Downscaling Rainfall Records via a Deterministic Fractal Geometric Approach

AGU 2009, San Francisco
Paper Number: NG33B-1085

Carlos E. Puente[1*] and Andrea Cortis[2]
[1]University of California, Davis, USA; Department of Land, Air, and Water Resources
[2] Lawrence Berkeley National Laboratory, Berkeley, USA; Earth Sciences Division
* Corresponding Author, E-mail: cepuente@ucdavis.edu

Abstract

With the proliferation of climate global change studies, downscaling of hydrological information (e.g., rainfall intensity time-series, spatial precipitation radar-images) from coarse to finer scales is one of the most important challenges hydrologists and meteorologists face today. Although a variety of stochastic and chaotic downscaling procedures reproduce correctly some of the characteristics of these hydrological records such as moments, correlations, and/or fractal and chaotic statistics, other important characteristics such as precise peak sequences and intermittency structures are often not correctly represented. In this work, we present a deterministic downscaling model based on a fractal-multifractal projection approach. Our downscaling model is capable of producing rainfall-like patterns both in time and in space. It is shown how the fractal-based approach may be used to simulate rainfall time series at multiple time-scales which, while sharing the statistical and fractal statistics of observations at the finer scales, yield suitable scenarios for global climate change analysis.

Introduction

With recent technological advances and the development of sophisticated mathematical techniques, such as those based on fractal geometry, modeling of rainfall complexity has substantially improved. Although these ideas have resulted in a new language to describe the intricacies of data sets at hand, oftentimes such tools are insufficient to study, on an individual basis, the incredible variety of patterns available to us.

Since rainfall distributions are typically erratic, noisy, intermittent, complex, or in short seemingly “random,” it has become natural to use stochastic (fractal) theories in order to model them. This has given rise to a variety of approaches that even though yield modeled sets, i.e. realizations, that preserve relevant statistical and physical attributes of the records (e.g. autocorrelation function, power spectrum, moments, etc.), such are often unable to capture the specific details and textures found in observed data sets.

As studies of nonlinear dynamics and deterministic chaos have revealed to us that details indeed matter, the following questions naturally arose: (1) Could it be possible to find suitable models of individual patterns that capture not only the overall trends and statistical features of the records but also their inherent details? (2) Could such a modeling approach help explain deterministically what otherwise appears to be random, as in deterministic chaos? and (3) Could such ideas, by capturing details, be helpful in studying the underlying dynamics of such sets?

Encouraged by the success in defining certain deterministic fractal sets via iterations of simple maps (e.g. Barnsley 1988), this work employs a fractal geometric approach aimed at capturing the complexity of rainfall patterns as deterministic derived measures obtained transforming simple multifractal measures via fractal interpolating functions (e.g. Puente 1992). As shall be demonstrated herein, the geometric approach produces a vast class of patterns, defined over one and higher dimensions, that resemble rainfall sets, and that allow to define suitable downscalings of sets as required when studying plausible climate change scenarios.

The Fractal-Multifractal Approach

The graph of a fractal interpolating function, from x to y and passing by N+1 points on the plane

$$\{(x_n, y_n): x_0 < x_1 < \dots < x_N\},$$

is obtained iterating a set of N affine maps as follows:

$$w_n \begin{matrix} x \\ y \end{matrix} = \begin{pmatrix} a_n & 0 \\ c_n & d_n \end{pmatrix} \begin{matrix} x \\ y \end{matrix} + \begin{pmatrix} e_n \\ f_n \end{pmatrix} \qquad (1)$$

subject to the conditions:

$$w_n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_{n-1} \\ y_{n-1} \end{pmatrix} \qquad (2)$$

$$w_n \begin{matrix} x_N \\ y_N \end{matrix} = \begin{matrix} x_n \\ y_n \end{matrix} \qquad (3)$$

and

$$|d_n| < 1\,, \qquad (4)$$

where the parameters a_n, c_n, e_n, and f_n, are found in terms of the scalings d_n and the coordinates of the interpolating points (e.g. Barnsley 1988; Puente 1992). At the end, a unique deterministic attracting set is found that turns out to have a fractal dimension between 1 and 2.

