DETERMINISTIC GEOMETRIC MODELING OF PRECIPITATION PATTERNS

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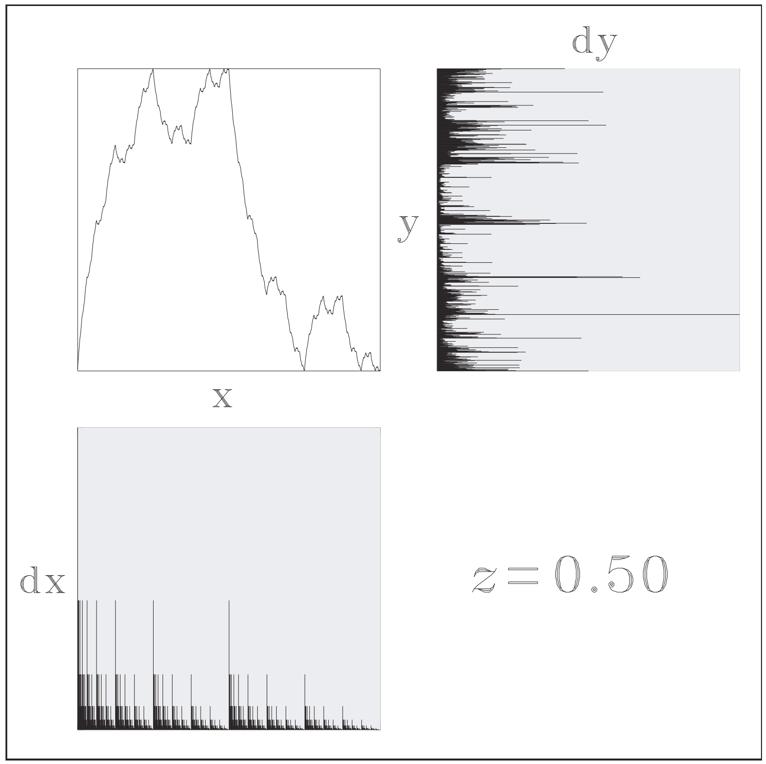
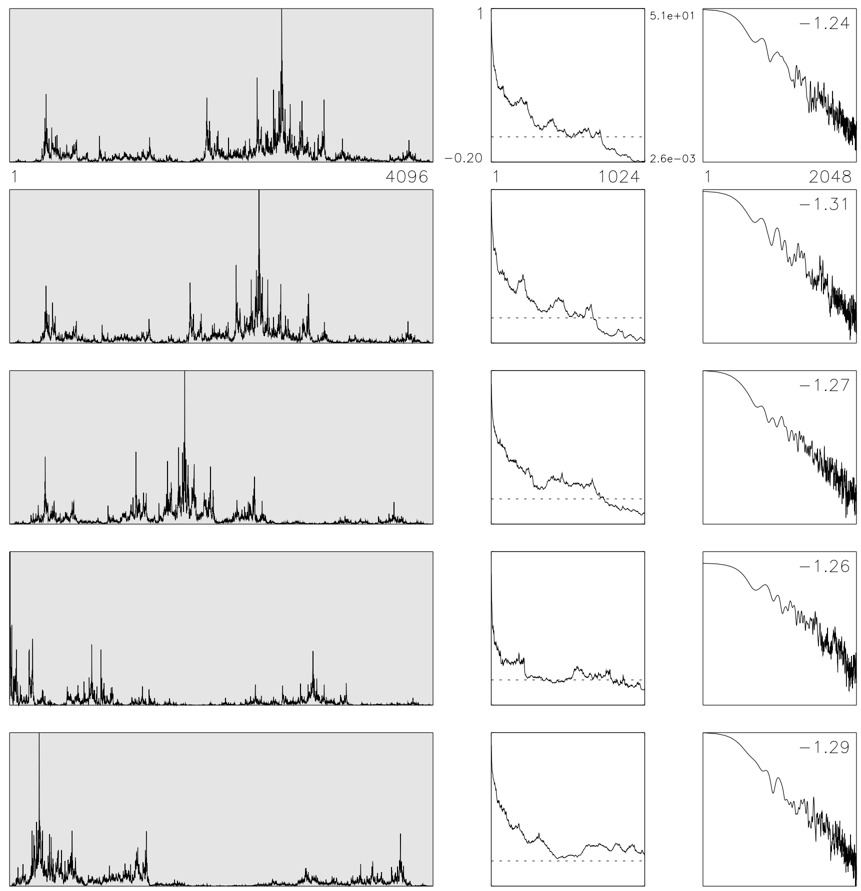
A Platonic approach to complexity

$$w_1(x,y) = (x/2,\ x + z\cdot y),\ w_2(x,y) = (x/2 + 1/2,\ 1 - x - z\cdot y)$$



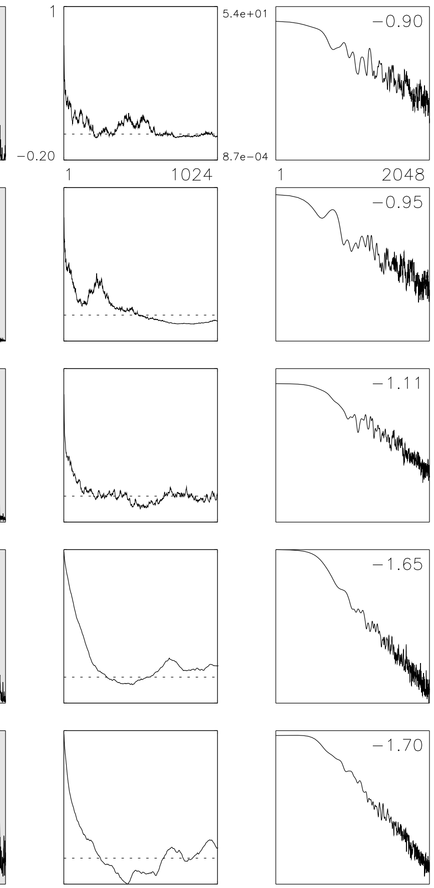
iteration of simple maps produces a "**wire**" from x to y
shadows yield **complex** sets, dy, based on a **multifractal**, dx
patterns appear to be **random** but they are not...

Some sample shadows
(Puente, 2004)



varying the height of a middle point by which a wire passes
sets have autocorrelations and spectra as found in natural data

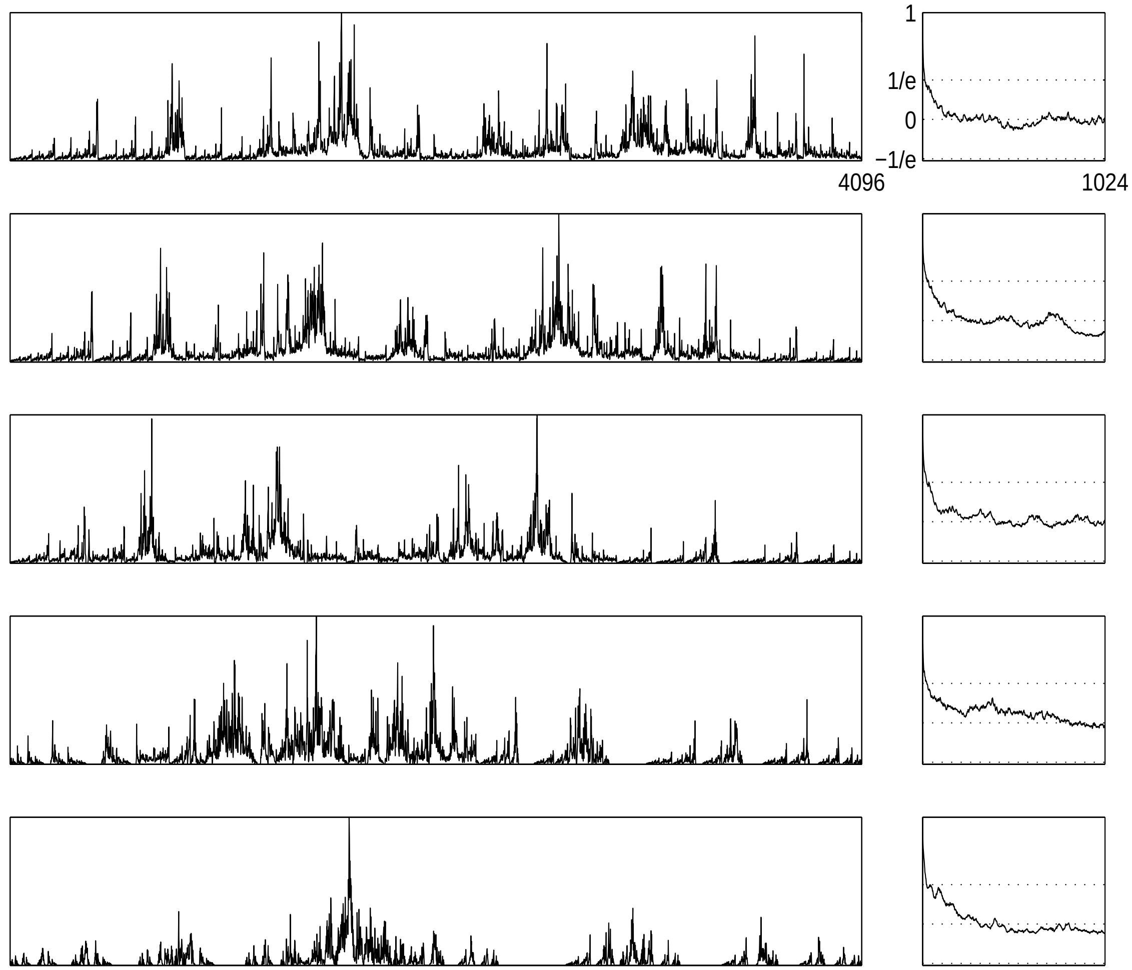
More sample shadows
(Puente, 2004)



sets with diverse shapes and statistics may be found
all sets are fully **characterized** via few “geometric” parameters

Yet more Platonic designs

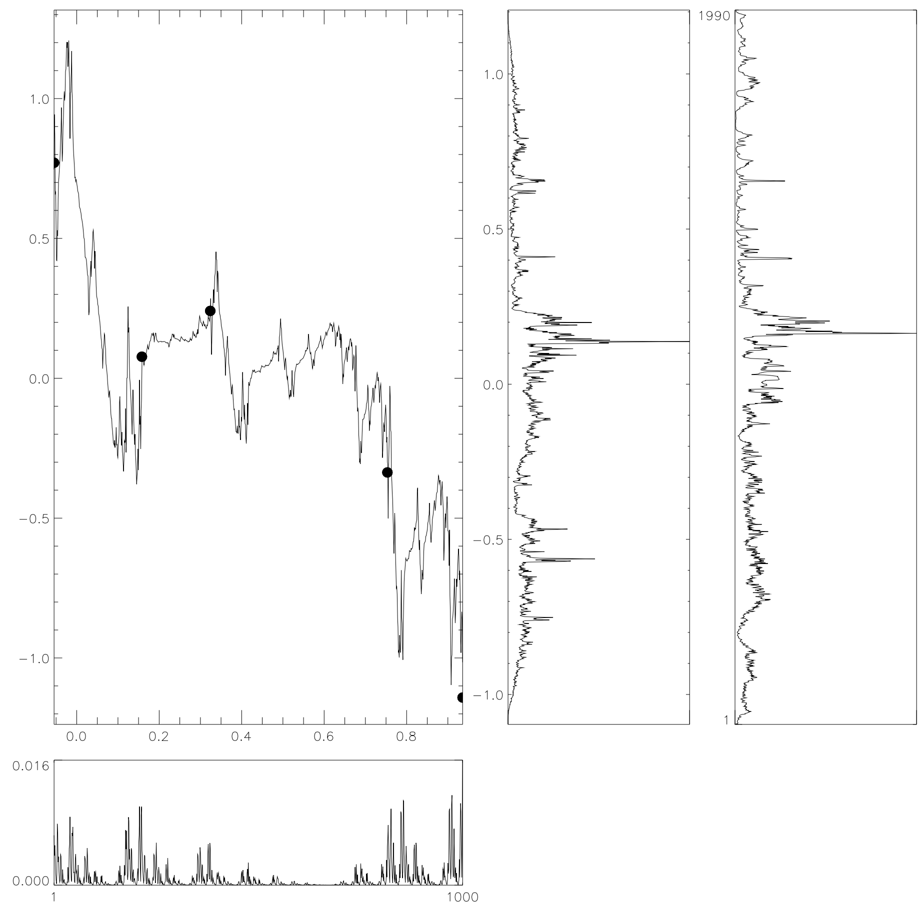
(Cortis et al., 2008)



adding a nonlinear **cosine** perturbation on y component...

