# On the Unitive Art of the Holy Trinity Carlos E. Puente



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From a common parlance, religion is experiential, subjective, heart oriented and personal, but science is experimental, objective, mind oriented and impersonal. Basically the difference between science and religion lies in the approach. A believer and a scientist may approach the reality differently. Contemporary science provides us with a dynamic worldview which blurs the apparent contradictions between science and religion. Science describes the reality of existence whereas religion inspires the meaning of its existence. It is said in general that science is a matter of reason; religion is a matter of faith. The nature of faith and its relation to reason are often misunderstood as a leap from reason to faith. It implies that when all the available evidences fall short of establishing the truth of religion, we must leap the gap between evidence and truth. But faith is not a leap within the order of the intellect, a leap which violates the very essence of that order. The leap of faith is a leap from the order of the intellect to the order of the heart. We recognize the reality of the order of the heart when we realize that we cannot achieve the well-being we seek from possessing the order of the body or the order of the intellect. Such a conviction points toward an order that is above the order of the intellect. Thus there is a leap, which is both methodological and metaphysical, between the orders of the body/intellect and the order of the heart. Even though faith is not produced by reason, our faith is reasonable because faith claims illumine the mind on matters that otherwise baffle us. Without such illumination faith is improper. An authentic understanding of the 'leap' based on the criterion of reasonableness becomes a striving force to foster dialogue between science and religion.

Being aware of the methodological and metaphysical leap, the current issue of *Omega* presents nine scholarly articles affirming the reasonableness of interfacing science and religion for a holistic understanding of reality. In the first article Dr. Ashok Vohra makes a critical

enquiry into the question what constitutes 'religious point of view', compares and contrasts it from what is commonly called scientific standpoint or scientific temper. The inadequacy of language in articulating scientific phenomenon and religious experience is supported and substantiated by the views of Ludwig Wittgenstein and Chomsky. Bringing to light the methodological differences in science and religion, Dr. Vohra argues for the position that there is not just one or any unique paradigm of explanation but that there are varieties of explanation – each one of them as good and efficacious as the other, if not better. The context as well as the subject matter, individually or collectively, determines which is the most appropriate model of explanation, and at what point it has to come to an end. In other words, at what point 'saying' is to be replaced by 'showing' would be determined by the context of discourse.

Dr. John Vattanky attempts a hermeneutical appropriation of interfacing the Nyâya System of Philosophy with science and religion. Nyâya makes use of predominantly logical reasoning as a method of philosophical enquiry whereas modern science is heavily dependent on experiments to understand the nature of the universe. And yet Dr. Vattanky believes that the philosophical insights and conclusions of Nyâya can be enormously enlightening to the scientist in as much as they would provide invaluable orientation about the wider context of the whole of reality in which the scientists have to conduct their researches. They would set the parameters which the scientist should always respect in their attempts to understand the world and interpret it. In fact the author introduces a perspective that our universe is not a system that is closed upon itself but it is very much open to transcendence. If this perspective is clearly understood then no scientist would dare to step out of his boundary and start making unwarranted statements about what is beyond the scope of science. He further argues that the idea of God is the central principle of the Nyâya system and this principle provides the hermeneutic key for the Nyâya system as a whole. Since Nyâya system of philosophy is an interpretation of the universe, this hermeneutic key of the system will reveal to us the inner structure of the reality which no scientist can ignore except at the cost of intellectual prejudice and opaqueness.

After having analysed the Indian situation as scientific forwardness, economic backwardness and spiritual inwardness, Dr. Kuruvilla Pandikattu

goes on to emphasize the spirit of dialogue found in India. Then he deals on the prospects of science-religion dialogue mainly in terms of the widening of collective human consciousness, which calls for a renewed vision of God, humans and the world. Proposing some correctives, the author focuses on the future of science-religion dialogue leading to purified religions, sciences with human face and societies which are open and vibrant.

Acknowledging the methodological and paradigmatic differences between scientific language and mythical language, Dr. Paul Thelakkattu explores the mythical underpinnings of science with reference to well known scientists like Karl Popper and Albert Einstein. Can we give a logical and rational foundation to theology? The article sounds an affirmative response since the modern science is a contribution of religious scientists to the world. Analyzing the Medieval theologians he shows how reason played a vital role in shaping the theological worldview of those times. According to Thelakkattu, the correlation between theoretical construction and the absolute reality is a matter of faith which he calls the faith in rationality which is founded in God the Supreme Reason. Reason creates meaning and science is a meaningful pursuit because it is a search for truth. The real danger of science as well as of religion is to loose faith in human rationality – the critical spirit.

