# From Dissipation to Conduction and from Darkness to Light

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**Abstract**

Mathematical transformations capable of transmuting spiky distributions, including those produced by natural turbulence, into smooth Gaussian densities, associated with calmness via Fourier law, are introduced. It is shown that such functions may be built in rather simple ways and that their graphs fill-up two-dimensional space in such a way that any small portion of such transformations yields, universally, Gaussian bells. It is explained how these limiting space-filling transformations represent unexpected remedies for the prescribed dissipation of turbulence into a desired condition of serenity based on heat conduction, and how one, based on ever-positive additions, defines a veritable antidote to disorder that ends up exchanging darkness into precious light, as its associated bell concentrates at infinity, inspiring us to exclaim "Where is o death your victory?, Where is o death your sting?" (1 Cor 15:55). Surprisingly, it is also elucidated how the ever-positive case helps us visualize key attributes of the Most Holy Trinity and how such a scientific construct invite us to find order, peace and love, but only in the artful and spiritual limit, in which the curious story of St. Augustine and the child at the beach are harmonized.

## 1. The Iteration of Simple Maps

The notions in this article spring forth from the iteration of two maps, defined from the plane to the plane, namely: $w_1(x,y) = \left(\frac{x}{2}, x + d_1 \cdot y\right)$ and $w_2(x,y) = \left(\frac{x}{2} + \frac{1}{2}, 1 - x + d_2 \cdot y\right)$.

These simple equations may be described as follows. First, the $x$ coordinates are decoupled from the $y$ values. While $w_1$ just divides the input value by 2, $w_2$ divides by 2 and then adds to it one half. Second, the $y$ components, for both maps, are linear combinations of the input $x$ and $y$ values. While a parameter $d_1$ multiplies the $y$ value on the first map, a parameter $d_2$ does it on the second map.

Simple calculations show that while $w_1$ takes the point $(0,0)$ into $(0,0)$ and $w_2$ $(1,0)$ into $(1,0)$, $w_1$ takes the point $(1,0)$ into $(1/2,1)$ and $w_2$ $(0,0)$ also into $(1/2,1)$. This means that iterating the maps includes the three points $\{(0,0),(1/2,1),(1,0)\}$ irrespective of $d_1$ and $d_2$ and that $w_1$ operates to the left and $w_2$ to the right of the domain, which goes from 0 to 1, (Barnsley, 1988).

To illustrate what is produced by the iterations, select $d_1 = -d_2 = 0.5$. Then, start the process at the middle point $(1/2,1)$, flipping sequentially a coin to decide which map to use in succession, say $w_1$ if heads and $w_2$ if tails. As illustrated in Figure 1, and as if by magic, the bombardment of dots so produced arranges into a "wire," a function from $x$ into $y$, in this case a "mountain" profile, and such occurs irrespective of chance (the actual coin tosses) and of the type of coin used, either fair or biased.

It happens that the process of iterations also generates unique textures over an ultimate wire, which reflect how such a set (known as the attractor of the maps) is filled up dot by dot. As seen in Figure 1, such textures are either a uniform (left) or a spiky histogram (right), corresponding to the usage of a fair or a biased coin.

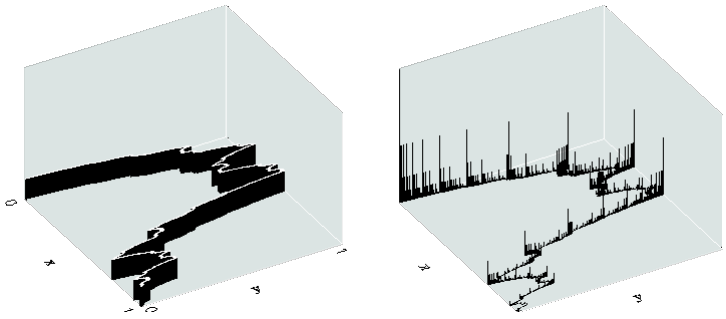

Figure 1. Outcomes of the iterations of the simple maps for $d_1 = -d_2 = 0.5$ and respective histograms of successive points. Usage of a fair coin (left) and a biased $70-30$ coin (right).

