A Deterministic Approach to Simulate and Downscale Hydrological Records

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Abstract

Application of a deterministic geometric approach for the simulation and downscaling of hydrologic data, daily rainfall and daily streamflow over a year, is presented. Specifically, it is shown that adaptations of the fractal-multifractal (FM) method, relying on only 6 to 10 geometric parameters, may do both tasks accurately. The capability of the FM approach in producing plausible synthetic and disaggregated sets is illustrated using rain sets gathered in Laikakota, Bolivia and Tinkham, Washington, USA, and streamflow sets measured at the Sacramento River, USA. It is shown that suitable deterministic synthetic sets, maintaining the texture of the original records, may readily be found that faithfully preserve, for rainfall, the entire records' histogram, entropy and distribution of zeros, and, for streamflow, the entire data's autocorrelation, histogram and entropy. It is then shown that the FM method readily generates daily series of rainfall and streamflow over a year based on weekly, bi-weekly and monthly accumulated information, which, while closely preserving the time evolution of the daily records, reasonably captures a variety of key statistical attributes. It is argued that the parsimonious FM deterministic simulations and downscaling may enhance and/or supplement stochastic simulation and disaggregation methods.

Introduction

Simulation and disaggregation (downscaling) of hydrologic records are key for the planning and design of water resource infrastructures. Although a host of (stochastic) procedures do exist for such tasks, the very nature of using realizations, preserving only some statistical/physical features, suggests that improved approaches may perhaps be developed.

Trying to capture the intricate details of geophysical records, Puente (1996) developed a deterministic fractal geometric method, the fractal-multifractal (FM) approach, which approximates a data set as a fractal transformation of a multifractal measure. As previous efforts have demonstrated that such geometric notions are useful for encoding various hydrological processes, e.g., daily rainfall and streamflow sets gathered over a year (Maskey et al., 2015, 2016a), this work explores the possibility of using the FM approach to simulate and downscale rainfall and runoff, that is, highly intermittent rainfall sets gathered at Laikakota, Bolivia and Tinkham, Washington, USA and also mildly intermittent streamflow sets measured at the Sacramento River, USA.

The Fractal-Multifractal Approach

Figure 1 shows how a ***fractal*** *interpolating function* $f: x \to y$ transforms a ***multifractal*** *measure* dx into a ***derived*** *measure* dy (Puente, 1996). As seen, such a function f, constructed iterating two simple contractile affine maps of the form $w_n(x,\ y) = (a_n x + e_n,\ c_n x + d_n y + f_n)$ (Barnsley, 1988), passes by the three points marked in blue and yields, while doing the calculations using a 47-53 proportion for the two maps, a classical multifractal measure dx (Mandelbrot, 1989) and a deterministic projection dy, whose smoothed version on the right, dy_s, resembles river discharges in time (Maskey et al., 2016a; Puente et al., 2016). At the end, dy is a set that is uniquely based on the interpolating points, the vertical scalings d_n and the iteration's proportion, a deterministic pattern that possess a physical interpretation as a non-trivial cascade of a conservative constituent (Cortis et al., 2013).

Figure 2 depicts a generalized version of the FM approach in which the interpolating points are replaced by end-points yielding, in the process, more general attractors. As illustrated, the iteration of two simple affine maps as above, with successive end-points marked by blue circumferences and according to 26-74 proportion, gives, in this case, a Cantorian measure dx and a sparse attractor from x to y that produces a highly intermittent derived measure dy, which, when trimmed below a threshold ϕ, as seen on the right, looks like rainfall records (Maskey et al., 2015).

