their work in order to verify for themselves how a simple parabola defining the logistic map produced an object that not only contained oscillations according to any natural number, but also had many instances in which the dynamics never stabilized and diverged, giving rise to the term chaotic.

When Albert Libchaber and collaborators showed that the same order a la Feigenbaum occurred in the progressive heating of fluids, the field of non-linear science bloomed in earnest and I took notice.

Page 96. So that you may appreciate the nature of the spiritual parable that echoes ancient fig trees, this slide contains, from below, three increasingly tall parabolas whose reiteration gives rise to rather distinct destinations within the tree.

As seen, when the parabola is below the one-to-one line, the succession of vertical-horizontal lines starting at a given x sub 0 show that the dynamics converge to the origin, portrayed in the straight root of the tree. When the parabola crosses the line, as seen in the middle, the dynamics repel the origin and the vertical-horizontal lines show convergence to a positive fixed-point, as seen in the tender branch on the tree.

As the parabola keeps increasing in height, the dynamics change and exhibit all the possibilities drawn in the tree, passing first through a cascade of bifurcations, that is, progressive periodic behavior in powers of two, and a plethora of other repetitions that are not powers of two and most commonly the dusty foliage of the tree reflecting the strangeness of chaos, as seen on the top right at the highest heat.

How sad is to end up trapped forever in such a hellish state, says the parable, as the crux of the manner rests in recognizing goodness in not crossing the line.

Page 97. This is the cover of my book The Fig Tree & The Bell, designed by Duartecito using a lovely picture by my friend and fellow Colombian Alejandro Gil. The book contains the following poem that I gave in person to Mitch Feigenbaum with the dedication “not by chance”. It says: *Foliage of disorder/trapped in empty dust,/jumps astir forever/enduring subtle thrust./Crossing of the outset/leaving faithful root,/looming tender offset/failing to yield fruit./Cascade of bifurcations,/increasing heat within,/inescapable succession/of branches bent by wind./Sprouting of dynamics,/attracted to the strange,/o infinity reminding/at the origin the flame.*

Page 98. As the title of the book mentions the bell, here is yet another parable reflected in the most beautiful diagram I know, for although it may be said: what good may come from a hydrologist living in a small town?, life is really full of surprises.

Page 99. As you know, for almost 20 years I shared on campus a seminar titled like the sub-title of my last book: Chaos, Complexity and Christianity. Here you see the key message of such a clandestine class: follow Y = X, the one whose silhouette ended up crucified on a cross, live avoiding chaos and complexity, become a saint as explained on the left, in other words love one another.

I did my seminar a record 44 times. Although I am sad not to do it during my retirement, I am very grateful to have contributed to students from many disciplines providing at least some food for thought.

By the way, the image shown comes from the Shroud of Turin, and the geometric inscription J X Y appears on a stone under his chin that may faintly be seen, as recently discovered via holographic analysis.

Page 100. Now, close to the end, I would like to tell you a bit about my future dreams.

Page 101. Here is my dear friend Steve Bennett, Benito, appointed to support me for years, particularly during the prescribed tribulations I have experienced by writing and teaching about simplicity at UC Davis.

While dreaming on what to do, he himself came out with a perfect gift for me as shown on the left and titled Jacob under the fig tree. You see, the patriarch fell sleep and had a faithful dream with stairs and angels and upon waking up he realized he was on holy ground and such propelled him to follow God more so than before. Such is the wish of my good friend for me and I do hope to fully reawake and do as many lectures I can in two languages and also record a collection of songs that have come my way to adorn my understanding.

Page 102. In regards to the lectures, here there are some friends that would be helping me. Father Rafael Pascual with whom I shall teach my seminar at a master's program in science and faith at Ateneo Pontifico Regina Apostolorum in Rome; Fray Nelson Medina who publishes in his popular blog some of the entries on a blog of mine he called me to do and titled Campanitas de Fe, little bells of faith; Father Wilson Mejia who teaches me much and at the same time invites me to share and sing; and last but surely not least, Juan Sebastián de Plaza, a civil engineer that gladly organizes lectures for me.

Page 103. The music side of things is looking very promising already as a Cuban musician Lázaro Alemán, Lazarus, resurrected into my life recently and is doing some lovely arrangements of my songs and as a Mexican singer Fabiola Jaramillo may perform one of my compositions to Mary, the mother of Jesus, when she performs at her mañanitas in Mexico City, next December.

As the album cover on the left shows, the dream is to have a band Shanti Setú, a Bridge to Peace in Sanskrit, and I promise to keep you informed when that happens.

Page 104. Before I sing to you to end this swansong, I would like to show you my finest composition: Marta, Cristina standing and Mariana, all gifts from above.

Page 105. So, here it is my song Pathways. I hope you like it: *Two options before us/two pathways ahead,/the one is the longest/the other straight./We journey directly/or go by the legs,/we follow intently/or end up in pain./By walking the flatness/or hiking the spikes,/we travel in lightness/or take serious frights./The incentive is unity/and the call proportion,/the key is forgiveness/and the goal true notion./In wandering wickedness/there is never fruit,/but in ample humbleness/one encounters the root.*

Page 106.

Page 107. *There is no excuse,/o listen my friend:/it's by the hypotenuse/or else by the legs./There is no solution/but walking straight:/the spikes of disorder/insinuate the way./There is a best pathway,/the palpably smooth./**It's by the hypotenuse/and walking in truth.**/There is one solution,/I tell you the truth./**It's by the hypotenuse/and walking in truth./**For any other pathway/will lead us astray./**It's by the hypotenuse,/there is no other way./**O listen, you brother,/let's brighten the day./**It's by the hypotenuse,/there is no other way.***

Page 108. *Otherwise, the devil/shall pull by the legs./**It's by the hypotenuse/or else by the legs./**For such road is fractal:/as long as it gets./**It's by the hypotenuse/or else by the legs./**O let's mend the broken,/growing to the root./**It's by the hypotenuse,/the one that yields fruit.**/Let's keep equilibrium,/avoiding dark soot./**It's by the hypotenuse,/the one that yields fruit./**O listen, you sister,/a counsel from science./**It's by the hypotenuse:/the simplest design./**I tell you integrating,/don't leave it to chance./**It's by the hypotenuse/the simplest design.***

Page 109. ***It's by the hypotenuse/the simplest design…***

Page 110. Dancing any one? Conga to infinity!

Page 111. It has been an honor…