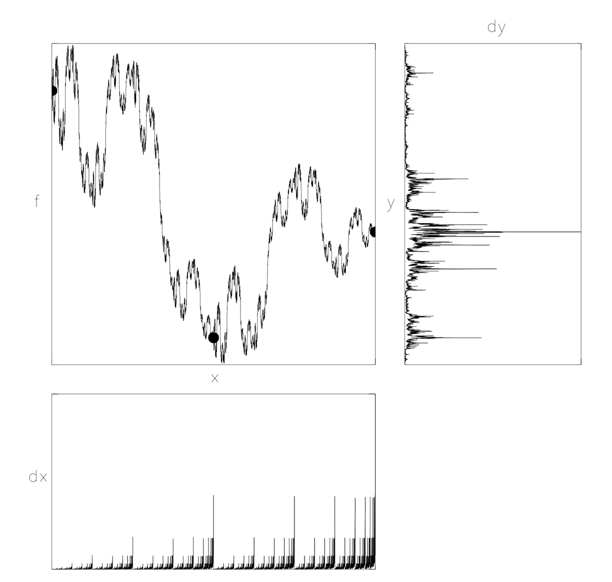

Figure 1. A multifractal measure dx is transformed via a fractal function f into a derived measure dy.

As the process of iterations is carried out, a unique invariant deterministic measure is induced over the fractal function that reflects how such an attractor is filled. The existence of such a measure allows computing its projections over the x and y coordinates, namely dx and dy (e.g. Puente 1996).

Figure 1 shows an example for a fractal function that passes by {(0,0), (1/2,-0.35), (1,-0.2)}, when the scalings are d_1 = -0.8 and d_2 = -0.6. In addition to the fractal wire f, the figure includes the implied projections dx and dy when the corresponding mappings w_1 and w_2 are iterated according to a biased coin having a 30-70% proportion, using “independent” pseudo-random tosses, starting the process from the mid-point (1/2,-0.35).

Due to the independent nature of y on x (Eq. 1) and the implicit contractions over x given by equations (2) and (3), the induced measure over x is simply a deterministic binomial multifractal. The measure dy, in turn, being related to dx via the fractal function, is just the derived distribution of dx via f (e.g. Puente 1992) and hence it is computed for a given value of y adding the corresponding “probabilities” in x for the values for which $f(x) = y$.

The ideas lead to very interesting and random-looking measures dy, which, as in the example, resemble rainfall data sets as a function of time (e.g. Puente and Obregón 1996). As (binomial) multifractal measures have been found relevant in studies of turbulence (e.g. Meneveau and Sreenivasan 1987), the projection sets given by these ideas, which turn out to perform a non-trivial ‘fractional integration’ of a simple parent multifractal measure over x, may be assigned an interpretation as reflections or transformations of turbulence.

Downscaling of Rainfall Sets in Time

To illustrate that the fractal geometric approach may be employed to downscale rainfall information, we present herein the results obtained while analysing two synthetic data sets generated via the fractal-multifractal approach, one resembling a detailed storm gathered every few seconds in Boston and another a set gathered every 6 hours at La Honda, California as reported in Cortis et al. (2009). Both of such sets, originally made of 2,048 data points, were aggregated to 64 values and then a suite of optimization programs was employed to approximate such coarse data set, its cumulative distribution and also other relevant statistics.

Figure 2 presents the results for the high resolution example. The pattern labelled **a** is the “real” set generated via the fractal-multifractal approach iterating 2 simple maps, using a binomial multifractal input with a 60-40% proportion and a fractal function passing by 3 points and having as scalings d_1 = -0.5 and d_2 = 0.3. Patterns **b**,**c** and **d** show examples of fractal-multifractal representations based on 2, 3 and 4 mappings that were selected such that their cumulative distributions differ from the original data (at the coarse resolution) by less than 6% maximum. As is seen for the graphs at the finer resolution and their corresponding cumulative distributions (with the real pattern shown in red), the downscaled patterns produced via the fractal-multifractal approach represent suitable simulations of the “original” set which besides sharing the overall location and texture of the mass, also possess similar statistical characteristics as indicated by close autocorrelation functions, power spectra and multifractal.spectra.