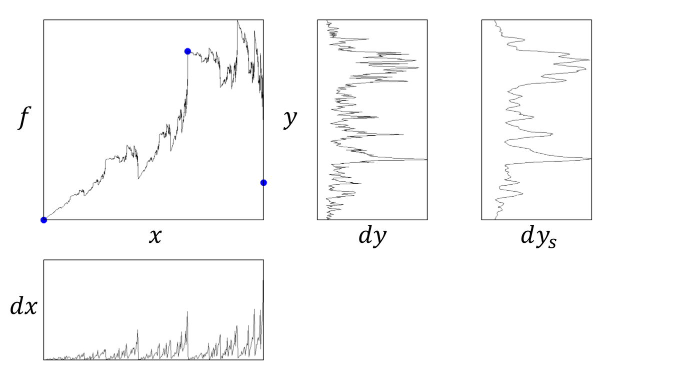

Fig. 1. The FM method: from a multifractal dx to a projection dy, via a fractal interpolating function f, followed by a smoothed output dy_s.

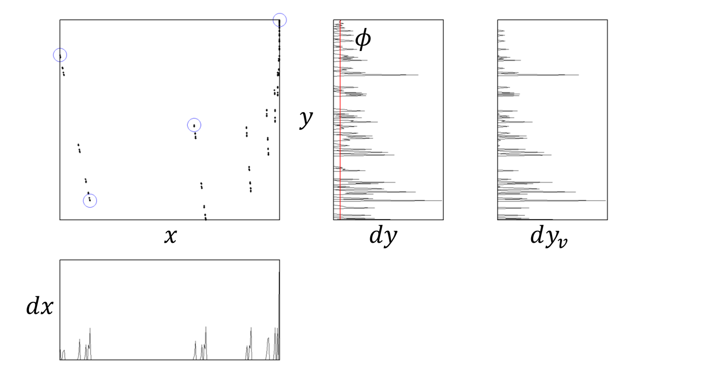

Fig. 2. A generalized FM approach: from a Cantorian measure dx, to a projection dy, via a disperse attractor from x to y, followed by an output dy_ν found pruning dy below a threshold ϕ.

Deterministic Simulations of Rainfall

The ability of the FM notions in simulating (highly intermittent) rainfall sets, containing substantial periods of inactivity (many zeros throughout the year), is illustrated for Laikakota, Bolivia in Figs. 3 and 4 and for Tinkham, Washington, USA in Figs. 5 and 6. Such sets were found running an optimization program, over the set of FM parameters, aiming at the preservation of the data's histogram, entropy and zero values (i.e., number of zeros and length of consecutive zeros).

For the Bolivia site, while Fig. 3 shows FM sets that capture the record's histogram and distribution of zeros, Fig. 4 includes FM simulations that preserve the entropy and zero values and the histogram and entropy combined.

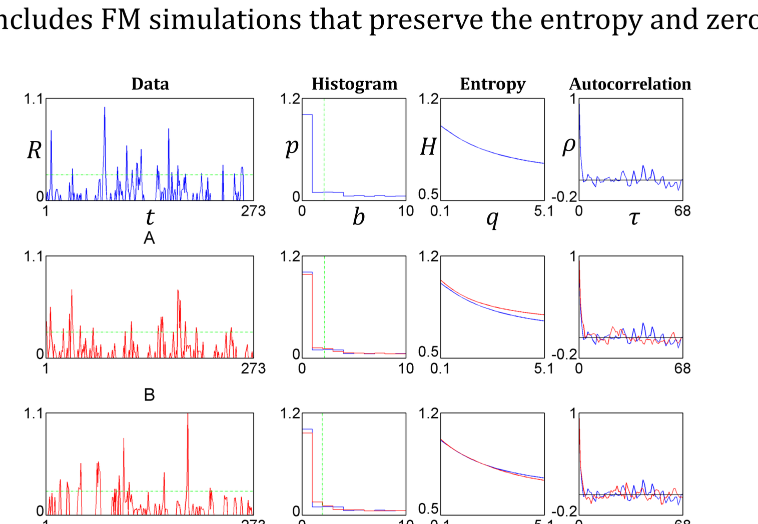

Fig. 3. A rainfall set at Laikakota, Bolivia (Sept. 1965–May 1966) (blue) and two FM simulations (red) preserving: **A** the record's histogram, and **B** the distribution of zeros (total number and consecutive). The green lines correspond to 90% masses.