A storm in Boston

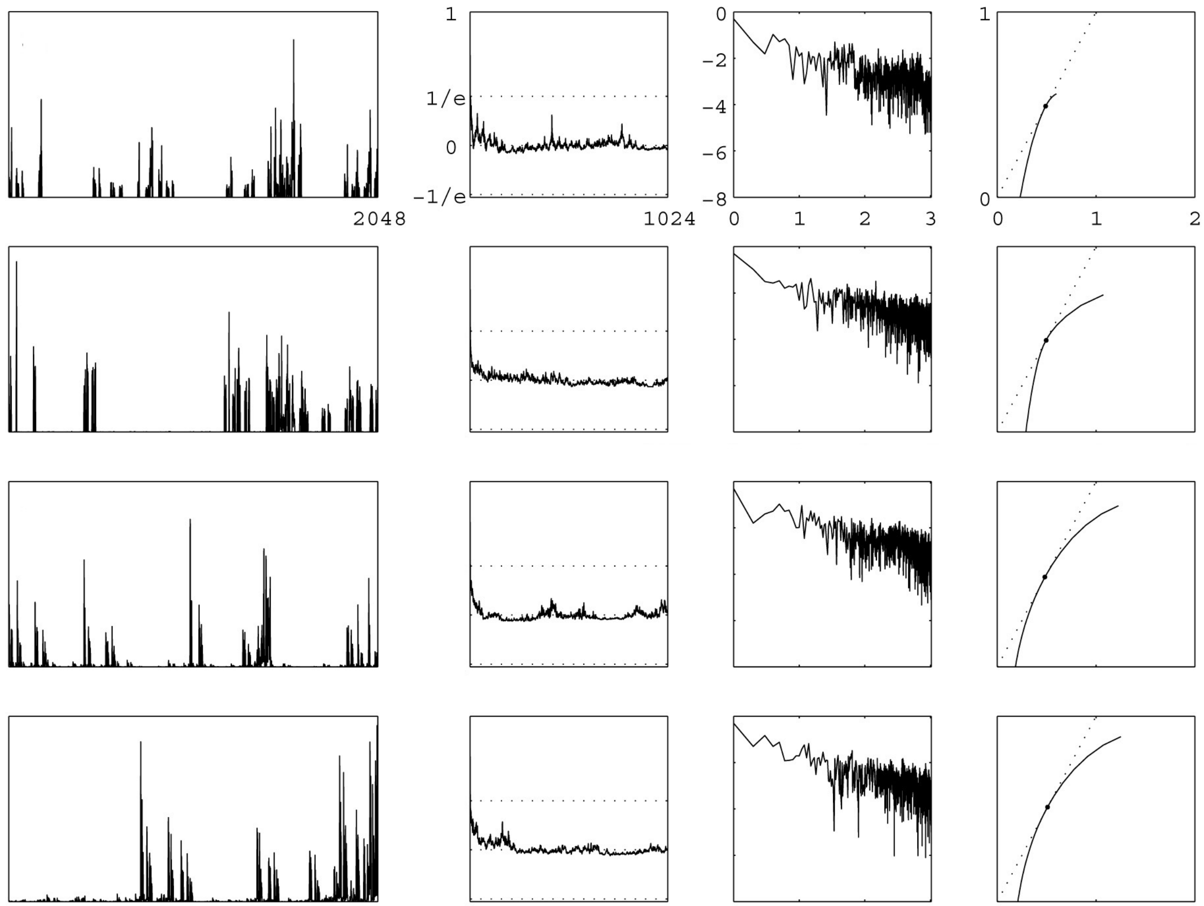
(Puente and Obregón, 1996)



iterating 4 maps maintaining data's moments and multifractality
geometric model also preserves spectra and chaotic nature of data

Real and simulated rainfall

(Puente and Cortis, 2009)

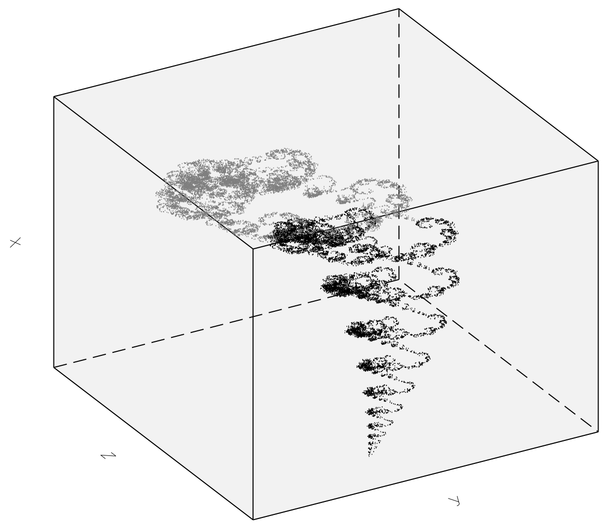


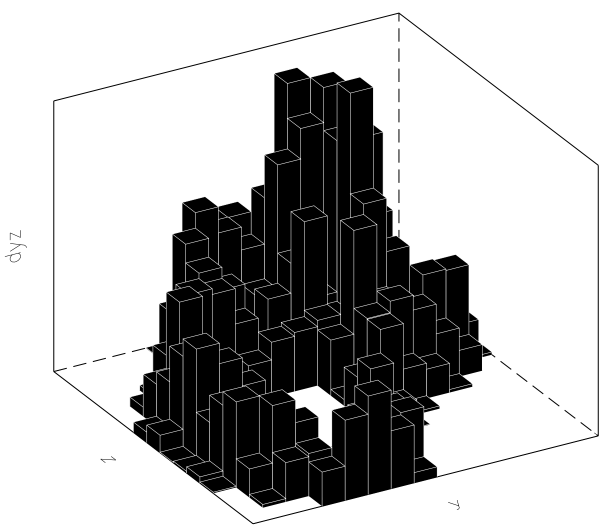
above: **real** data at La Honda, California

below: simulations via **Cantorian** dx's

Extensions to two dimensions: wires from 1D to 2D

(Puente, 1994)



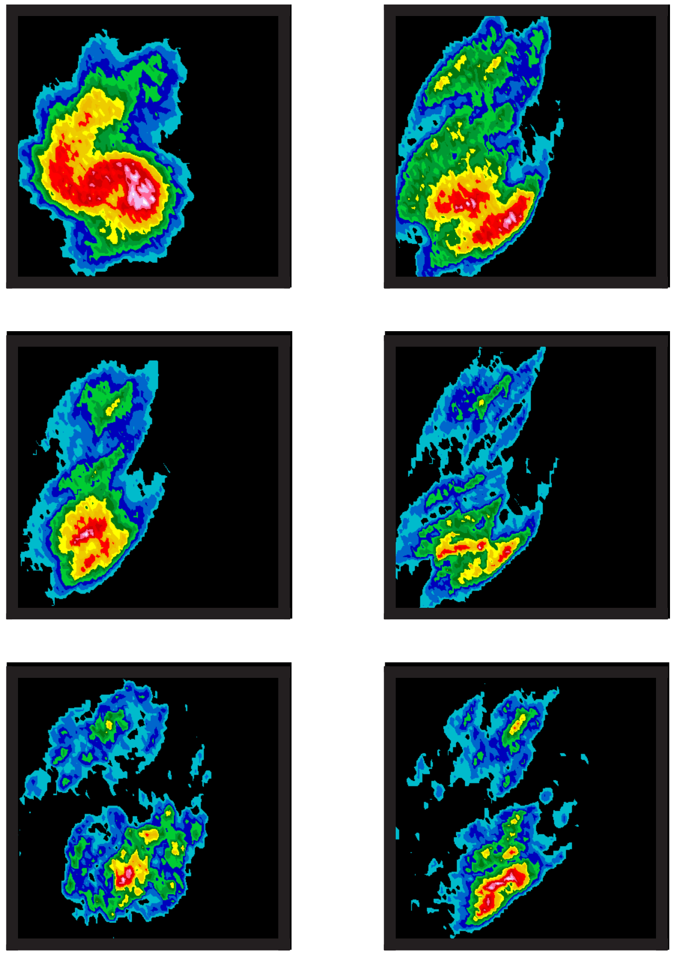


Mappings:

$$w_n \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_n & 0 & 0 \\ c_n & r_n^{(1)} cos\theta_n^{(1)} & -r_n^{(2)} sin\theta_n^{(2)} \\ k_n & r_n^{(1)} sin\theta_n^{(1)} & r_n^{(2)} cos\theta_n^{(2)} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} e_n \\ f_n \\ g_n \end{pmatrix}$$

more parameters, but similar...

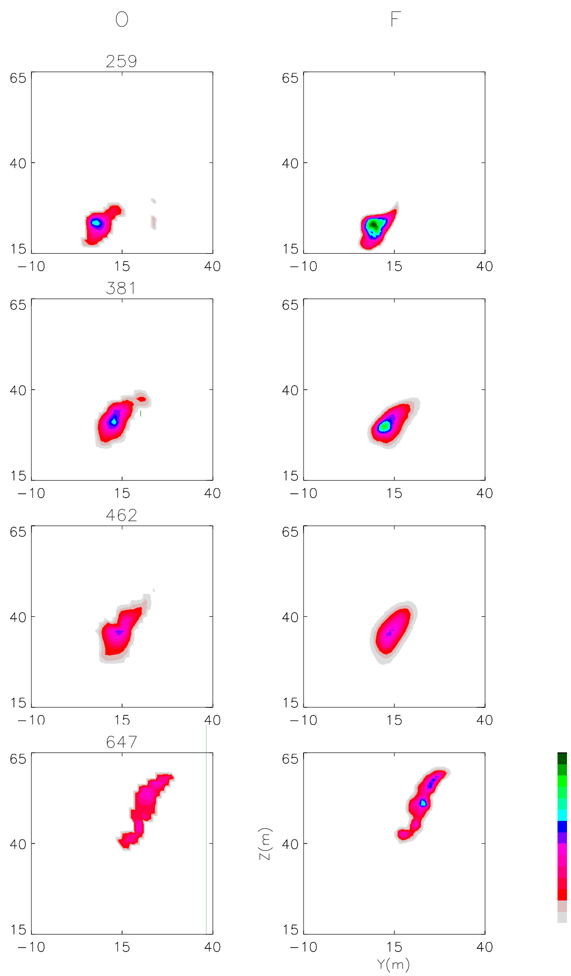
Deterministic precipitation patterns in space
(Puente, 2004)



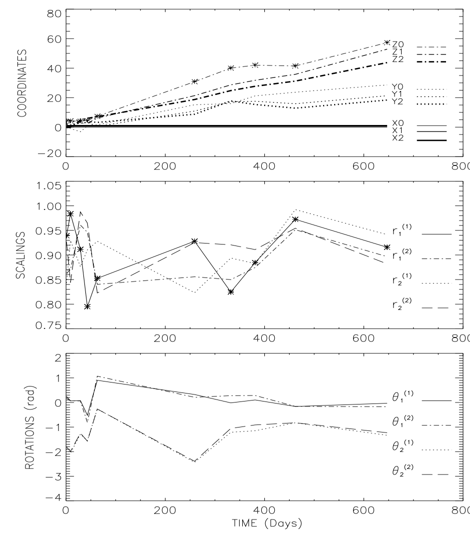
from single wires obtained iterating two linear maps