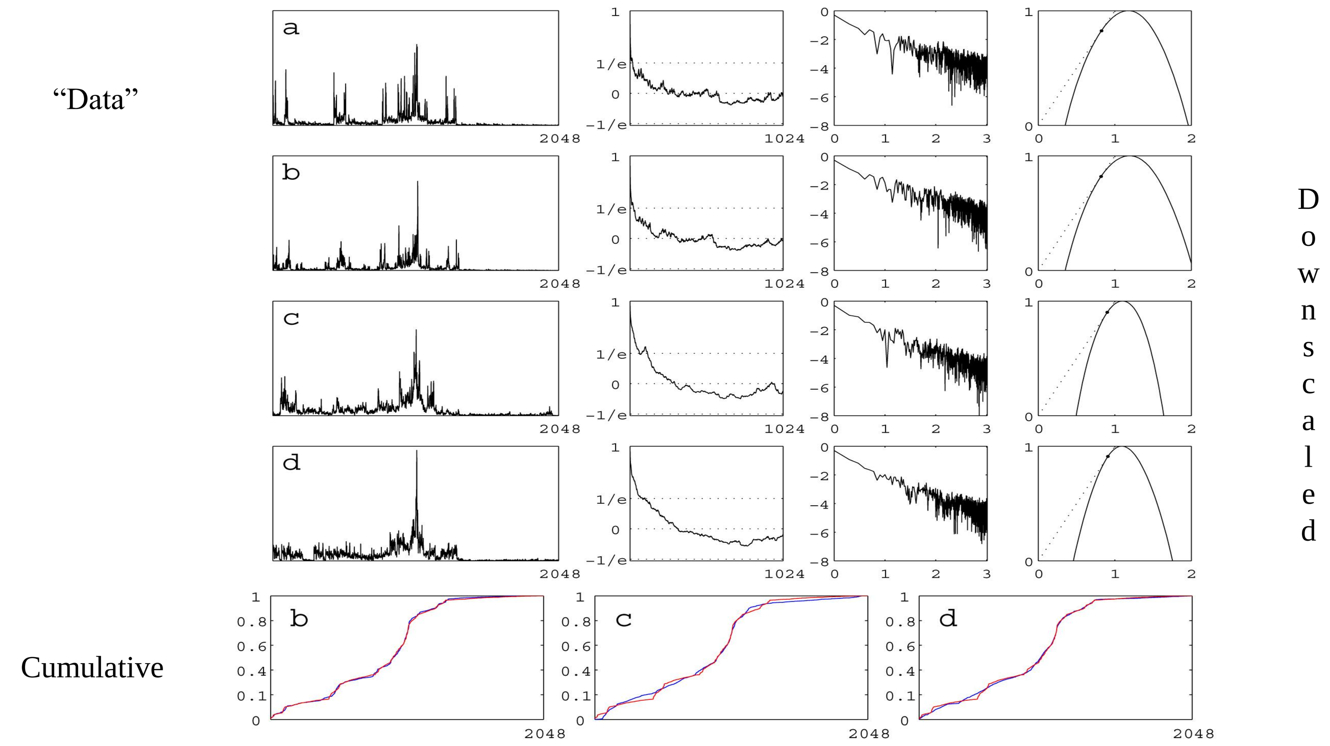

Figure 2. A “real” set **a** and three approximations via downscalings computed via fractal-multifractal representations based on 2, 3 and 4 mappings. The key statistics are, in order, i) lag to 1/e: 20, 25, 65, 85; ii) power spectra exponent -0.92, -1.15, -1.23 -1.25; and iii) multifractal spectrum entropy dimension: 0.82, 0.82, 0.90, 0.91.

Figure 3 presents the results for the other example which is based on a fractal interpolating function generated via 3 maps and a Cantorian multifractal input leading to periods of no rain. The downscaled patterns **b**,**c** and **d** correspond to other Cantorian or close to Cantorian fractal-multifractal representations based on 3 mappings but with different scaling signs than the real data set. They were selected such that their cumulative distributions differ from the original data by less than 8% maximum. As is seen,, the downscaled patterns for this case also yield suitable representations of the “original” set in overall textures and statistics.