In the next article Prof. D. N. Yadav searches for the possibility of relating science with religion and vice versa. Taking into account the views of Ian G. Barbour and Eugenie Scott he addresses the methodological issues involved in this process of relating the two diverse and unique disciplines. The author finds an adequate model in the integral vision of Ken Wilber, namely "All Quadrant, All level." Such a model comprises of the exterior and interior realties which are both subjective and objective in nature. It integrates deep religion with broad science by showing that deep spirituality is in part a broad science of the farther reaches of human potential. It also integrates deep religion with narrow science, because even deep spiritual data and experiences can be carefully investigated and interpreted with narrow science. The richness of the integral vision is further fostered and nourished by the insights of J. Arthur Thomson, Fritjof Capra, Swami Vivekananda and Bhagavad-Gita. The article ends by portraying an integral approach to science and religion for a holistic understanding of reality.

Jeevan Mendonsa delineates some theological implications of new physics from the Catholic viewpoint. Introducing the special theory of relativity and quantum mechanics, he takes the readers into the mystery of the interrelatedness of the fundamental reality of the universe and shows how the new physics offers a new understanding of reality through cosmic interrelatedness, an understanding which is holistic and not anthropocentric. The insights of contemporary science especially the new physics challenges the implicitly held absolutist and rigid understanding of theological dogmas and invites us to redefine the dogmatic categories and terminologies. It also offers scope for reflection on some of the basic Catholic beliefs like the immanence and transcendence of God, the natures of Jesus Christ and the concept of transubstantiation. The article serves as a platform that offers immense possibilities to open further interesting avenues of dialogue between science and religion and also calls us to be ever open to the fascinating and mysterious ways in which God interacts with us and the universe

The belief that the God head consists of persons: the Father, the Son and the Holy Spirit, is a fundamental doctrine in Christianity. Carlos E. Puente introduces a mathematical construction having three united components that, in a special limiting case, helps us visualize key attributes of the Blessed Trinity: the Father in heaven via a loving and light conducting bell curve concentrated at infinity, the Son in a uniform and serene histogram that satisfies the adagium "cut the mountains and fill the valleys," and the Holy Spirit in a space-filling transformation built by mid-point additions of unity that joins the Father and the Son and proceeds from both of them. This work explains how the scientific construct and contrary notions regarding power laws in natural and man-made complexity provides a general framework that allows us to appreciate the essential options we all face in our lives regarding order and disorder and shows how the ideas invite us to find order, peace and love, but only in the artful limit. The specific connections of the ideas with a variety of Biblical citations are emphasized.

Prof. Y.V. Satyanarayana explores human attitude to nature specifically from Hindu and Christian perspectives. He begins with portraying three different attitudes in relating man and nature such as manunder-nature attitude, man-over-nature attitude and man-with-nature attitude. Making a critical enquiry into the philosophical and religious reasons for the development of the first two kinds of attitudes, Satyanarayana proposes the third attitude ideal in which humans realise themselves as an integral part of nature, and the biotic community, which includes all animate and inanimate things of the ecological system. He acknowledges the richness of Hindu tradition that lays more emphasis on the unity between man and nature. According to Hinduism, humans are neither separate, nor independent of nature, but constitute a part and parcel of the natural world. The author provides a new paradigm for environmental ethics calling for a radical shift from a human centred anthropocentric morality to a nature centred non-anthropocentric morality.

Going against the dualistic approach, Dr. Joseph Marianus Kujur takes a middle path and identifies ways and means of bridging the gap between science and religion. Hence, the present paper explores the possibilities of dialogue and integration, first, in the areas of the worldviews of the modern science and tribal religion; and second, in the context of the tribal development thereby accompanying the marginalised masses to realise that another world is indeed possible through the intervention of the modern science and technology. While the discussion on cosmology, values, beliefs and practices will constitute the first domain; areas such as medicine, biotechnology, biology, environment, etc., will form the second. To this end, the case study of the Aeta tribe gives us some insights into the interrelations between the modern science and tribal spirituality. Based on this study the author shows that there is no dichotomy between modern science and tribal spirituality as has been conceptualised by the zealots in each of the two camps. Science and spirituality have the same function but different means.

The fact that there are always aspects of life and nature that can not be explained, certain kinds of questions that can not be answered implies there is an aspect of reality beyond all scientific inquiry. Neither science nor religion provides us with a complete or absolute description of reality. Hence both together may give us a more authentic glimpse of what reality is like.