**2. A Universe of Shadows**

Realizing that the iterations generate not only a wire but also a unique texture, it becomes relevant to ask how such texture looks when seen from the $x$ and $y$ axes. In this spirit, Figure 2 shows what is found when the spiky object over the mountain profile, $f$, is drawn together with their "shadows" named $dx$ and $dy$.

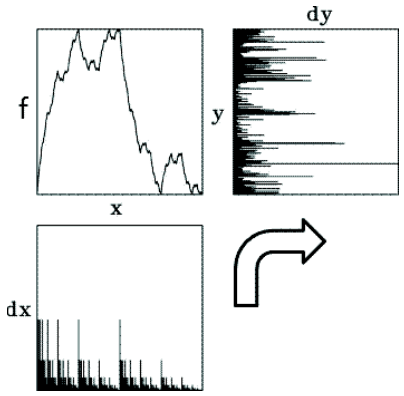


Figure 2. Unwinding the spiky profile in Figure 1 (right).

Clearly, the graph $dx$, over $x$, is just the object in Figure 1 (right) seen from such an axis. The spikes come from the iterations, as follows. At first, once the first coin toss happens, the left half is more likely given the proportion 70-30 on $w_1$ and $w_2$ and this yields a texture made of two uniform sides having distinct heights. Then, as the same coin is tossed again (independently from the first toss), the texture splits into four parts following the same proportion, generating successively a so-called multiplicative cascade that eventually yields the spiky $dx$ biased towards the left. As hinted, such an object ultimately contains infinitely many infinite "thorns" arranged by layers, spikes that are so sparse (on a given layer) that they are supported by fractal dust.

It happens that these very ideas have been found useful in describing the way turbulence happens in the air. When the inertia subjugates the fluid's cohesion, as indicated by the Reynolds number (the ratio of the velocity times a characteristic length divided by the fluid's viscosity), the air breaks into an irreversible chain (cascade) of inwardly rotating eddies that progressively divide into eddies, which leads to an arrangement of energies that conforms, by layers, precisely to the 70-30 ratio. Strikingly, observations of turbulence along a line are universally consistent with a permutation of the spiky object $dx$, leading to thorns of energy that eventually dissipate in the form of heat (Meneveau and Sreenivasan, 1987).

Curiously, but not surprisingly, this process—driven by the selfish and devilish inward (negative) spirals of the power of the air (Eph 2:2)—and yielding an object so fractured that it is well-named a multifractal, is also useful for modeling the genesis of wealth inequalities of nations, and in particular the one in the United States (Puente, 2011a).

While $dx$ is a blueprint of turbulence, the rather complex-looking $dy$, found adding $dx$'s for all values of $x$ for which $y = f(x)$, may be understood in Platonic and physical ways, as follows. If we think of the wire as a system happening from $x$ into $y$, its output $dy$ may be thought of as the "shadow" cast by the wire when "illuminated" by the input $dx$, in a manner that resembles Plato's notion in his famous allegory of the caveman. As the united mountain profile is a function from $x$ to $y$, a derived texture $dy$ may also be thought of as a physical transformation of turbulence, one that rearranges the energies in the original eddies.

This construction represents a novel vision to approach natural complexity (Puente, 1996), as a seemingly-random $dy$ is, in reality, an entirely deterministic set encoded by few parameters. The notions do represent a romantic Platonic idea in this day and age, that is, in the twenty-first century and after quantum mechanics, but we have been playing with these shadows (extending the ideas to more than three initial points and using more general attractors not shaped as wires) and we can indeed generate interesting types of complexity without invoking chance.

**3. Unexpected Limiting Cases**

These geometric notions have certainly sustained my scientific career, but there is even more, as the limiting cases provide interesting and surprising constructs. For a wire having as parameters $d_1 = -d_2 = z$, tending to 1, that is, the so-called +, - case, Figure 3 (left) shows the surprise that follows.