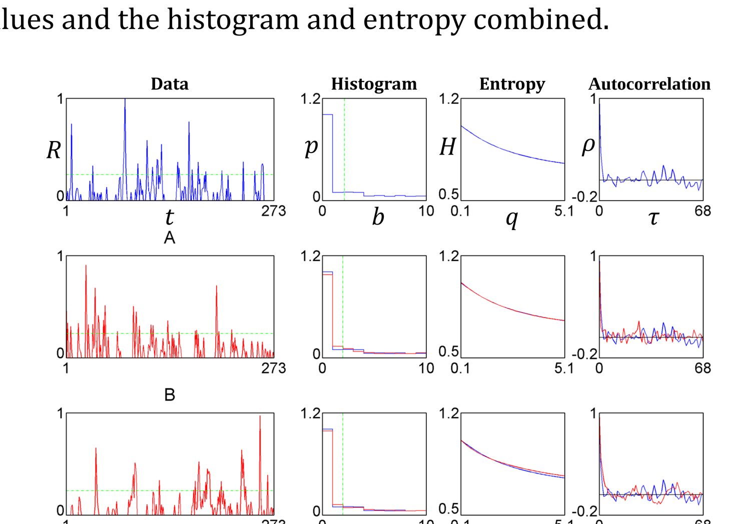

Fig. 4. A rainfall set at Laikakota, Bolivia (Sept. 1965–May 1966) (blue) and two FM simulations (red) preserving: **A** the record's Rényi entropy and distribution of zeros (jointly), and **B** the record's histogram and entropy (jointly).

Deterministic Simulations of Rainfall (cont.)

Figures 5 and 6 are the corresponding simulations for a water year at Tinkham, Washington.

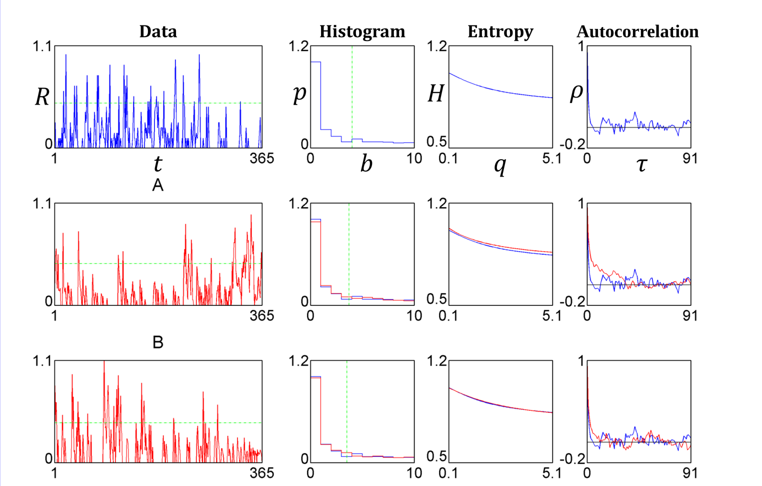

Fig. 5. A rainfall set at Tinkham, Washington (Oct. 2000–Sept. 2001) (blue) and two FM simulations (red) preserving: **A** the record's histogram and **B** the distribution of zeros (total number and consecutive). The green lines correspond to 90% masses.

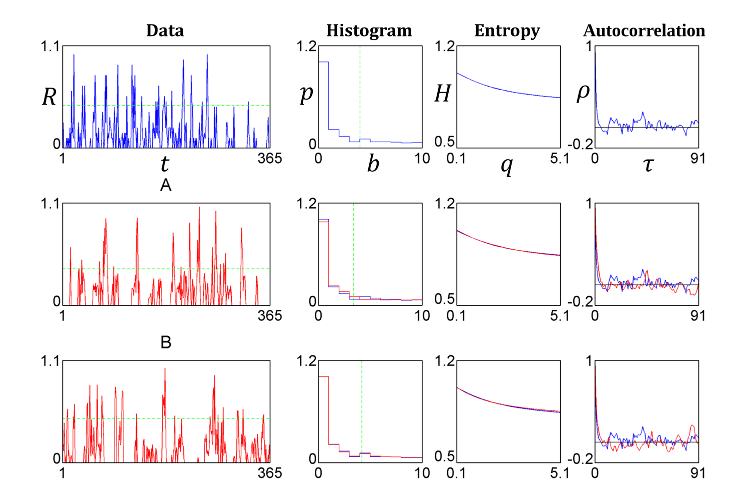

Fig. 6. A rainfall set at Tinkham, Washington (Oct. 2000–Sept. 2001) (blue) and two FM simulations (red) preserving: **A** the record's Rényi entropy and distribution of zeros (jointly) and **B** the record's histogram and entropy (jointly).