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# On the Unitive Art of the Holy Trinity<sup>1</sup>

- Carlos E. Puente<sup>2</sup>

*Abstract: The belief that God is made of three distinct persons: the Father,* the Son and the Holy Spirit, is a fundamental doctrine in Christianity. This article introduces a mathematical construction having three united components that, in a special limiting case, helps us visualize key attributes of the Blessed Trinity: the Father in heaven via a loving and lightconducting bell curve concentrated at infinity, the Son in a uniform and serene histogram that satisfies the adagium "cut the mountains and fill the valleys," and the Holy Spirit in a space-filling transformation built by mid-point additions of unity that joins the Father and the Son and proceeds from both of them. This work explains how the scientific construct, and contrary notions regarding power laws in natural and man-made complexity, provides a general framework that allows us to appreciate the essential options we all face in our lives regarding order and disorder and shows how the ideas invite us to find order, peace, and love, but only in the artful limit. The specific connections of the ideas with a variety of Biblical citations are emphasized.

*Keywords:* Holy Trinity, Wholeness, Fragmentation, Turbulence, Peace, Love, Power Laws, Space-Filling Transformation, Gaussian Distribution.

## Introduction

As stated in the Catechism of the Catholic Church, "the mystery of the Most Holy Trinity is the central mystery of the Christian faith and Christian life."<sup>3</sup> As exemplified by the famous dialog between St. Augustine and a child at the beach, the one in which the Saint explains to the kid that he cannot empty the ocean into a small hole and then the child replies that Augustine, likewise, cannot comprehend with his own mind the mystery of the Holy Trinity, it is not possible to infer the triune God apart from divine revelation and our best efforts of understanding ultimately go beyond reason to enter into the realms of faith.

This article explores matters of order and disorder by studying the opposite concepts of unity and fragmentation, conduction and dissipation, and the infinite and the finite as they arise via a representation of complex natural patterns using the iteration of simple rules and in the modeling of complexity via power laws. As in previous efforts,<sup>4,5</sup> this work's premise is that we may learn from the ways of nature so that we, in our human undertakings, choose to avoid the cases associated with complexity in order to find peace. Arguing that science provides a suitable and impartial framework to reflect on peace, this work shows that there is indeed one and only one state, defined by a lovely limiting case that reflects key features and symbols of the Holy Trinity, in which we may achieve universally everlasting peace and love. It is explained, in a manner that supports the Christian faith and hence invites to abandonment to the divine providence, how the clearly superior limit is intimately related to fully heeding the teachings of Jesus Christ so that, by repenting of our sins and allowing the Holy Spirit to fully transform us, we may arrive to the Father

### The Iteration of Simple Rules

To set the stage, it is pertinent to introduce a simple game of chance.<sup>6</sup> To start, select a triangle and number the three vertices according to the sides of a die. Then, select the point marked by a little square in the middle of the line that joins vertices 1-2 and 3-4 and roll the die. Say that the outcome is a 5.

Now, mark the point that lies in the middle of the initial little square and the vertex in the triangle marked with the 5:



and keep repeating, moving to the middle of the vertices given by the outcomes of the die.

Clearly, this game is better played not by hand but on a computer, translating the dynamics via three simple *rules* that point toward the vertices  $(x_v, y_v)$ :

$$w_n(x, y) = \left(\frac{x + x_n}{2}, \frac{y + y_n}{2}\right), \qquad n = 1, 2, 3$$

After several *iterations* of such rules, there appears the celebrated *Sierpinski triangle*:



made of all the points in the big triangle not located on the white middle triangles, *ad infinitum*.

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As such a set is eventually found irrespective of the outcomes of the die, the type of die used either fair or not, and the location of the initial little square, the iterations game, known as the *chaos game*,<sup>7</sup> turns out to be an effective procedure for finding the *attractor* of the rules, one that, as in this case, does not depend on chance itself. The Sierpinski triangle is a prototypical *fractal* object whose pieces, when magnified, look similar to the entire set.<sup>8</sup> As such an object contains infinitely many straight lines but also infinitely many holes, its *fractal dimension* is a number between 1 and 2, namely  $\ln(3)/\ln(2) \approx 1.58$ .

#### Fractal Interpolating Functions, Mountains and Clouds

Replacing the rules to iterate gives rise to other interesting fractal attractors.<sup>9</sup> For instance, the two simple linear maps, with the *x*-component decoupled from the *y*-component,

$$w_1(x, y) = (x/2, x + d_1 \cdot y)$$
 (1)

$$w_2(x, y) = (x/2 + 1/2, 1 - x + d_2 \cdot y)$$
(2)

and iterated using coin tosses, result in attractors shaped as convoluted *wires* that pass by the three points  $\{(0,0), (1/2,1), (1,0)\}$ .