Pollution dynamics and predictions

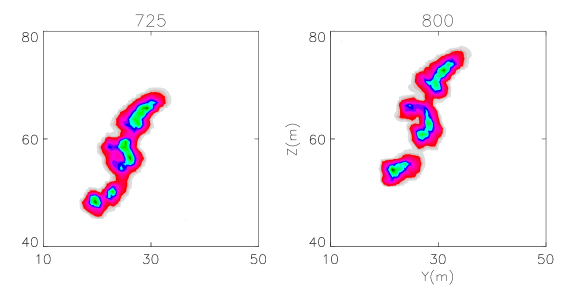
(Puente et al., 2001)



Parameters:



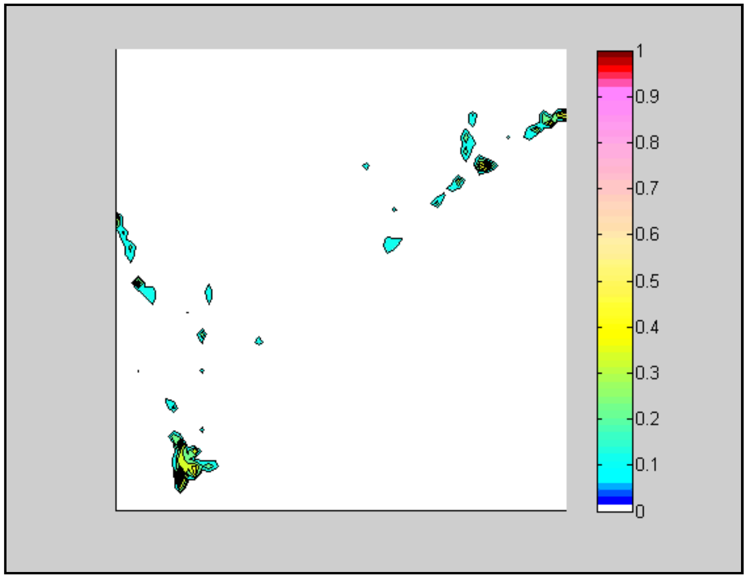
Predictions:



the geometry of pollution may be captured via successive wires

such allowed computing reasonable predictions from trends...

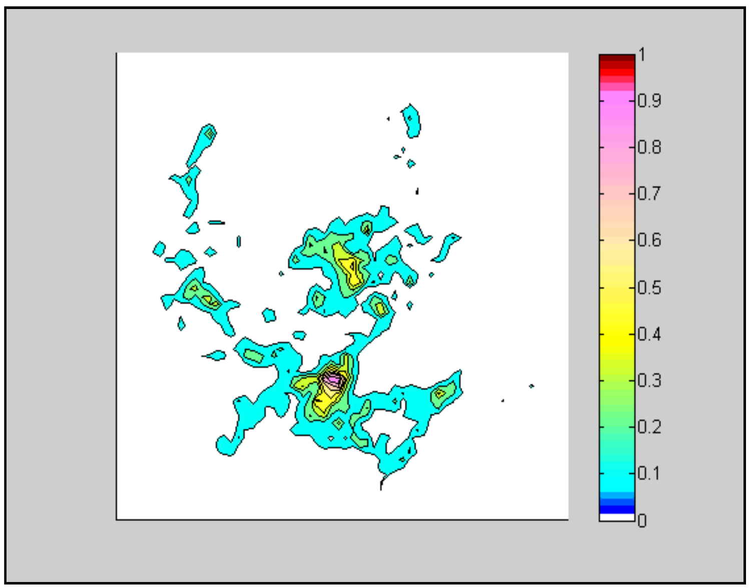
Simulations over two dimensions



From a **3D wire**:

scalings, 0.75 ± 0.05, -0.9 ± 0.08

angles, set ± 20



From a **4D surface**:

scalings, $\approx 0.59, 0.54 \pm 0.03$

angles, set ± 5

variable heights, weights

Inverse problem via modified particle sworm

(After Juan Luis Fernández Martínez)

Set	Model	$ob \leq 0.01$	$0.01 < ob \leq 0.02$	$0.02 < ob \leq 0.03$	$0.03 < ob \leq 0.04$	$ob > 0.04$
Boston	3p	5	52	29	10	4
	4p	10	85	4	1	0
	5p	13	63	9	3	12
Data 1	3p	16	21	54	7	2
	4p	10	71	13	5	1
	5p	14	76	7	1	3