All FM rainfall simulations, based on the iteration of two maps and requiring from 6 to 8 FM parameters, preserve key statistics in their entirety, including 90% extremes. Nash-Sutcliffe efficiencies for histograms and entropy are above, respectively, 99% and 81%, the number of FM zero values are above 92% of the zeros in the records, and the maximum length of consecutive zeros on FM sets differ from those in the sets by less than 4 days (Maskey et al., 2016b).

Deterministic Simulations of Streamflow

The capability of the FM approach in simulating (mildly intermittent) streamflow sets is illustrated in Figs. 7 to 10 for two distinct years of records at the Sacramento River, 2002-03 and 2004-05. While Figs. 7 and 9 show simulations preserving the records' autocorrelation and histogram for the two water years only, Figs. 8 and 10 include simulations aimed at fitting the autocorrelation and histogram combined and the autocorrelation, histogram and entropy combined.

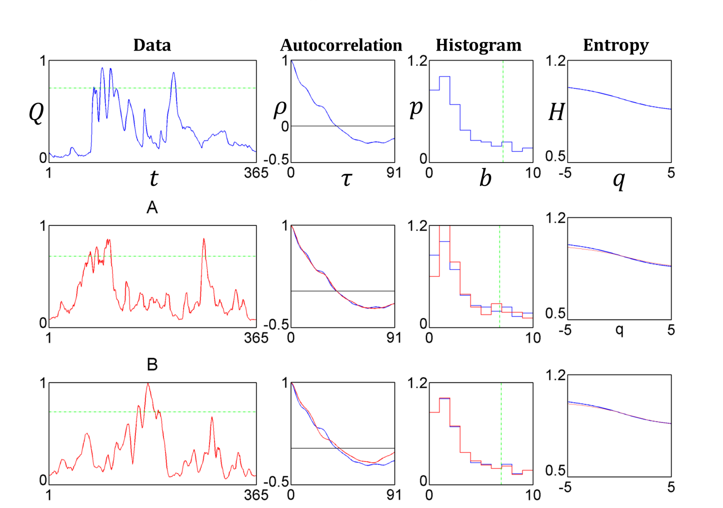

Fig. 7. A streamflow set at the Sacramento River (Oct. 2002–Sept. 2003) (blue) and two FM simulations (red) preserving: **A** the record's autocorrelation, and **B** the record's histogram. The green lines correspond to 90% masses.

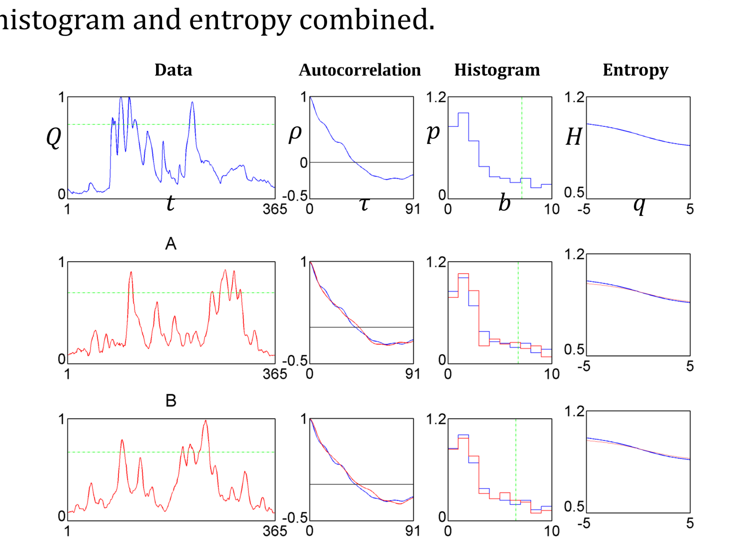

Fig. 8. A streamflow set at the Sacramento River (Oct. 2002–Sept. 2003) (blue) and two FM simulations (red) preserving: **A** the record's autocorrelation and histogram (jointly), and **B** the record's autocorrelation, histogram and Rényi entropy (jointly).