As shown for  $z = |d_1| = |d_2|$ , when z = 0.5 and z = 0.8:



while the shapes of such objects depend on the signs of the *vertical scalings*  $d_1$  and  $d_2$ , their fractal dimensions depend on the magnitude of such parameters.

As is seen for z = 0.5, these attractors are functions from x, in the horizontal, to y, in the vertical, and their curious shapes may be described as follows. When  $d_1$  is positive and  $d_2$  is negative, the + - case, the attractor looks like the boundary of a *mountain* that is higher on the left side. Likewise happens when  $d_1$  is negative and  $d_2$  is positive (not shown), for such a case simply gives a flipped version of the same mountain. When  $d_1$  and  $d_2$  are negative, the - - case, the wire also looks like the boundary of a *mountain*, but now such a profile is symmetric. Finally, when both  $d_1$  and  $d_2$  are positive, the + + case, the wire resembles the boundary of a symmetric *cloud*.

As illustrated by the additional ink required for z = 0.8 relative to those for z = 0.5, these attractors, whose fragments look similar to the whole under suitable magnification, fill distinct degrees of space within the plane and hence have fractal dimensions that range from 1 to 2.<sup>10</sup> When z < 0.5, the wires are finite in length and, as a consequence, their fractal dimension equals 1. But as the magnitude of the vertical scalings is increased beyond z = 0.5, the attractors become infinite and they progressively fill more space in the plane, yielding a fractal dimension of 1.68 for z = 0.8 and a limiting *plane-filling* dimension of 2, when tends to the value of 1.

As with the Sierpinski triangle, the attracting wires, known as *fractal interpolating functions* even when their fractal dimension is 1,<sup>11</sup> may also be described in simple terms and without chance. For the + + case, the resulting cloud (and also angel's wings) may be understood as follows. Join the three interpolating points {(0,0), (1/2,1), (1,0)} from left to right. Find two new points going *up z* from the middle of those lines. Join from left to right and get four new intermediate points going *up*  $z^2$ . Then, repeat the process, *ad infinitum*, in powers of *z*:



For the mountain profiles, there are similar constructions.<sup>12</sup> For the + - case, these are the defining steps. Join the three initial points from left to right. Find two new points going *up* and *down z* from the middle, respectively. Join from left to right and get four new intermediate points going *up*, *down*, and the mirror image *down*, *up* a quantity  $z^2$ . Then, repeat the calculations in powers of *z*, using the same sequence of ups and downs from the previous step and completing the sequence using the previous' step mirror image.

For the case, the attractor is found as follows. Join the three initial points from left to right. Find two new points going *down z* from the middle. Join from left to right and get four new intermediate points going all  $up z^2$ . Then, repeat the process in powers of *z* alternating the points' placements *down* and *up*.

#### **A Platonic Universe of Projections**

The chaos game paints the attracting wires point by point and, as it happened with the Sierpinski triangle, such sets appear irrespective of the type of coin guiding the iterations. However, usage of *fair* or *biased* coins results in distinct stable textures defined over the wire, as shown below for the + - case with z = 0.5:



When the coin used is fair, the attractor is filled evenly and such results in a *uniform* histogram over the wire. This happens because the *x*-components of the two rules  $w_1$  and  $w_2$  in equations (1) and (2), namely  $w_1(x) = x/2$  and  $w_2(x) = x/2 + 1/2$ , equally split the wire's domain [0,1], as they operate in a balanced fashion to the left and to the right of 1/2.

When the coin employed is biased, say when  $w_1$  is used 70% of the time and  $w_2$  the remaining 30%, a *multifractal* texture is generated over the wire.<sup>13</sup> This happens due to the precise structure of the rules in *x* that yields a progressively biased concentration of points to the left of the attractor via a *multiplicative cascade*:<sup>14</sup>



The peculiar nature of the textures over the wires may be further explored by defining their *projections* over the *x*- and *y*-coordinates.<sup>15</sup> This entails moving along such lines, identifying all points within the

wire that are perpendicular to a given location and adding the textures of all those points. Such an operation happens to be easily made, for the sought projections are simply histograms of the points generated via the chaos game.

The projections over x are easily foreseen. As the wires are functions from x to y and hence there is only one value of y for a given value of x, the histograms over x are either *uniform* or *multifractal*, depending on whether the coin used in the iterations is fair or biased.