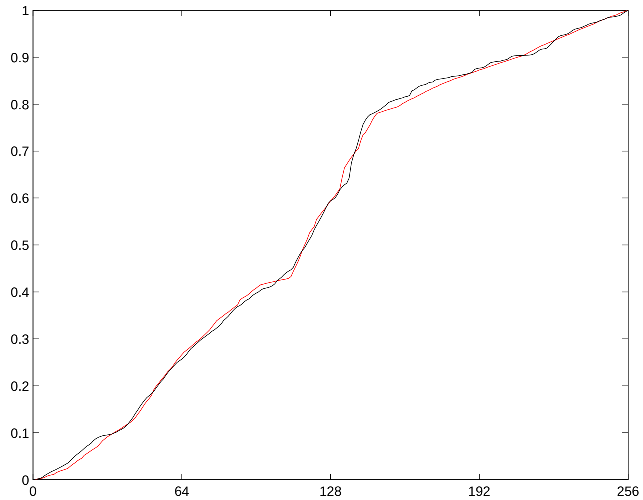
From 100 random initial conditions

Data 1 is synthetic via a 3p projection

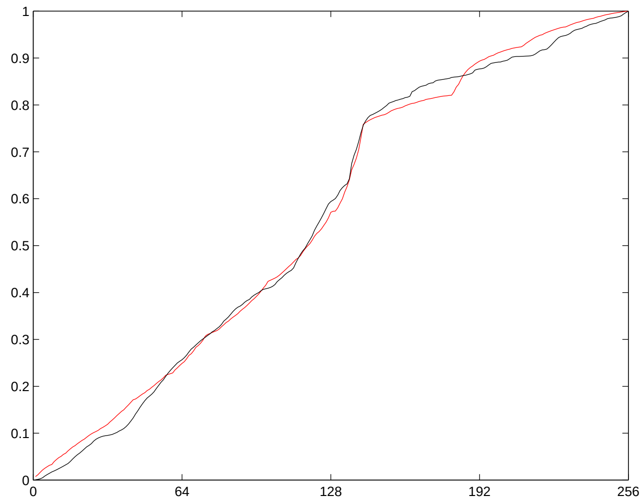
A storm in Boston revisited

Cumulative distributions

3p, 5 par, 0.0096



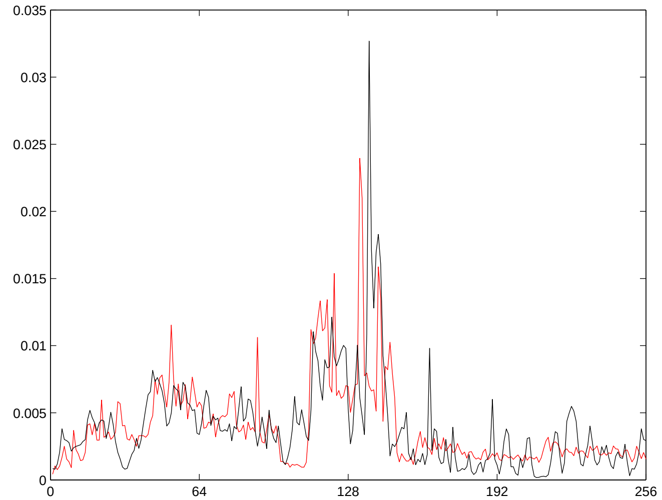
3p, 5 par, 0.0194



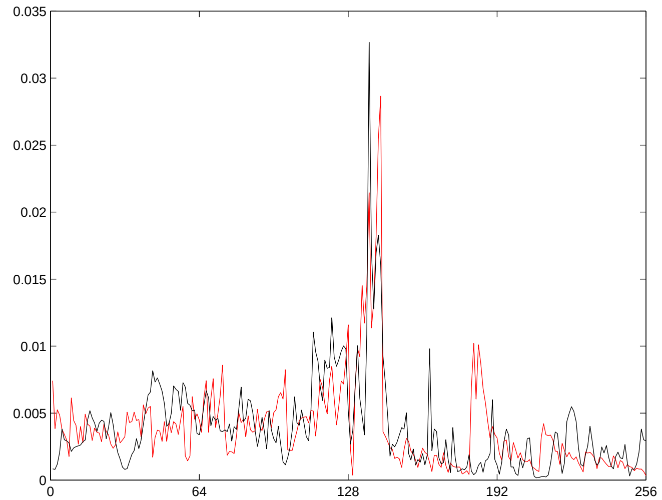
A storm in Boston revisited

Data sets

3p, 5 par, 0.0096



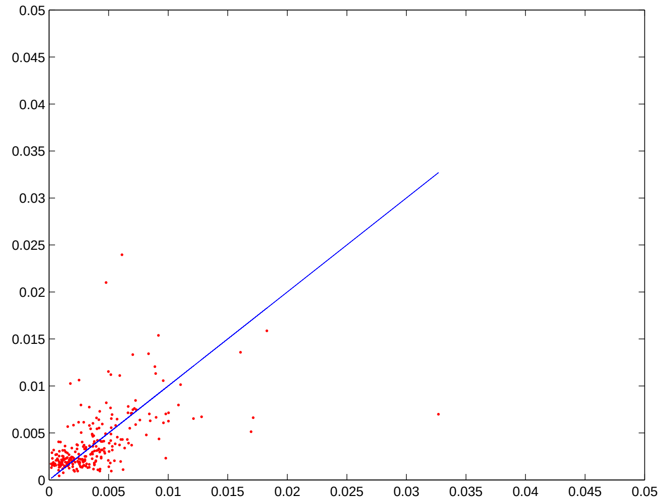
3p, 5 par, 0.0194



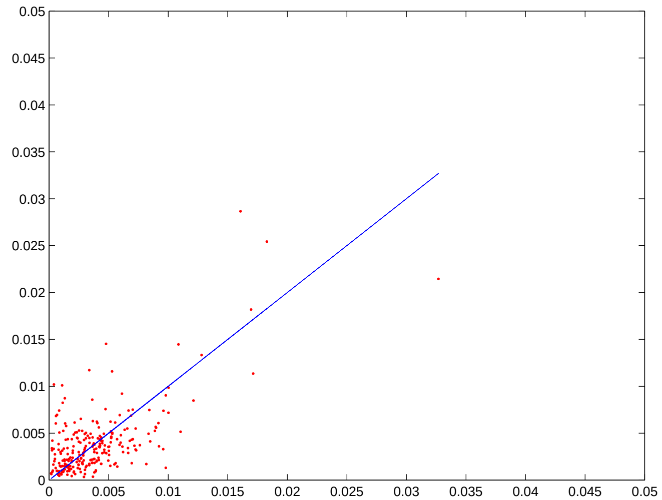
A storm in Boston revisited

Scatter

3p, 5 par, 0.0096



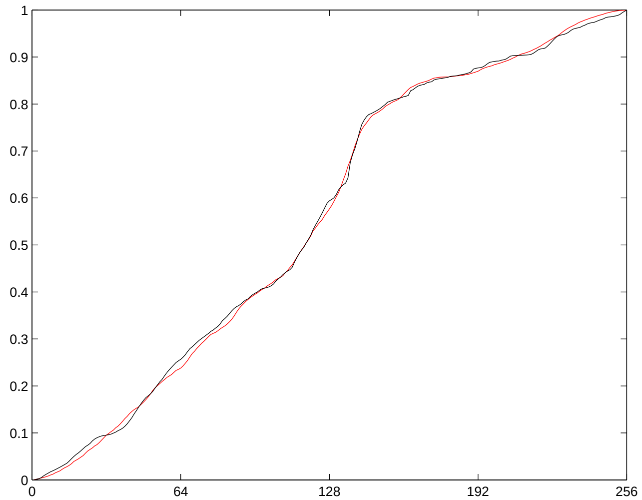
3p, 5 par, 0.0194



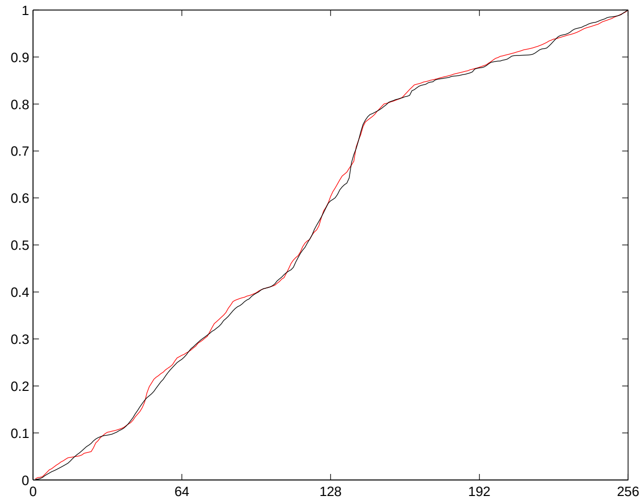
A storm in Boston revisited

Cumulative distributions

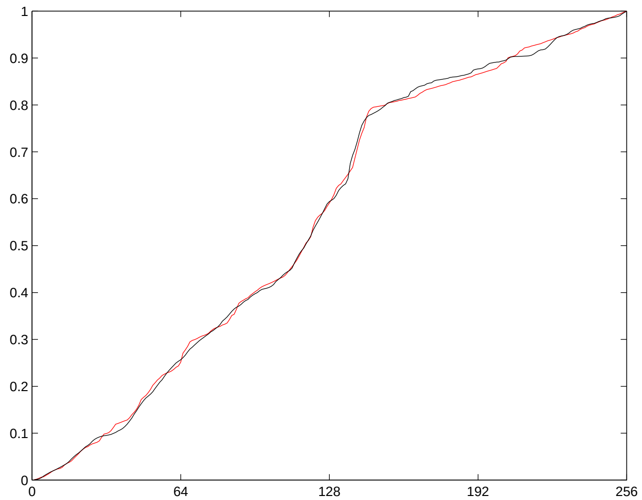
4p, 8 par, 0.0078



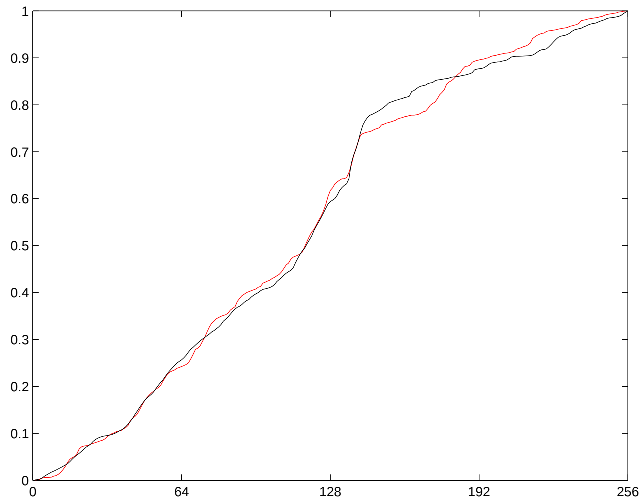
4p, 8 par, 0.0082



4p, 8 par, 0.0087



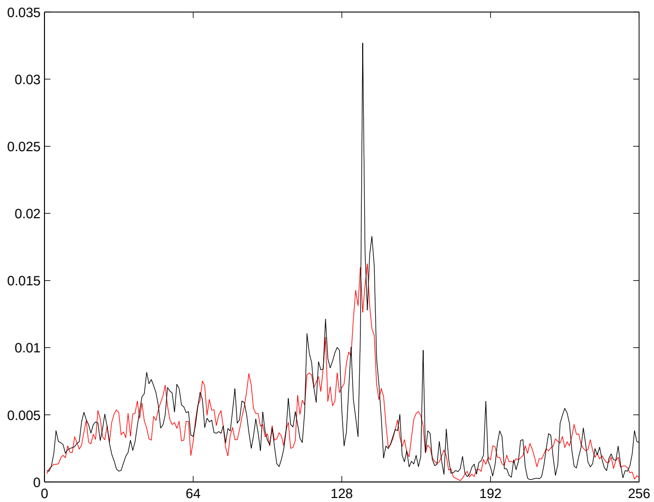
4p, 8 par, 0.0212



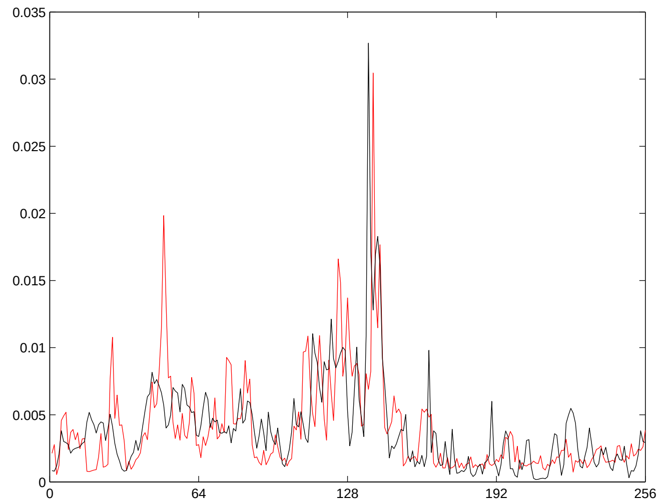
A storm in Boston revisited

Data sets

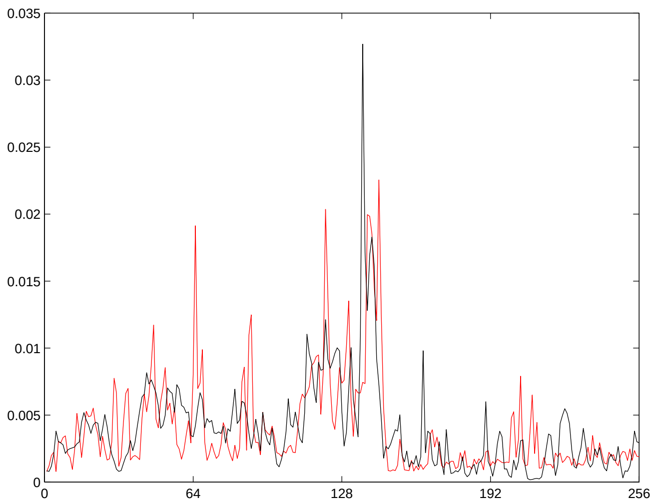