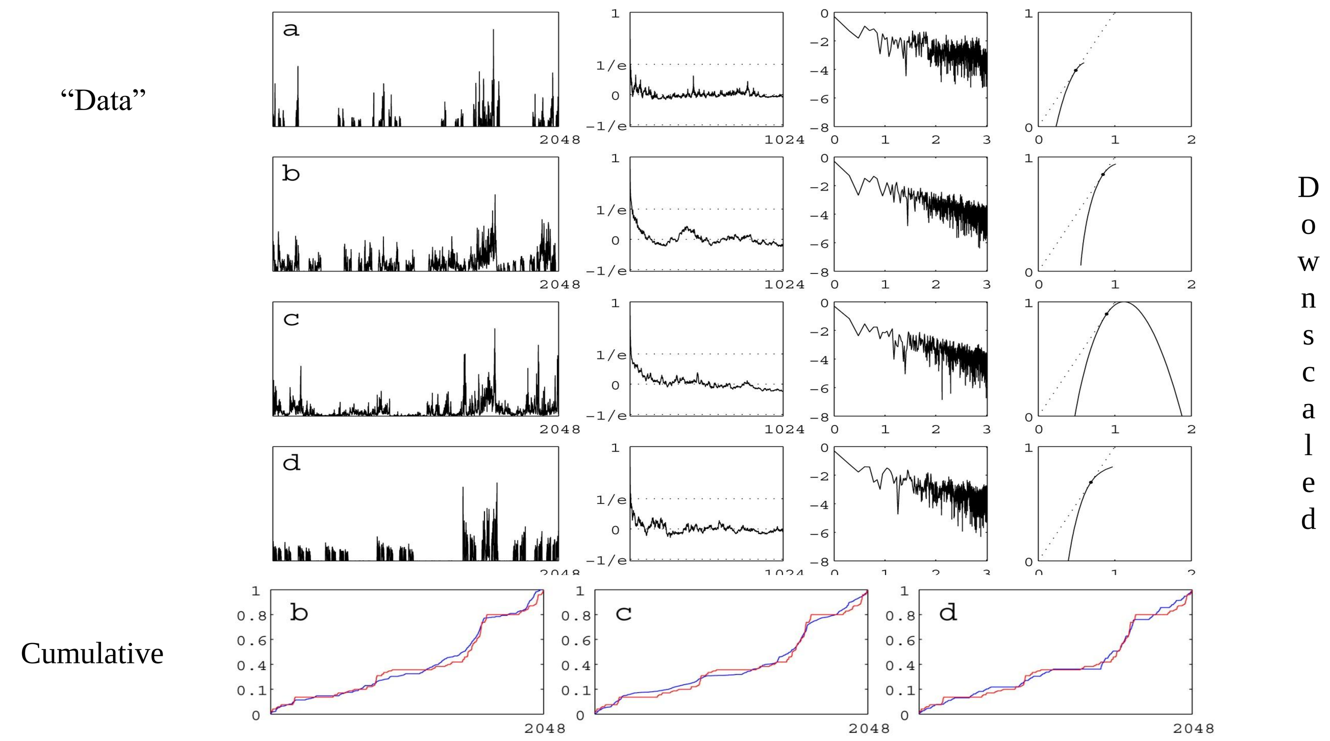

Figure 3. A “real” set **a** and three approximations via downscalings computed via fractal-multifractal representations based on 3 mappings for alternative signs on the scalings. The key statistics are, in order, i) lag to 1/e: 3, 15, 9, 4; ii) power spectra exponent -0.69, -1.05, -0.98, -0.81; and iii) multifractal spectrum entropy dimension: 0.49, 0.85, 0.89, 0.70.

Other Plausible Rainfall Sets

The fractal-multifractal approach may also be employed to simulate more general rainfall data sets that while not maintaining the specific cumulative distribution of the data still share the same overall statistics. Figures 4 and 5 show some examples of such simulated patterns that correspond to the “real” sets used in Figures 2 and 3. As may be seen, it is not difficult to find interesting patterns having diverse geometries that may also be employed in global climate change studies.