The projections over y, however, are not easily anticipated. As the wires typically link several values of x with a single value of y, the histogram over y requires non-trivial additions of textures that depend on the specific local geometry of the wires. As such, the nature of these projections can only be ascertained by performing all calculations, as illustrated next.

When z = 0, the wire simply joins the three initial points by straight lines and the projected histograms over x and y, denoted as dx and dy, yield for fair and biased coins:



As is seen, a fair coin gives a uniform histogram in x that is *trans-formed* into another uniform histogram in y, and a biased coin provides a spiky multifractal histogram in x that is *transformed* into another histogram in y containing many *thorns*.<sup>16</sup> The projections in y reflect the fact that horizontal lines cross the wire precisely at two locations (except for the very top tip), yielding for dy the sum of two textures. For the uniform

case, such textures are equal, and hence a uniform histogram appears in y. For the multifractal case, such textures are different, and adding them gives in y a wrapped around sum of the multifractal in x.

For the + - case and when z is increased to 0.5, the chaos game generates the familiar mountain profile and the stable textures shown below:



As is seen, both fair and biased coins generate complex-looking projections that reflect the specific *ups* and *downs* of the wire and the precise nature of the histograms over x. While the uniform case yields a symmetric dy that represents the horizontal crossings of the mountain, the multifractal case results in a random-looking histogram dy that resembles a natural set, like rainfall, measured over a line, say in time or in space.<sup>17</sup>

As the wire represents a function from x into y, one may interpret the attractor as a *system* that transforms an *input dx* into an *output dy*. In such a spirit, the notions define a *Platonic* approach to natural complexity, one in which, paraphrasing Plato, "reality" (as defined via a set dy) is but a "shadow" (technically here a projection) of an object (the wire) subject to a suitable "illumination" (the histogram dx).<sup>18</sup> This notion is supported by the fact that multifractals and multiplicative cascades are relevant to study *turbulence*, hence ascribing to the projections over *y* a "physical" interpretation of "transformations of turbulence," as is the case in many natural phenomena. Seeking to understand complexity without invoking the notion of chance is also rather Platonic in a general romantic sense, but varying the parameters herein and also the points by which wires pass (and further extensions) indeed define a vast universe of interesting patterns represented via few relevant parameters.<sup>19</sup>

When *z* is increased to 0.8, the following wires and projections are found:





As is seen, the wire's increase in fractal dimension happens due to additional variations in ups and downs and such leads to projections over *y* that are increasingly smooth, even if the input is a spiky multifractal histogram.

When z is increased to 0.999, near the limiting value of 1, the diagrams become:



As is seen, the wire's further increase in fractal dimension towards 2 results in rather dense variations in ups and downs in the nearly spacefilling wire that now stretches vertically between -31.6 and 32.4, while maintaining the domain from 0 to 1. This gives, for both inputs, smooth projections over *y* that tend to *Gaussian* or *normal* distributions,

$$dy(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-(y-\mu)^2/2\sigma^2}$$

with distinct finite means  $\mu$  and with variances  $\sigma^2$  that grow to infinity as *z* tends to 1.<sup>20</sup>

This surprising result turns out to be *universal* as it is valid not only for the shown inputs, but also for arbitrary multifractal illuminations corresponding to any bias on the coin and also for any non-discrete histograms in *x*, including smooth densities over [0, 1] and arbitrary spiky textures over uncountable *dusts* as generated via multiplicative cascades containing holes.<sup>21</sup> This happens because all restrictions of the infinite wire over domains that are arbitrarily small (say between two consecutive inverse powers of 2) are also infinite space-filling wires yielding Gaussian densities with variances tending to infinity that, when arbitrarily weighed, give, extraordinarily, yet other Gaussian densities with variances tending to infinity. Remarkably, the limiting wire universally *filters* great many inputs into a standard *normal* distribution, subtracting the mean  $\mu$  and dividing by the standard deviation  $\sigma$ , and such happens only in the spacefilling case when the attractor becomes maximally infinite.<sup>22</sup>

This limiting result turns out to be particularly noteworthy as it establishes an unforeseen connection between space-filling fractal wires and the famous *bell curve*, leading to a new version of the celebrated central limit theorem of key relevance in science but in terms of sums of dependent (not independent) variables that reflect the geometry of the wire and that do not rely on the concept of chance.<sup>23</sup> The universality of the limit is also decidedly significant, as it demonstrates, in a physical setting, that it is possible to define a system that *exchanges* the *violence* and *dissipation* of heat embedded in turbulent multiplicative cascades into the gentle processes of *diffusion* and *conduction* of heat associated, via Fourier's law, with the *normal* distribution.<sup>24</sup>