4p, 8 par, 0.0078



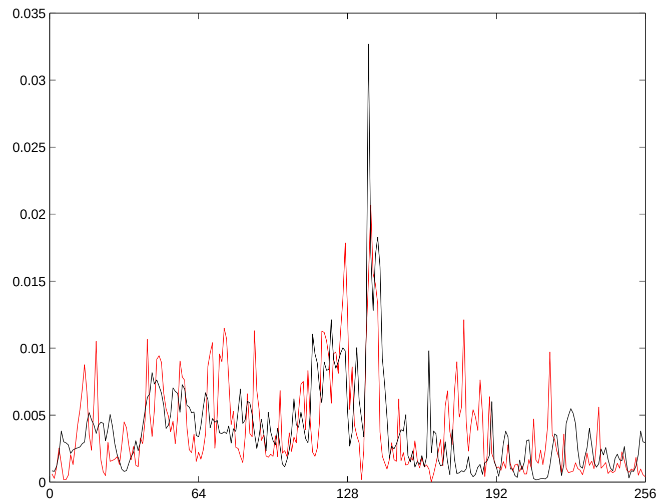
4p, 8 par, 0.0082



4p, 8 par, 0.0087



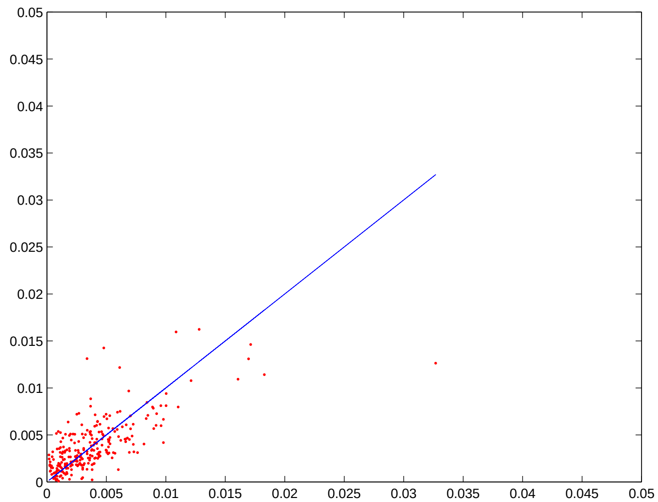
4p, 8 par, 0.0212



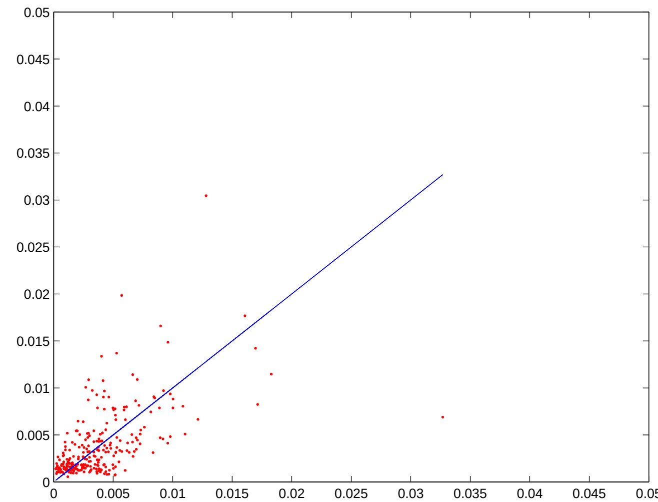
A storm in Boston revisited

Scatter

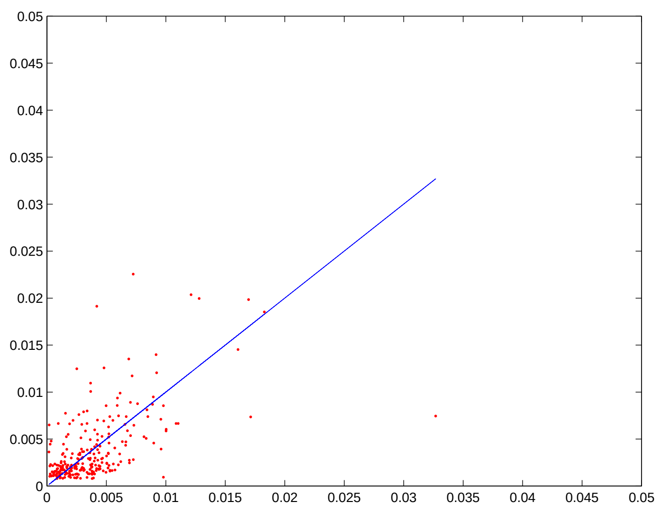
4p, 8 par, 0.0078



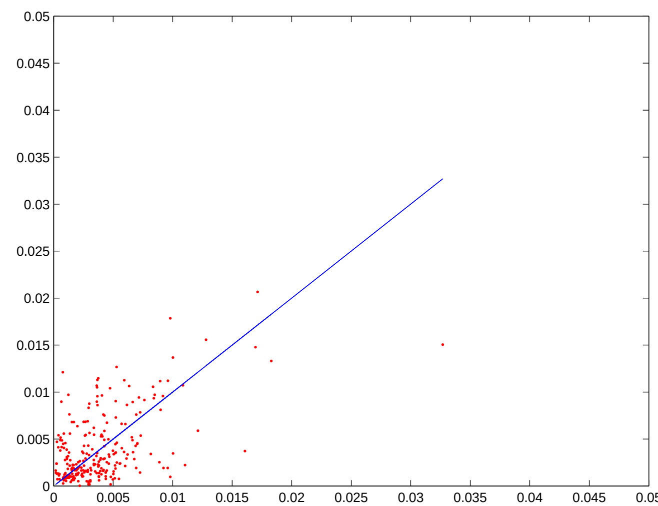
4p, 8 par, 0.0082



4p, 8 par, 0.0087



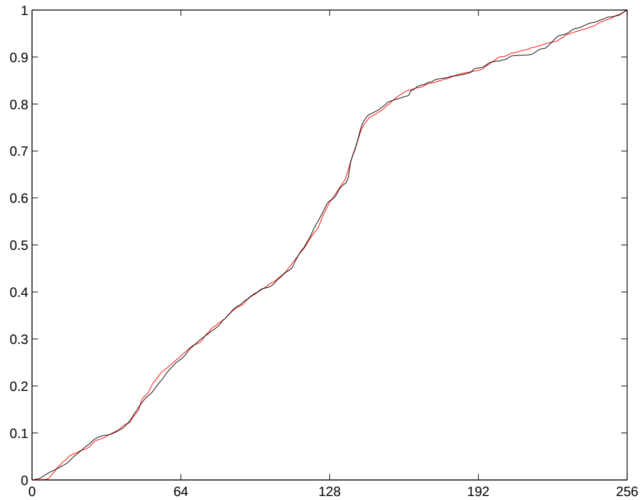
4p, 8 par, 0.0212



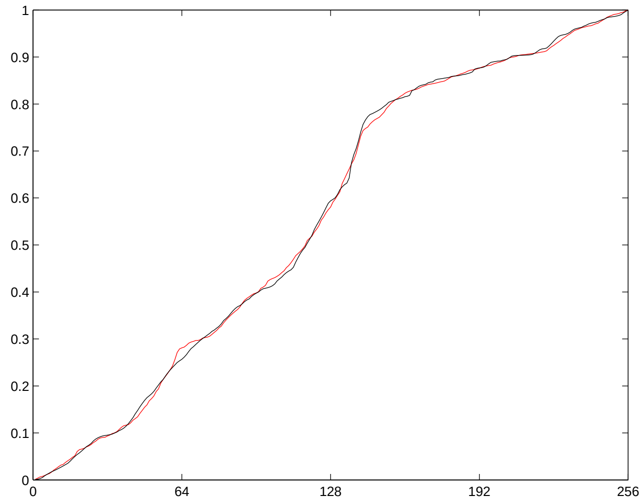
A storm in Boston revisited

Cumulative distributions

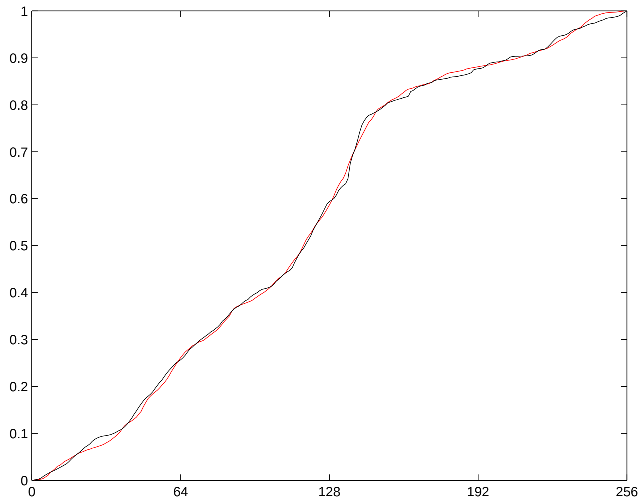
5p, 11 par, 0.0063



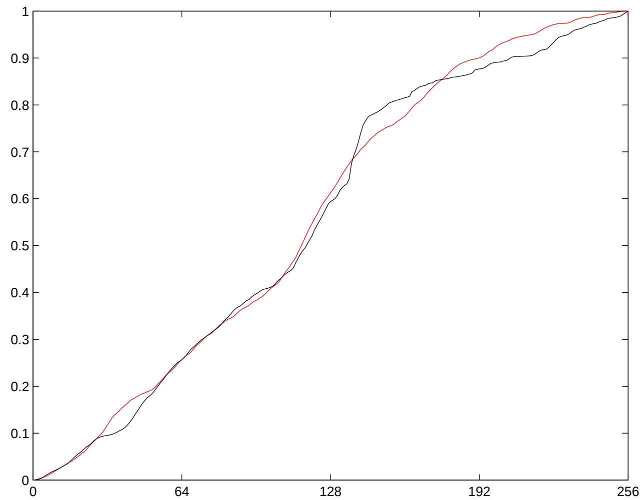
5p, 11 par, 0.0077



5p, 11 par, 0.0080



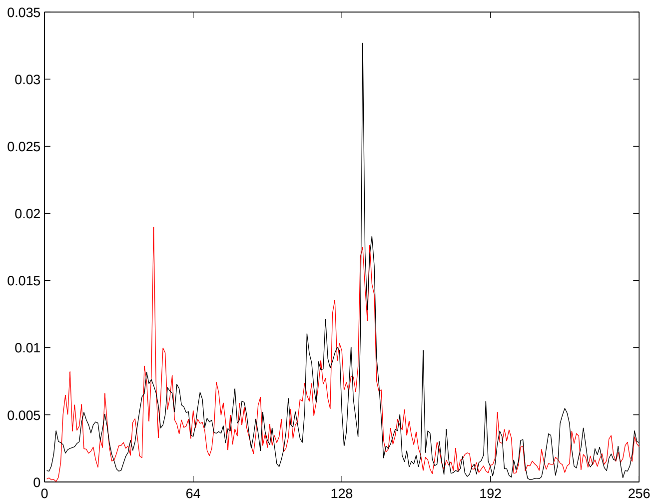
5p, 11 par, 0.0218



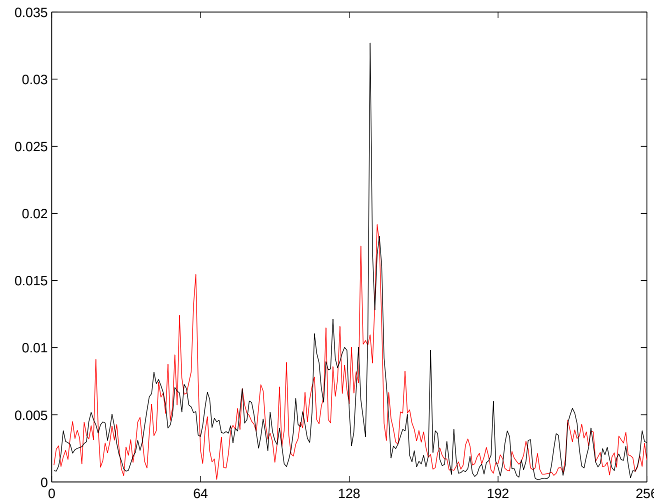
A storm in Boston revisited

Data sets

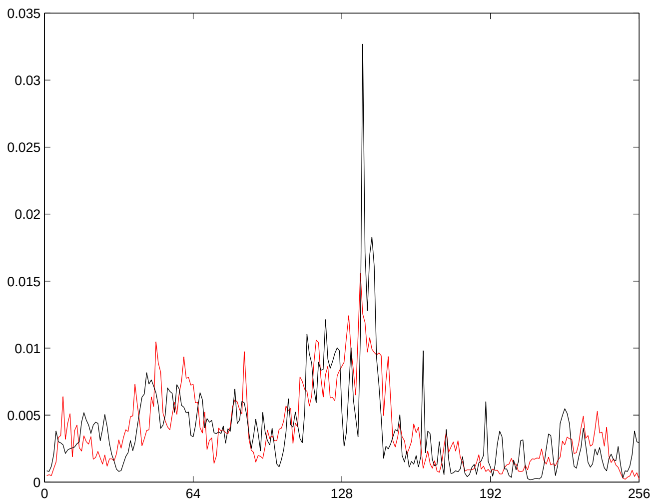
5p, 11 par, 0.0063



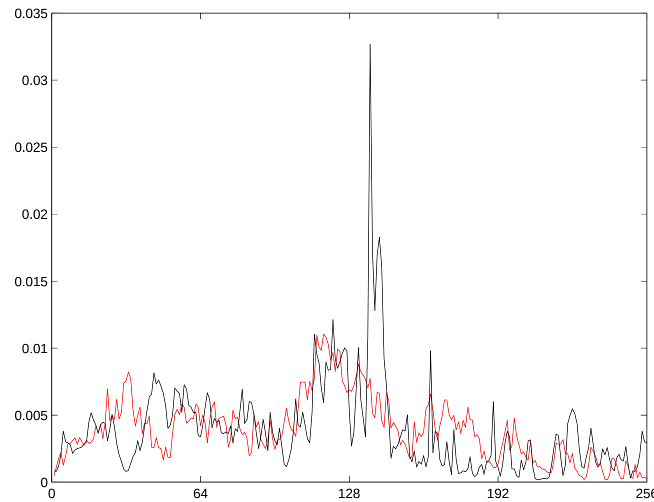
5p, 11 par, 0.0077



5p, 11 par, 0.0080



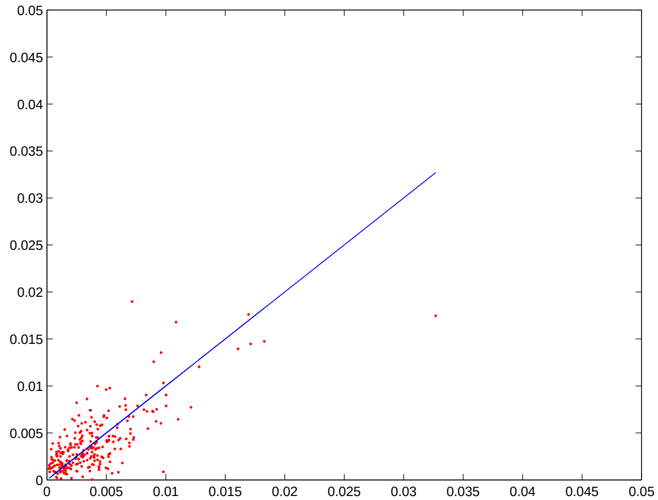