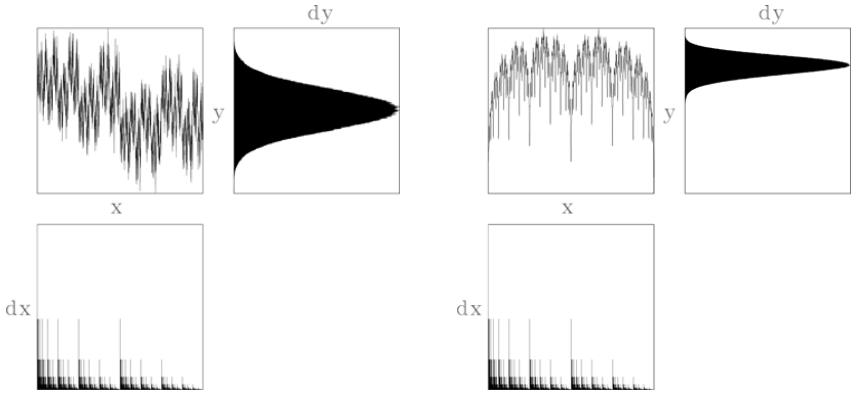


Figure 3. From dissipation to conduction (left) and from darkness to light (right).

The ultimate wire (shown for $z = 0.999$), defined by mid-point additions from the original three points via a sequence of increasing powers of $z$ that travel up and down (Puente, 2011b), grows and fills the

plane from minus infinity to infinity, its graph acquires a (fractal) dimension that tends to two, and, in the process, it transmutes the thorns over dust in the input multifractal into a smooth Gaussian bell curve with a finite center. This wondrous result (also valid for the case -, +) happens to be universal, as bells are always found from the same wire, not only using fair and arbitrarily biased coins, but also for any non-discrete input $dx$, including objects defined over fractal dusts.

As these results, from $x$ into $y$, imply a non-intuitive transformation of the *dissipation* of turbulence into the *conduction* of heat, implicit in the bell via Fourier's law—and not the other way around as in the common passage from order into chaos, this discovery urged me to think about the ultimate meaning of a function capable of converting spiky *violence* into diffusive *calmness*. As dividing the limiting wire into successive pieces (by halves) also yields projections that are bells, I finally concluded—though resisting the "unscientific" thought at first—that such a special transformation was somehow related to "love." For, what else could transform dust and dissipation into something harmonic, normal and conducting but love?

When both $d_1 = d_2 = z$ approach plus 1, a fractal wire shaped as a symmetric cloud is found, and the Platonic ideas define yet another bell universally (for any non-discrete $dx$), but now centered at infinity, as hinted in Figure 3 (right). What is shown there is not the limit, but what is found when $z = 0.99$. As such tends to 1, the bulk of the cloud, once again found by mid-point additions of the three original points in increasing powers of $z$ but now placed always up, elevates to infinity, and the mean—the center—and the variance—the spread—of the projection over $y$ both tend to infinity. But the square root of the variance over the mean tends to zero, indicating, with all probability, the limiting presence of an infinite spiky bell at infinity (Puente 2011b), in a manner that resembles the big bang, but in reverse!

While the case -, - leads to confusing oscillations forever, in Figure 3 (right) we may see how, in a "mystical" manner, the ever-positive wire generating the same bell from any of its parts (as for the mountain), and also filling two-dimensional space, raises it all to the clouds, filtering any kind of disorder, thorns, and dust—except for a discrete input—into an improbable condition of plenitude without entropy, yet reflected by the melodic bell. How not to recognize here, in a majestic limit that takes the ultimate *darkness* of death (below) to *light* (at infinity), a manifestation of freedom and divine LOVE? How not to appreciate

here an essential call to the eternal? For, in the directionality of the diagram, from $x$ into $y$, we may exclaim with Saint Paul: “Where, O death, is your victory? Where, O death is your sting?” (1 Cor 15:55).