# A Rather Special Space-Filling Limit

The mountain wire corresponding to the - - case turns out to yield not one but two oscillating bells. This happens as the cyclical downs and ups in powers of z defining such an attractor lock in the limit when z tends to 1, yielding two alternating means.<sup>25</sup>

The cloud wire corresponding to the + + case ends up being quite special:



As shown for z = 0.99, with a scale in y that in reality is about 57 times larger than the one in x and with the scales in dy not reflecting the equal

histogram areas for aesthetic reasons, when *z* tends to 1, there appears over *y* a bell concentrated at *infinity*.

Calculations for the uniform case show that the mean and variance for the ++ case are  $\mu = 1/2(1-z)$  and  $\sigma^2 = 1/12(1-z^2)$ , which both tend to infinity as z tends to 1. However, as the coefficient of variation, that is,  $\sigma/\mu$ , contains the factor  $(1-z)^{1/2}$ , such a quantity tends to zero, and the limiting projection over indeed exhibits no variation as it gives a rather *singular* and special bell that coalesce all the mass into a single point, at infinity.<sup>26</sup>

This peculiar result turns out to be *universal* for any non-discrete input dx. This happens because arbitrarily small restrictions of the limiting wire, as explained for the + - case, give infinite space-filling wires yielding Gaussian densities with means and variances tending to infinity and coefficients of variation tending to zero, which when arbitrarily weighed have no other place to go but to infinity, even if the uniform case is more efficient and multifractal cases result in larger variations prior to the limit, as when z = 0.99.

Vastly exceeding the + - case, the limiting cloud, when z = 0.999..., provides also a rather special system, one capable of *transmuting* violence into diffusion and dissipation into conduction, one that, by concentrating it all, cancels the *entropy* of any input into a united whole at infinity.<sup>27</sup> What else but **love** could exchange any type of non-discrete disorder into the incomparable and unitive sound of freedom and the exquisite heavenly joy of cloud *nine*?

Quite extraordinarily, the *ever-positive* cloud built by mid-point additions of powers of z tending to 1, that is, eventual additions of 1 everywhere, is a mighty transformation capable of moving the entire "sea" of any non-discrete input dx into a single point at infinity, and such happens in a way that reminds us of the story of St. Augustine and the child at the beach and invites us to reflect, with due humility, in the mystery of the Blessed Trinity.

### A Representation of the Holy Trinity

As hinted from the previous section, the maximally infinite and limiting + + wire driven by a uniform histogram and giving a normal density concentrated at infinity:



provides a truly exceptional and unitive diagram, which, in its three fully interrelated components generated all at once, allows us to ponder about the ineffable nature of the triune God, as follows.

In the ever-conducting bell at infinity we may appreciate the unfailing and pure light of the *Father* enthroned in heaven; in the ever balanced and straight input we may visualize the sinless and perfect *Son*; and in the ever positive transformation, also shaped as angel's wings, we may contemplate the remarkably sacred bond of the *Holy Spirit* who fully connects the Father and the Son and proceeds from both of them.

That the bell at infinity symbolizes God the Father, the one who categorically defined Himself as "*I am who am*" (Ex 3:14),<sup>28</sup> may be seen in the amazing and singular concentration of everything within Him and in His majestic placement all above (Ps 8:2, Mt 6:9–13). No doubt, this is consistent with Him being eternal and unchangeable in nature, and with Him being able to do what He wills (Ps 113:3), and, in particular, with Him being the mighty creator of all, as expressed in the book of Genesis.

As all mass and energy are focused in the always *conducting* bell, such a state contains no entropy whatsoever, and hence it may be appreciated that God is *light* without any darkness (1 Jn 1:5). The Father's power is, no doubt, truly miraculous and His connection with light and clouds may be seen in various Biblical passages. For instance, He spoke to Moses via a *burning bush* not being consumed (Ex 3:1–6), guided the Israelites from Egypt via a column of *fire* to give them *light* (Ex 13:21), appeared as a column of *cloud* when meeting Moses at the tent (Ex 33:9–10), and was present at the dedication of the temple by Solomon, even if the *cloud* was dark (1 Kgs 8:10–12).