5p, 11 par, 0.0218



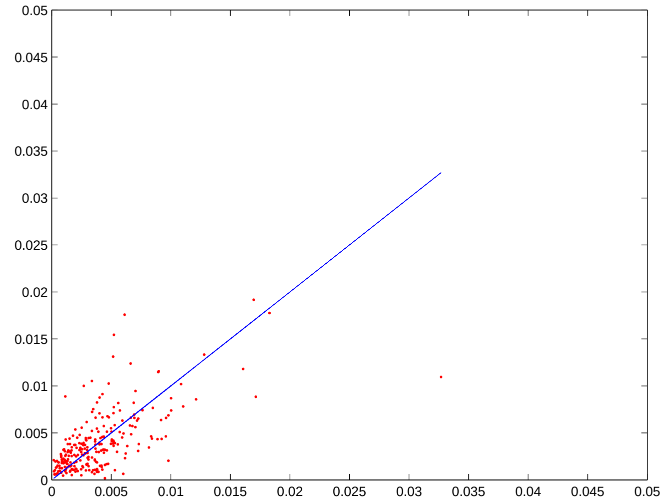
A storm in Boston revisited

Scatter

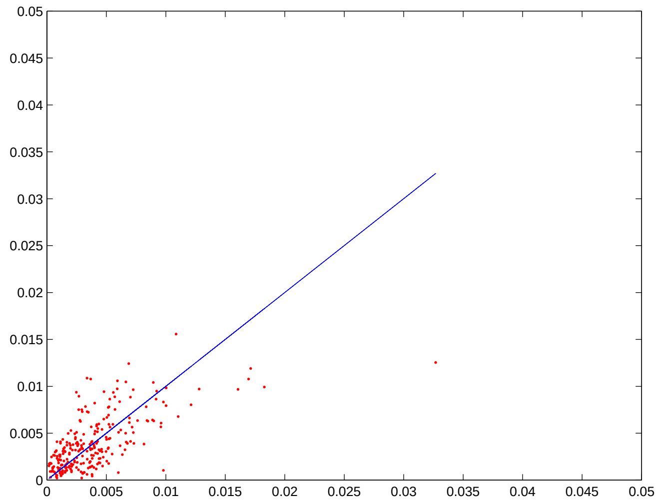
5p, 11 par, 0.0063



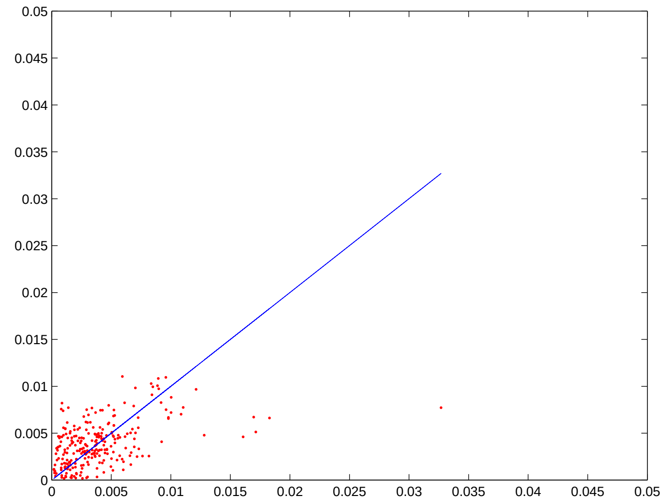
5p, 11 par, 0.0077



5p, 11 par, 0.0080



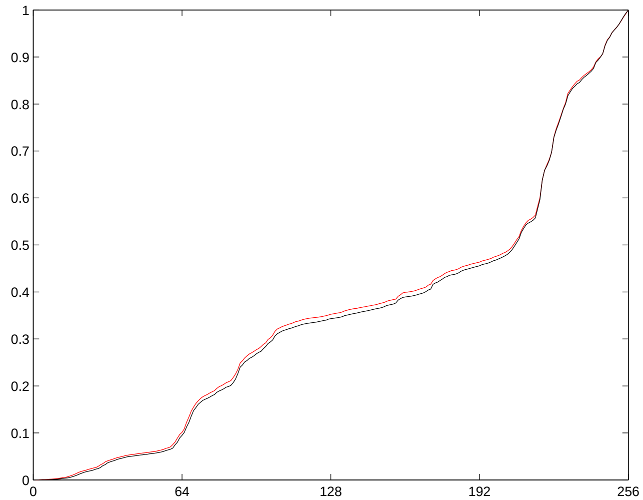
5p, 11 par, 0.0218



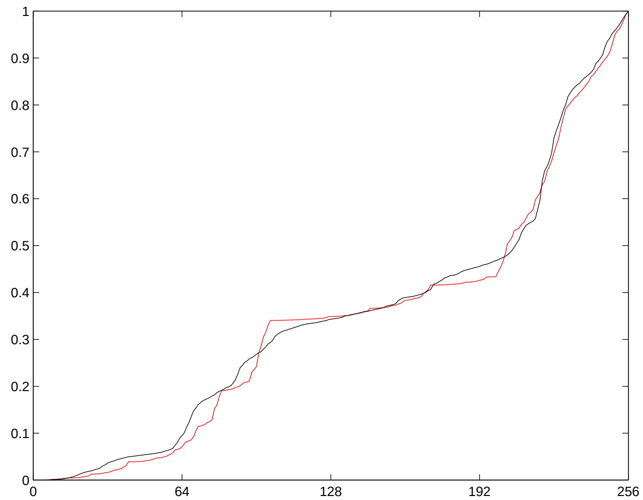
A Synthetic data set

Cumulative distributions

3p, 5 par, 0.0012



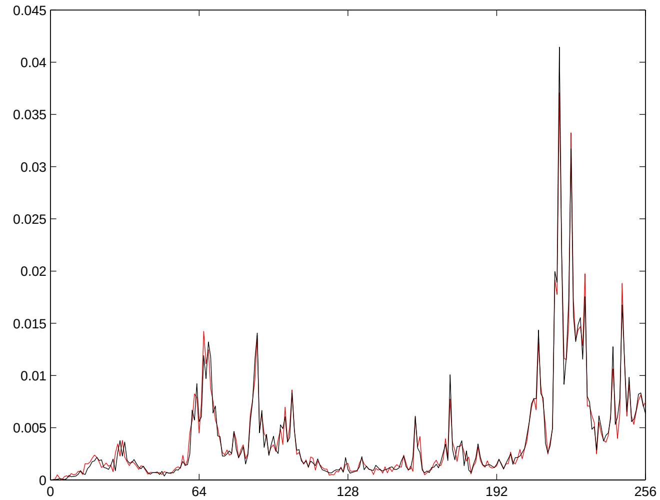
3p, 5 par, 0.0202



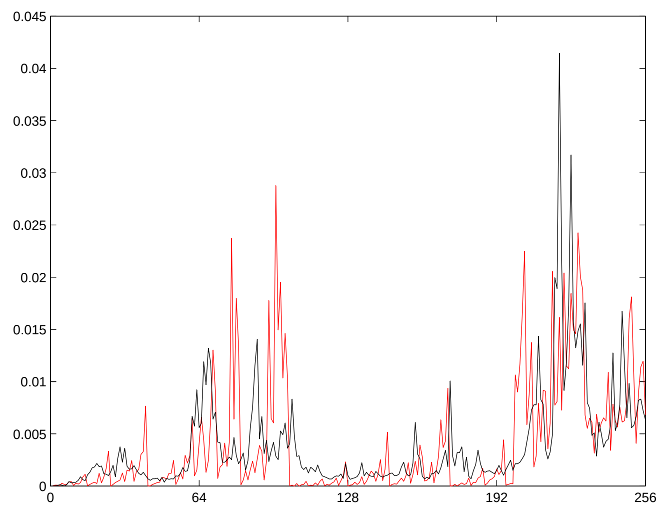
A synthetic data set

Data sets

3p, 5 par, 0.0012



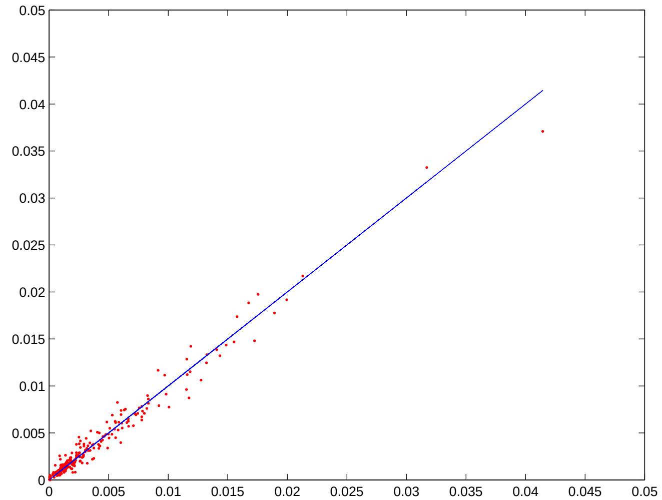
3p, 5 par, 0.0202



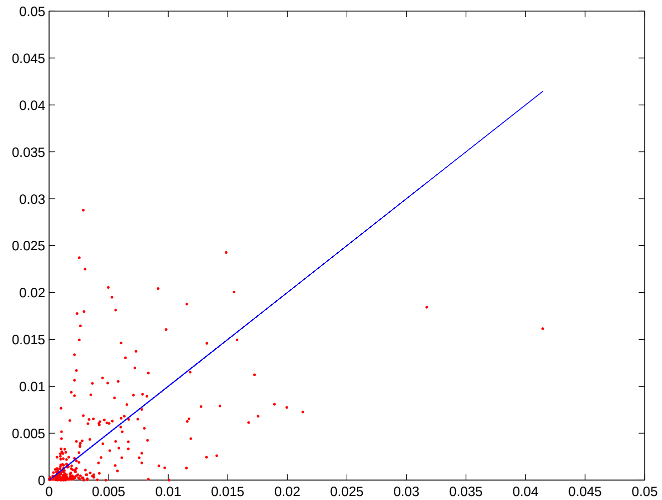
A synthetic data set

Scatter

3p, 5 par, 0.0012



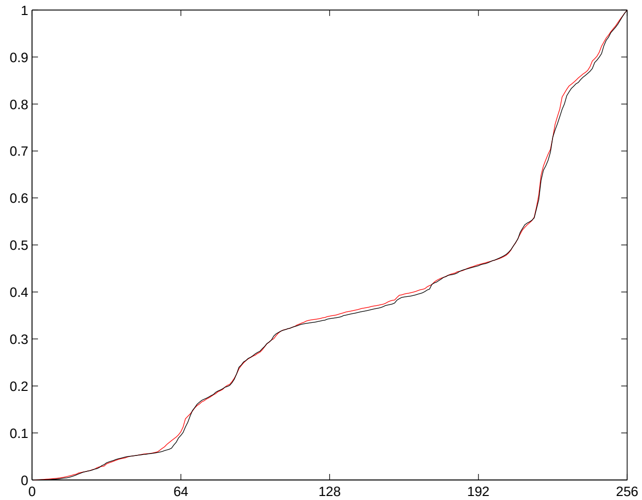
3p, 5 par, 0.0202



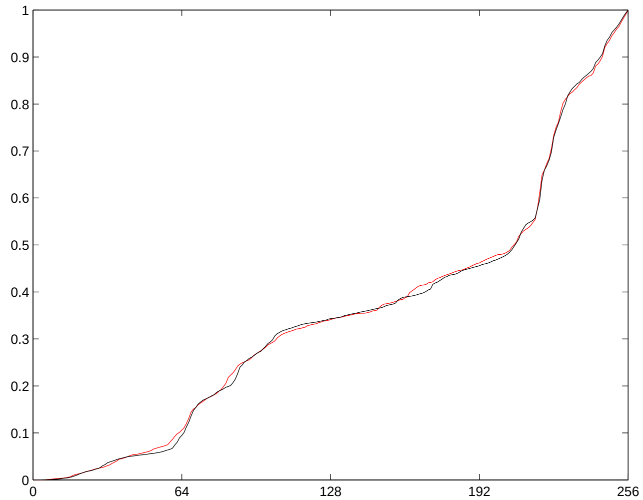
A synthetic data set

Cumulative Distributions

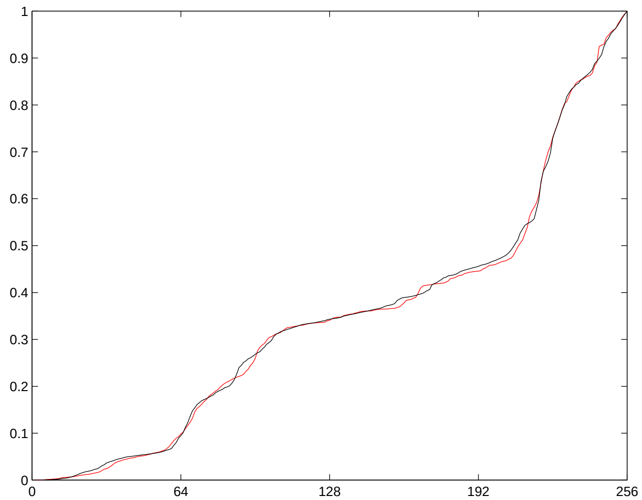
4p, 8 par, 0.0053



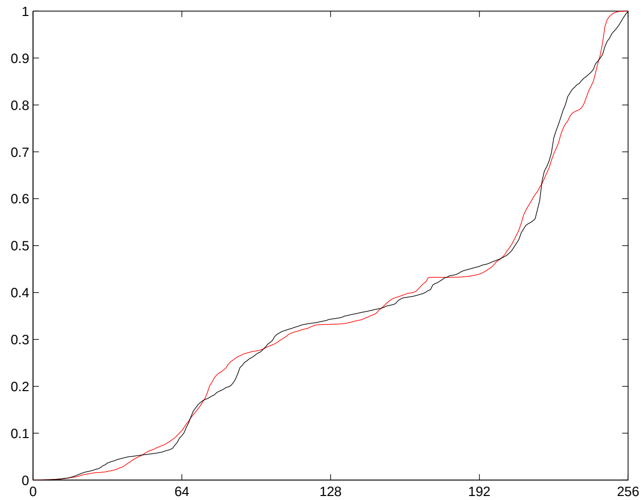
4p, 8 par, 0.0067