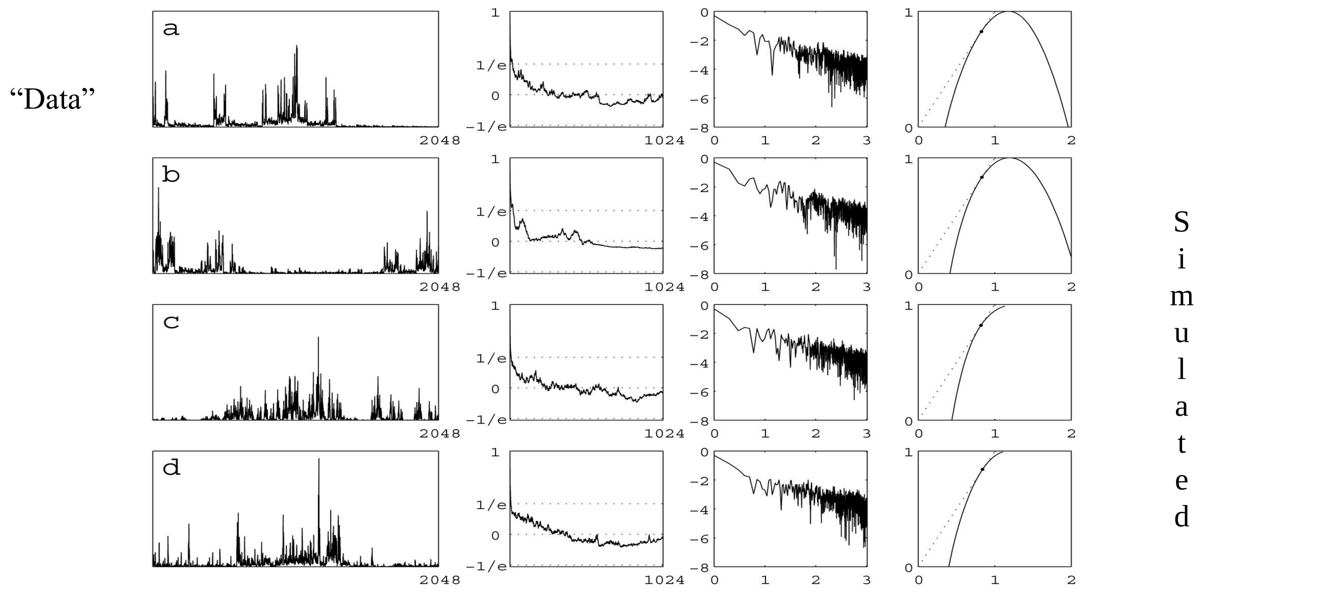

Figure 4. The data set in Figure 2 and simulations via fractal-multifractal representation based on 2, 3 and 4 mappings having similar autocorrelation function and identical power-spectrum scaling and entropy dimension.