**4. The Most Holy Trinity**

In this very spirit of redemption, how not to celebrate a truly splendid diagram, one illuminated by the just equilibrium of a fair coin, that completes a majestic trilogy in Figure 4?

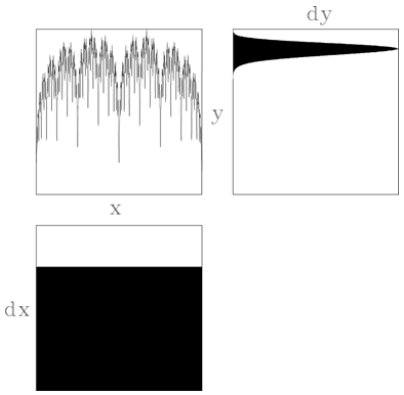


Figure 4. A representation of the Most Holy Trinity.

Here we may witness, symbolically and with due humility, of course, the *Father*, powerful in heaven, conducting and diffusing perfect love; the *Son*, always constant and the geometric “salvation of God” defined by the unique solution of “filling the valleys and cutting the mountains” (Lk 3:5-6); and the *Holy Spirit* that proceeds from them both and whose love truly transforms us, if we allow it.

Based on this limiting diagram, we may reflect, among other things, on the lovely encounter between St. Augustine and the little boy at the beach. As the transformation herein is capable of taking all the ocean—in the black rectangle—into a single point centered at infinity, the little boy was hinting at an unlikely possibility, little bucket by little bucket, in order to explain the mystery of the Most Holy Trinity to the Saint!

Curiously, the key transformation not only resembles geometrically a cloud, but also angel wings or the wings of a dove (Puente, 2011b). As in the limit the object is built adding powers of unity, we may appreciate that such an object contains infinite unity, and hence invoking the “unity of the Holy Spirit” is not a small thing! Furthermore, as $1 = 0.999\ldots$, such unity also reflects an infinite symphony of love in the outward and *positive* spiral of number 9, which opposes the selfish eddies of our sinful disorder. For,

coincidentally, Jesus died for us at the 9th hour and there was darkness from the 6th hour (Mk 15:33-34): the very geometry of selfish eddies that lead to cascading division and ultimate dust and death.

At the end, all limiting diagrams based on the +, + wire help us visualize God's infinite mercy through Jesus' unitive and loving sacrifice on the positive cross, which leads us to heaven. After all, the Eucharist, as a tiny piece of the cloud wire, takes us to infinity, as it is sensible from Figure 3 (right) to ask God not to look on our sins—on a spiky cascade—but rather on our faith—the unitive transformation. For, in the same spirit, we may see why blaspheming against the Holy Spirit (even if discretely by not believing in a superior dimension) is not forgiven (Mk 3:29), as it is only through Him that infinity is reached.

Consistently, it may be shown that the three members of the Holy Trinity may be associated to the irrational numbers $\pi$,$\sqrt{2}$ and $e$ (Puente, 2011b). As such are key in the formula of the bell and as the bell also symbolizes freedom, it may be argued that the ideas herein summon us to a rather central limit, in the consistent practice of picking up Jesus's cross to love and hence living away from sin (Mt 16:24, Jn 3:5).

**6. A Poetic Summary**

This work has exhibited the key antidote, as follows. From $x$ into $y$, as unnatural flow, mapping immensity, leaving dust below; from $x$ into $y$, inspiring all awe, o plus of liberty, forever aglow; from $x$ into $y$, only a tiny piece, wired to totality, o normal release; from $x$ into $y$, o infinite fleece, by packing vitality, no thorns but peace; from $x$ into $y$, singular the dough, symphony of unity, breeding single row; from $x$ into $y$, by breaking a spell, amazing simplicity, o refuge from hell; from $x$ into $y$, from holy plateau, perennial immunity, o spirit on the go; from $x$ into $y$, triune is the cell, omnipotent divinity, o sacred God's bell.

Acknowledgments. This work is dedicated to the only true Church.

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