4p, 8 par, 0.0079



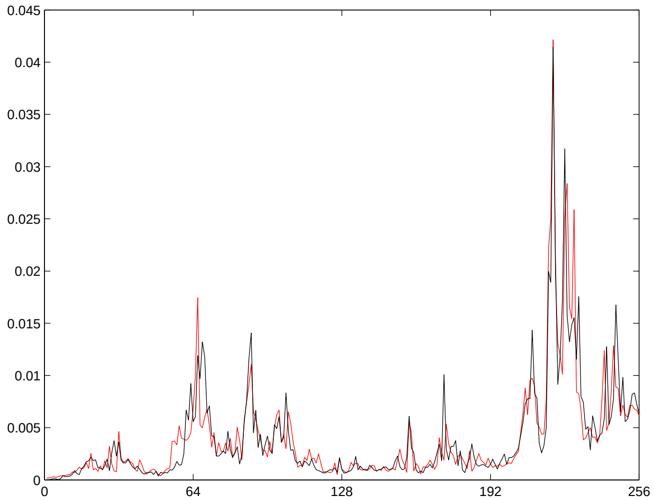
4p, 8 par, 0.0209



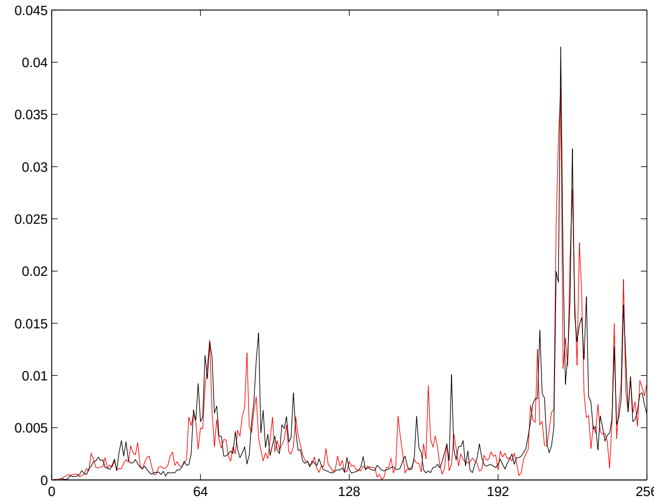
A synthetic data set

Data sets

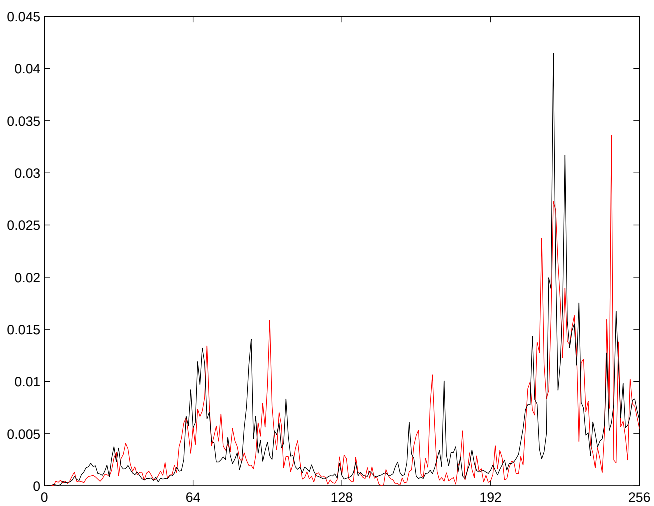
4p, 8 par, 0.0053



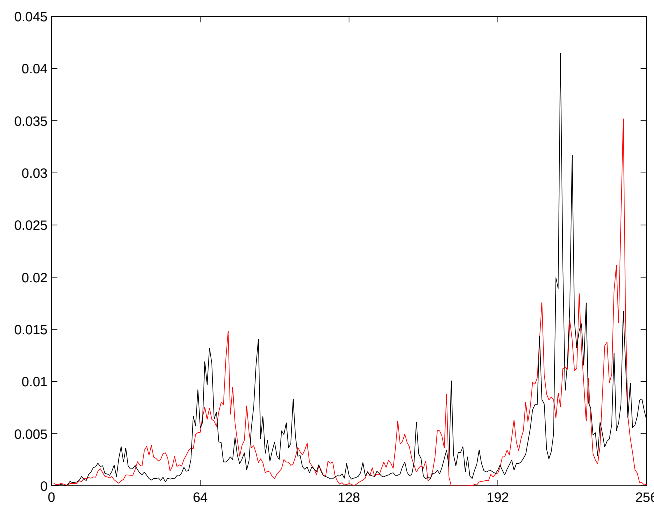
4p, 8 par, 0.0067



4p, 8 par, 0.0079



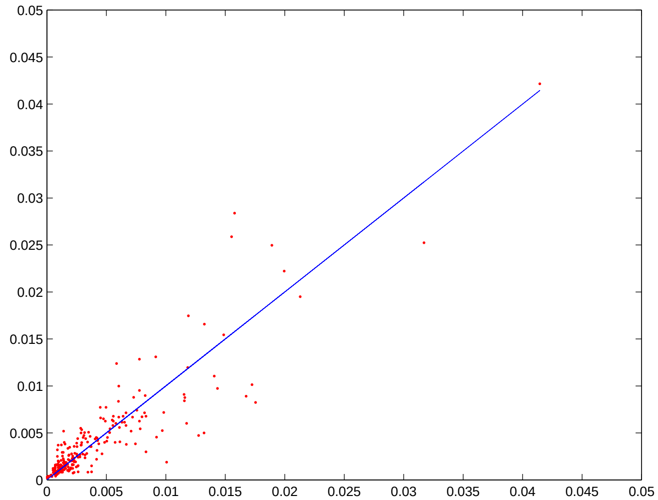
4p, 8 par, 0.009



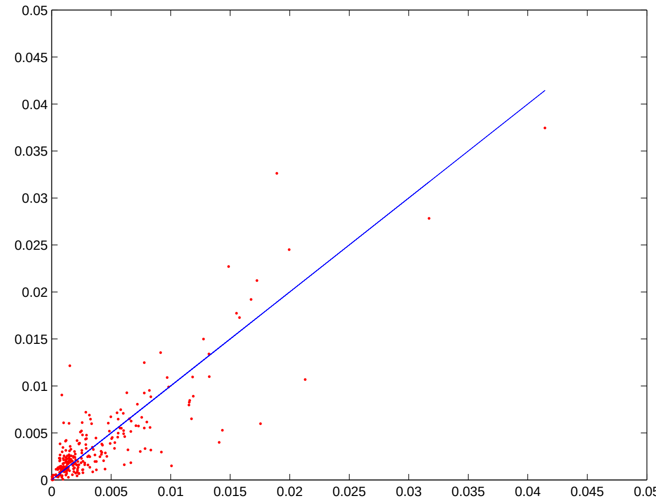
A synthetic data set

Scatter

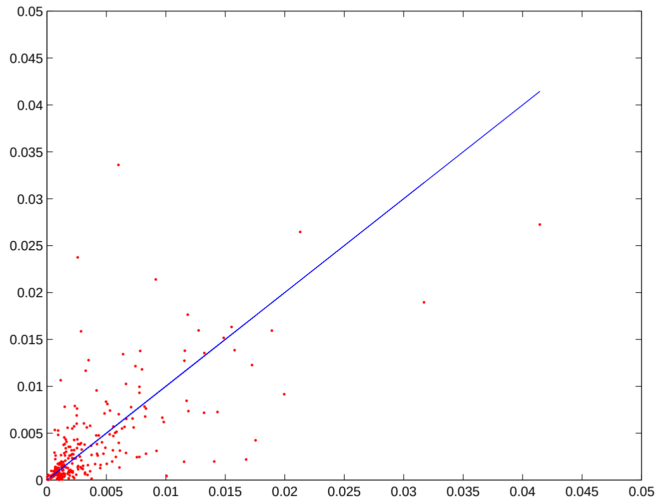
4p, 8 par, 0.0053



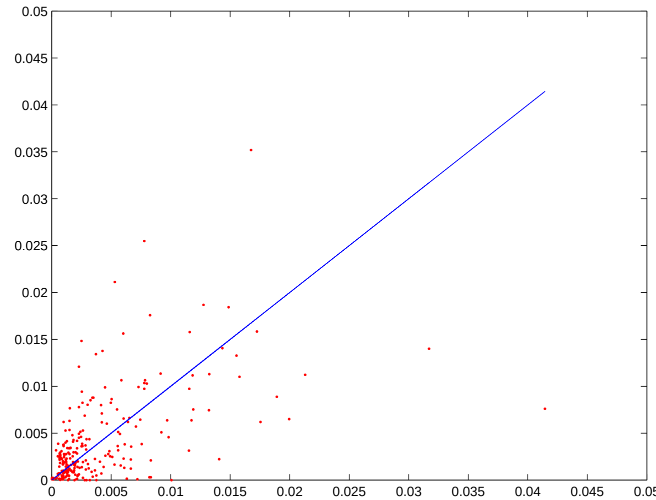
4p, 8 par, 0.0067



4p, 8 par, 0.0079



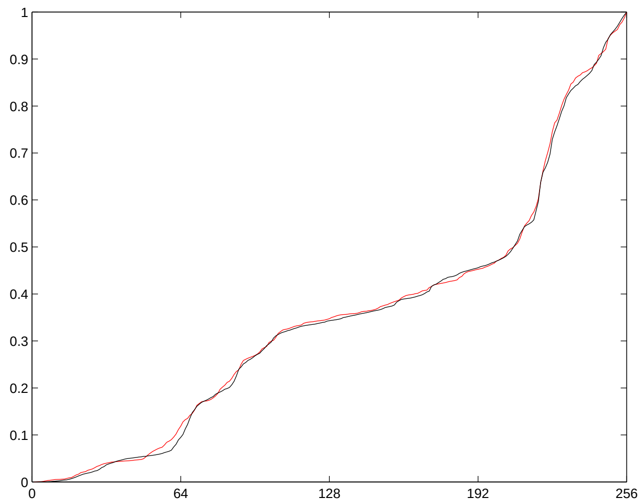
4p, 8 par, 0.0209



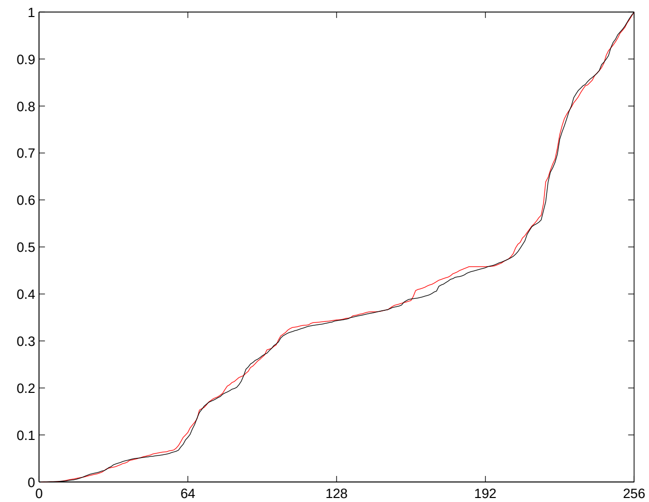
A synthetic data set

Cumulative Distributions

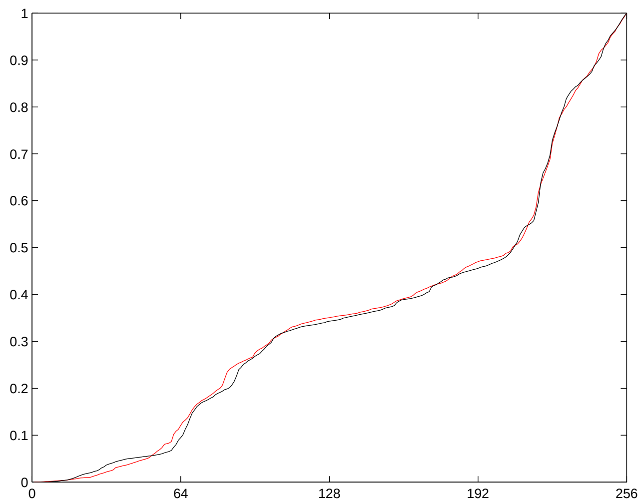
5p, 11 par, 0.0062



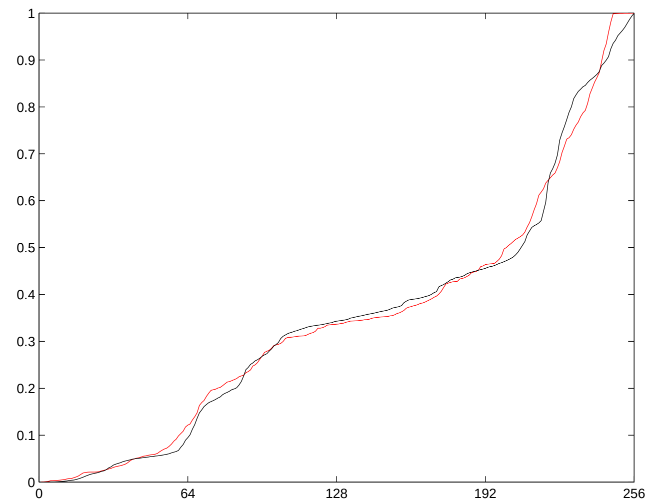
5p, 11 par, 0.0073



5p, 11 par, 0.0087



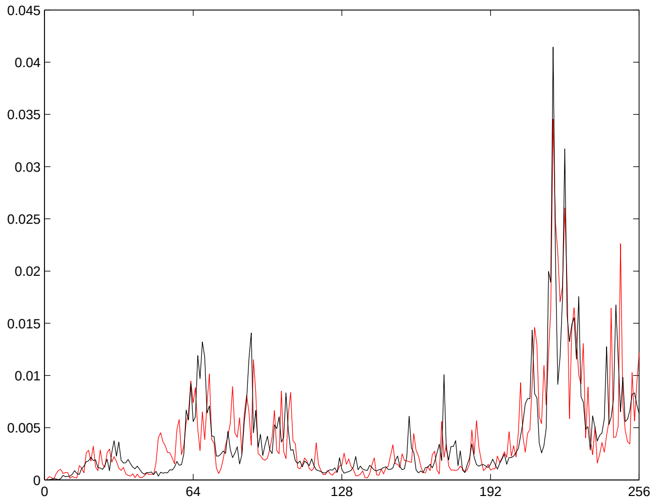
5p, 11 par, 0.0205



A synthetic data set

Data sets

5p, 11 par, 0.0062