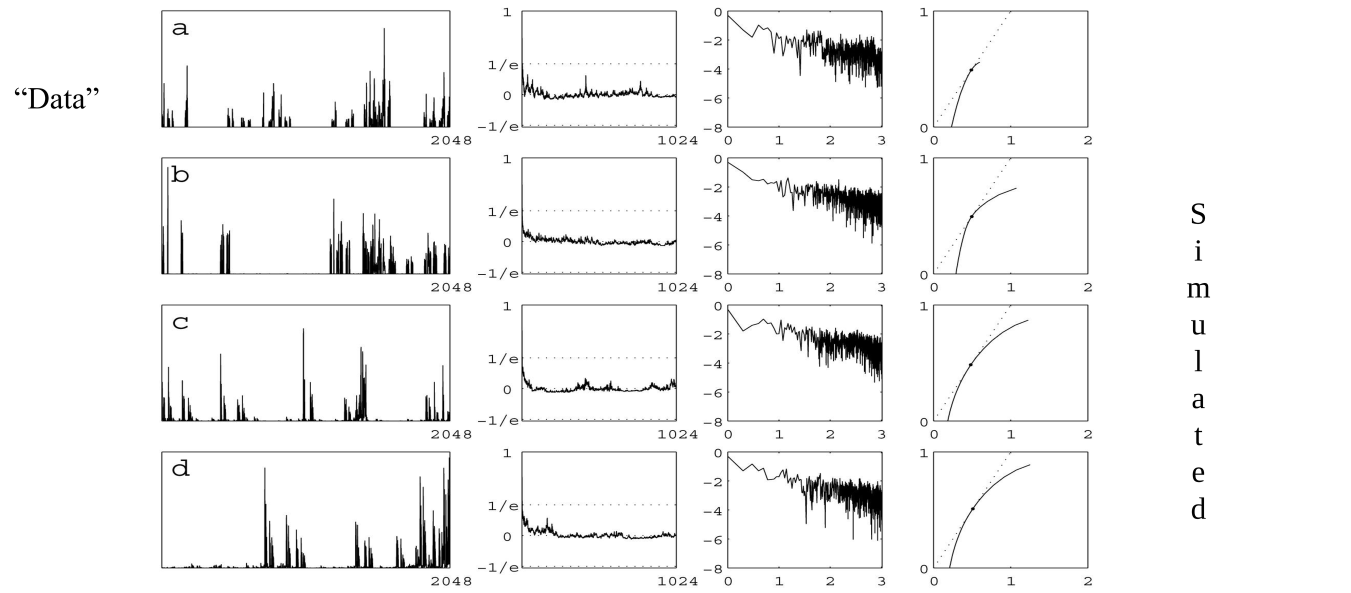

Figure 5. The data set in Figure 3 and simulations via fractal-multifractal representations based on 4 mappings having similar autocorrelation function and identical power-spectrum scaling and entropy dimension.

Conclusions

We illustrated via few examples how a novel deterministic geometric procedure may be used to downscale and simulate rainfall data sets defined over time as required in the analysis of global climate change studies. The approach not only preserves key statistical qualifiers but may also be used to capture the overall texture of a hyetograph including its intermittencies.

Current efforts concentrate on the analysis of rainfall spatial patterns for the same (downscaling and simulation) purposes and on the efficient solution of the inverse problem for hydrological data sets, that is, one in which the sets are not approximated as herein but rather fitted. The fractal-multifractal method, and its extensions (Cortis et al., 2009), holds promise as an important tool for hydrological and meteorological synthesis and dynamics (Puente et al., 2001) and as such it provides a viable alternative to existing stochastic procedures. Our approach, as illustrated in all figures, also hints, in a counter-intuitive fashion, at the possibility of hidden determinism in natural complexity (Puente, 2004; Puente and Sivakumar, 2007).

References

Barnsley, M.F. (1988), *Fractals Everywhere*, Academic Press, San Diego, California.
Cortis, A, B. Sivakumar & C. E. Puente (2009), Encoding hydrologic information via a fractal geometric approach and its extensions. *SERRA*, DOI 10.1007/s00477-009-0349-4.
Meneveau, C. & K.R. Sreenivasan (1987), Simple multifractal cascade model for fully developed turbulence, *Physical Review Letters*, 59, 1424-1427.
Puente, C.E. (1992), Multinomial multifractals, fractal interpolators, and the Gaussian distribution, *Physics Letters A*, 161, 441-447.
Puente, C.E. (1996), A new approach to hydrologic modeling: derived distributions revisited, *Journal of Hydrology*, 187, 65-80.
Puente, C.E. (2004), A universe of projections: may Plato be right?, *Chaos, Solitons and Fractals*, 19, 241-253.
Puente, C.E. & N. Obregón (1996), A deterministic geometric representation of temporal rainfall, *Water Resources Research*, 32(9), 2825-2839.
Puente, C.E., O. Robayo, M.C. Díaz & B. Sivakumar (2001), A fractal-multifractal approach to groundwater contamination, *SERRA*, 15(5), 357-371.
Puente, C. E. & B. Sivakumar (2007), Modeling hydrologic complexity: A case for geometric determinism, *HESS*, 11, 721-724.