God the Father may be associated with the *normal* process of *diffusion* related to the *bell* rather than with the *violent* process of *turbulence* that produces dissipation and death.<sup>29</sup> This is seen by realizing that "the Lord is a merciful and gracious God, slow to anger and rich in kindness and fidelity" (Ex 34:6) as He rejoices with the conversion of sinners and has no pleasure on their death (Ez 18:23,32). This is also consistent with His nature as He "diffuses" His invitation to reconciliation and love freely (Jr 3:14), rather than by force.

In the rather gracious bell of *freedom* at infinity we may also appreciate that God Our Father is fully filled with *love* (1 Jn 4:8). That this is the case may be seen from Jesus' explanations about our ultimate central limit in loving God with *all* we have --heart, understanding and strength-- and in loving others as ourselves (Mk 12:29–31). For, only in the exquisite accumulation of such "space-filling" actions, we may *conduct* the flame of God that allows us to experience "the glorious freedom of the children of God" (Rom 8:19–21).

That the uniform condition may be associated with Jesus Christ may be further appreciated realizing that multiplicative cascades growing imbalances or propagating holes:



may be used, even if combining them and adding "random" variations from time to time, to express our increased separation from the *straight* law of God, a situation that grows symbolic *thorns* over *dust* and leads us, by our disobedience, to our *dissipation* and *darkness*.<sup>30</sup>

If we heed the geometric exhortations by the prophets Isaiah and John the Baptist to "cut the mountains and fill the valleys" and to make the roads *straight* and *smooth* (Lk 3:5–6, Is 40:4–5), we find, in the unique solution of *equilibrium*, God's glory and salvation, that is, God the Son sent from heaven in the *uniform* histogram.<sup>31</sup>

This is consistent, as Holy Scripture tells us that Jesus did not sin (2 Cor 5:21) as He came not to abolish the *law* or the *prophets* but to fulfill them (Mt 5:17). The Bible also tells us that He said "come to me, all you who labor and are burdened, and I will give you *rest*" (Mt 11:28) and such may be achieved only in the most efficient state of *uniformity*, for He further explained that "whoever is not with me is against me, and whoever does not *gather* with me *scatters*" (Mt 12:30), as it happens when a multiplicative cascade destroys unity due to its lack of love.<sup>32</sup>

That the limiting and rather artful cloud wire (or angel's wings) may be related with the Holy Spirit may be seen in the particularly fascinating way by which infinite additions of *positive unity* beautifully unite the components associated with the Father and the Son in a manner that exchanges the concentrated energy above. For, as explained before, in the space-filling attractor we may recognize what we experience when *love* fully fills our hearts, and for, in the incredible transformation capable of raising any non-discrete input into the bell at infinity, or similarly, capable of exchanging bitterness into peace (Is 38:17), we may certainly appreciate *divine mercy* and *ultimate power*.

How not to visualize in the perfect communion of the triune diagram the words of the apostle Paul when he said, "Christ Jesus himself is the capstone and through Him the whole structure is held together and grows into a temple sacred in the Lord, in Him you also are being built together into a dwelling place of God in the Spirit"? (Eph 2:20–22). How not to recognize in the sublime sketch God's ultimate gift of love for us, in His Son coming down from heaven, from *y* into *x*, so that we, by *believing* in Him, including his death on a *positive* cross and his resurrection, may have *eternal life* with the Father, from *x* into *y*? (Jn 3:16).

For as we may see in the artful diagram, "the law," dx, "has only a *shadow* of the good things to come, and not the very image of them" (Heb 10:1), as the new covenant in Jesus' blood (Lk 22:20) takes us, by *faith* in Him, into the glorious and eternal bell, dy, that represents His *perfect* law of *freedom* (Jas 1:22–25).

## On the Life of Jesus Christ

The beautiful Trinitarian diagram, repeated here one more time:



allows us to contemplate key events in the life of Jesus.

To begin, and in the direction from *y* into *x*, we may observe the nature of Jesus' *birth*, for the Holy Spirit overshadowed Mary and hence made Him holy (Lk 1:34–35). Within the same direction we may appreciate the source of Jesus' power that enabled him to fulfill the law, perform his many *miracles* and, in particular, baptize with *Spirit* and *fire* (Mt 3:11).

In the junction of the two directions, from below and from above, we may visualize the ultimate and characteristic *unity* of Jesus and the Father (Jn 14:11), and in particular, the awesome event of Jesus' *trans-figuration* that so graphically links Him to the absolute purity of the light at infinity (Lk 9:28–36). This was a truly extraordinary event that consistently included a *cloud* that overshadowed Jesus, Moses, Elijah and the scared disciples, and that, as in Jesus' *baptism* by John the Baptist, prominently contained the voice of God confirming that Jesus was His chosen Son.