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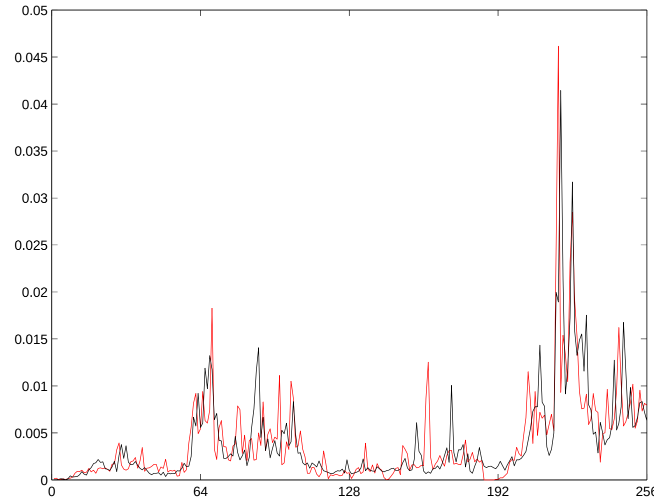

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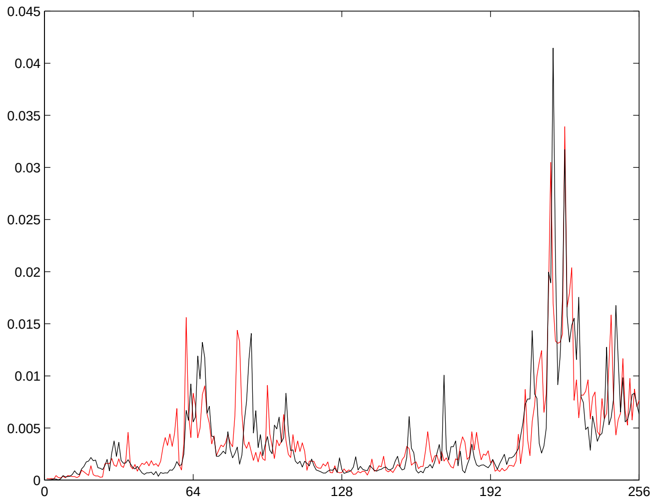

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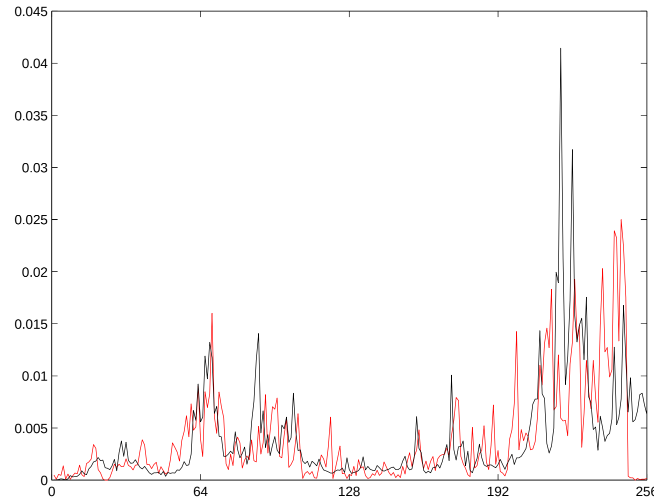
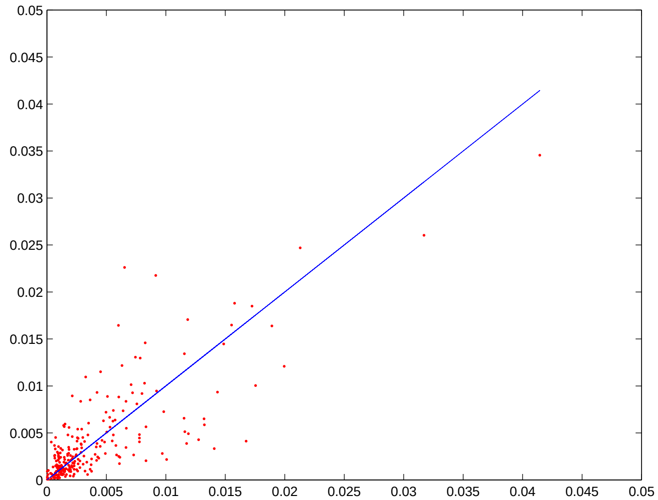


A synthetic data set

Scatter

5p, 11 par, 0.0062



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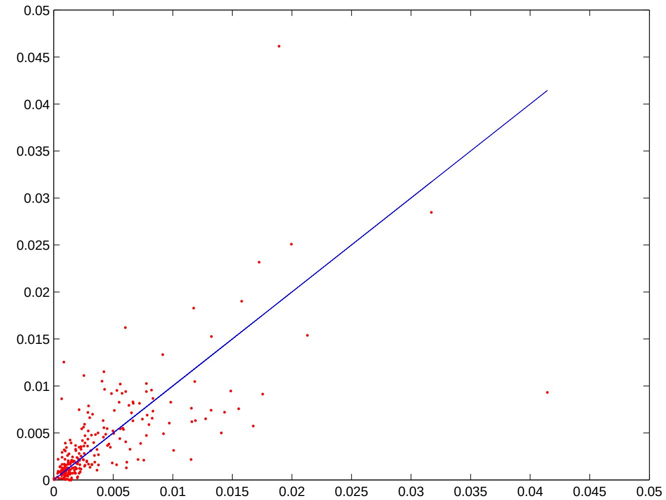

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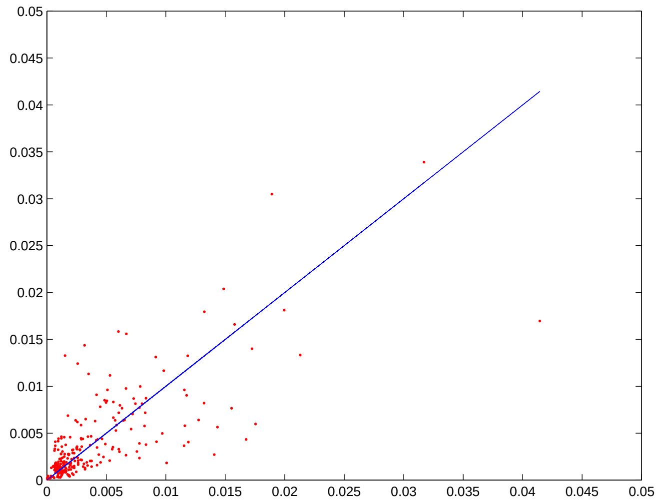

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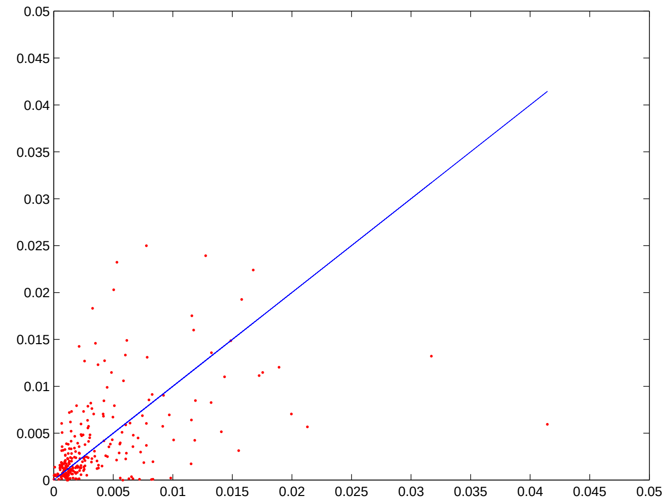

Conclusions

- The Platonic ideas may be used to simulate a host of rainfall patterns, both in time and in space.
- These geometric notions may ultimately provide a parsimonious deterministic "language" of natural complexity.
- The inverse problem for a given set is, for us, difficult to solve.
- But if the inverse problem is solved, the ideas may perhaps provide new vistas to understand rainfall dynamics.