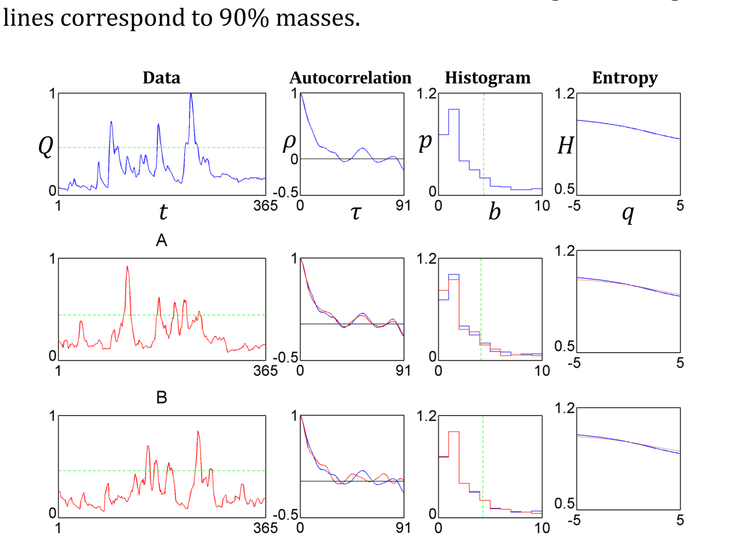

Fig. 9. A streamflow set at the Sacramento River (Oct. 2004–Sept. 2005) (blue) and two FM simulations (red) preserving: **A** the record's autocorrelation, and **B** the record's histogram. The green lines correspond to 90% masses.

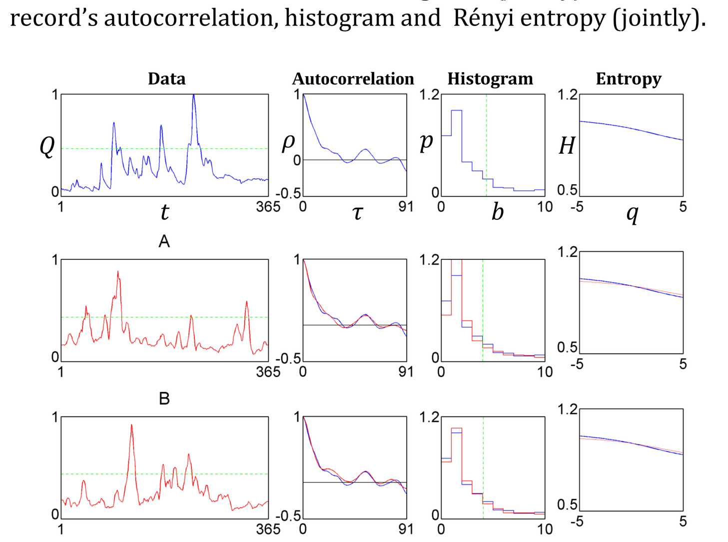

Fig. 10. A streamflow set at the Sacramento River (Oct. 2004–Sept. 2005) (blue) and two FM simulations (red) preserving: **A** the record's autocorrelation and histogram (jointly), and **B** the record's autocorrelation, histogram and Rényi entropy (jointly).

All FM simulations, requiring 6 to 8 FM parameters, preserve whole functions of the record's: autocorrelation, histogram and entropy, as used in the respective objective function. Nash-Sutcliffe indices on preserved statistics in Figs. 7 and 9 are always greater than 98%. The corresponding efficiencies in Figs. 8 and 10 are always above 87% (Maskey et al., 2016c).

FM Disaggregation of Precipitation and Streamflow

The ability of the FM approach as a downscaling method is demonstrated in Fig. 11 for rainfall and Fig. 12 for streamflow. The calculations involve finding suitable FM encodings of accumulated records at a coarse scale (weekly, bi-weekly and monthly) and using the obtained FM parameters to re-compute the output projection dy over a fine resolution (daily).

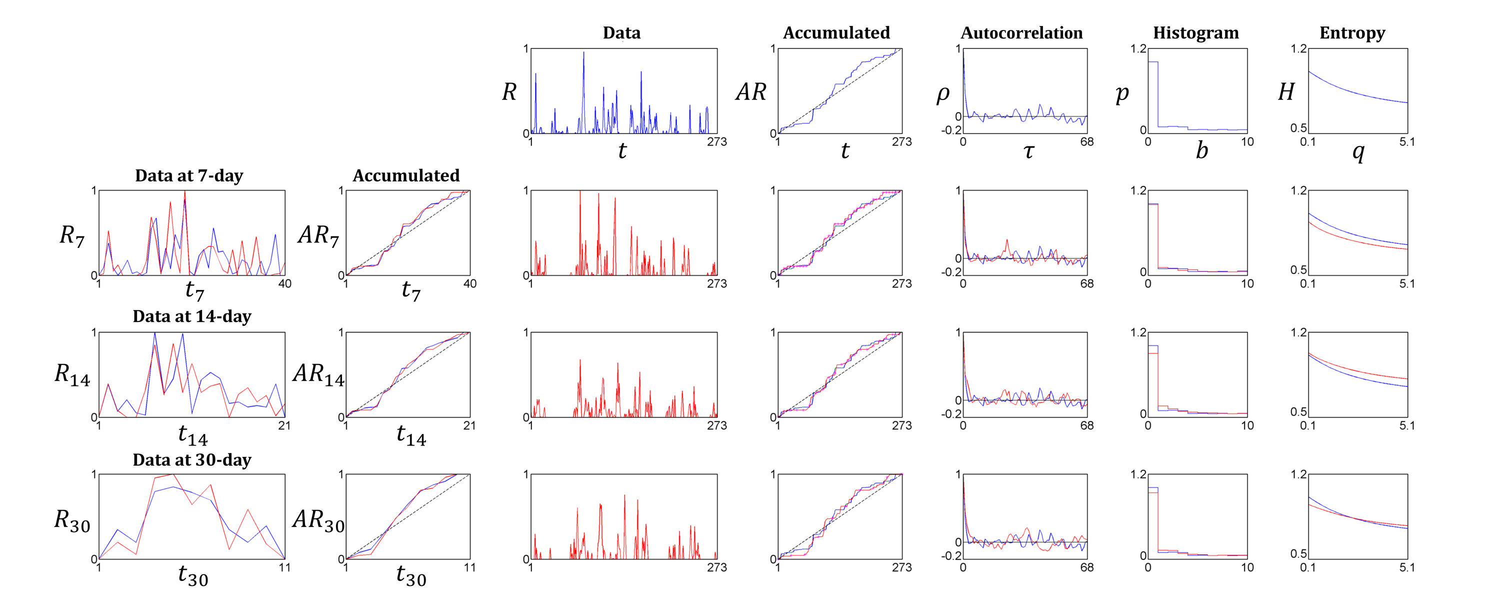

Fig. 11. A rainfall set at Laikakota, Bolivia (Sept. 1965–May 1966) (blue) and suitable FM downscales based on three affine maps (red), from weekly to daily, bi-weekly to daily and monthly to daily, followed by a comparison of statistics at the daily scale.

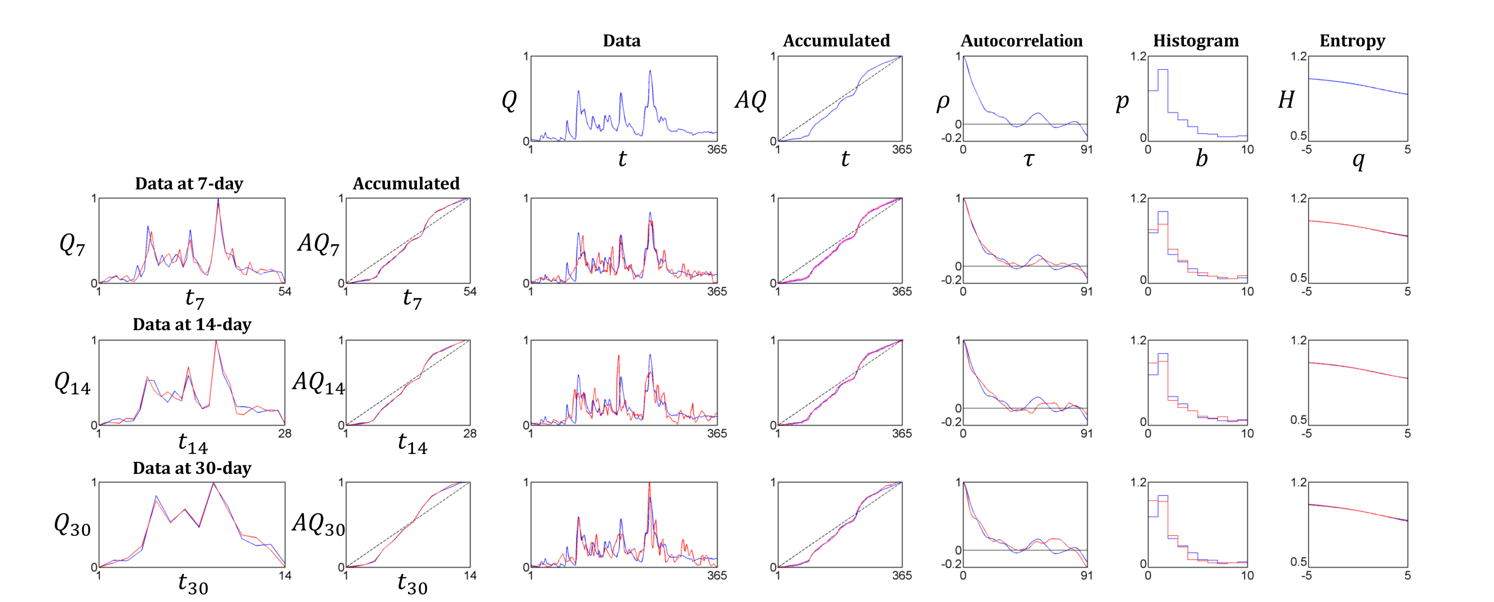

Fig. 12. A streamflow set at Sacramento River (Oct. 2004–Sept. 2005) (blue) and suitable FM downscales based on three affine maps (red), from weekly to daily, bi-weekly to daily and monthly to daily, followed by a comparison of statistics at the daily scale.

The FM disaggregation, based on the iteration of three affine maps and hence 10 FM parameters, yields textures that are indistinguishable, by the naked eye, from those present in the observations. While rainfall downscales exhibit inaccuracies in entropy –with Nash Sutcliffe indices as low as 45% in weekly to daily downscales–, their histograms (and therefore the moments) remain well preserved, even from the monthly scale. As expected, the downscales for the mildly intermittent streamflow set are even better, for the Nash Sutcliffe indices for autocorrelation, histogram and entropy are, for all disaggregation scales, above 84, 90 and 99%, respectively (Puente et al., 2016).

Concluding Remarks

The deterministic Fractal-Multifractal approach may be used to produce sensible simulations and disaggregation for both precipitation and streamflow sets, which preserve entire statistical information of relevance in applications. The ideas rely on the solution of an appropriate inverse problem, in the space of FM parameters, which easily yields a variety of plausible solutions at a fraction of the time required for performing encodings of daily sets. By defining sets that are indistinguishable from observed records, the notions supplement procedures based on stochastic methods. It is envisioned that the FM method may also be used to downscale other sets, such as outputs of global circulation models.

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