In the direction from x into y we may clearly grasp Jesus' *resurrection from death* (Lk 24:5–6), even if, as shall be shown later on, such is perhaps better visualized via a dissipative multifractal input rather than the uniform input. In any event, it is clear that the triune

diagram in its upward movement may raise almost anything into infinity, and of course that explains, again in a rather pictorial fashion, Jesus' *ascension into heaven* (Lk 24:50–51), the ascension of the prophet Elijah (2 Kgs 2:11), the assumption of Mary as affirmed by the Catholic Church,<sup>33</sup> and the future *rapture* of the Church (1 Thes 4:16–17).

Finally, and in the power of the reverse direction from y into x, we may visualize why Jesus' disciples have power to perform miracles (Mk 16:17–18), why they may indeed loose on earth what shall be loosed in heaven (Mt 16:19), and why through them, by the same power of the Holy Spirit that laid tongues of symbolic *fire* to the disciples (Acts 2:3), there happens the ineffable miracle of the transubstantiation during Holy Mass by which Jesus gives us *life* (Mt 26:26–28).

How beautiful it is to realize that a little piece of the ultimate attractor, just a little piece of the *Holy Eucharist*, often accompanied during the consecration by the sound of a *bell*, contains indeed all the infinite power of the whole attractor, as a little piece of love is just pure love! How lovely it is to recognize that the limiting wire may be split into a myriad of components that maintain the same majesty! In this same spirit of awe, and as all pieces of the attractor make up the wings of angels, we may foresee in the triune diagram (no doubt with due imagination) the return of Jesus in the company of angels, also from y into x, at the time He will justly separate the sheep from the goats (Mt 25:31–46).

### On Faith, the Spirit and Salvation

As the generic multiplicative cascade reflects natural *turbulence* universally, both in the precise partitioning of *eddies* into *eddies* according to the 70-30 energy split and also in the eventual *dissipation* of all energy:<sup>34</sup>

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and as such a process may be used to represent our increasing separation from God, that is, the growth of our *negative spirals* leading to *violent thorns* disperse as *dust* when we pay attention to the *ruler of the power of the air* (Eph 2:2),<sup>35</sup> who is the Devil, we realize that the limiting diagram based on a multifractal histogram and the limiting positive cloud:



may also be relevant to reflect on matters of salvation and faith.

As the wage of *sin* is *death* (Rom 8:13), and as the ultimate + + wire is capable of transforming disperse *thorns* over *dust* into the bell at infinity, we may realize via this diagram how *death* may be exchanged into *life* and *peace* (Rom 8:6), and how *mourning* may be converted into *joy* (Jer 31:13), in a matter that makes us exclaim with the apostle Paul, "Where, O death, is your victory? Where, O death, is your sting?" (1 Cor 15:55).

Clearly, the key for victory is in the ever positive attractor that Jesus unveiled, the Holy Spirit (Jn 14:16–18, 1 Jn 4:9), for via the same diagram we may appreciate why there is no condemnation for those who

are in Christ Jesus and His *Spirit* (Rom 8:1-2) and why if we *believe* that Jesus died and rose, so too will God, through Jesus, bring us back as well (1 Thes 4:14). For as further explained in Holy Scripture, if we live according to the *flesh*, we will *die*, but if by the *Spirit* we put to death the deeds of the body, we will *live* (Rom 8:13, Jn 6:63), and this can be seen realizing that the flesh and the Spirit are opposed to each other (Gal 5:17), as a divisive multifractal *dx* representing our sin is *perpendicular* to the bell at infinity *dy*.

*Salvation* is, no doubt, an ineffable gift that happens by *grace* and through *faith* (Eph 2:8–9) and, as such, it is very soothing to know that "all things work for good for those who *love* God, who are called according to his purpose" (Rom 8:28) and that "all the paths of the Lord are faithful *love* toward those who honor the covenant demands" (Ps 25:10). These citations are soothing indeed and quite reassuring for, as reflected in the limiting diagram based on a multifractal input, the heavenly destination may be achieved in the fullness of *faith* and the *Spirit*, even when we are not completely attuned with the perfectly uniform redeemer. For quite graphically, love indeed covers a multitude of sins (1 Pt 4:8).

The notions herein also reminds us of Jesus' mysterious words when He said, "Amen, I say to you, all sins and all blasphemies that people utter will be forgiven them; but whoever blasphemes against the *Holy Spirit* will never have forgiveness, but is guilty of an everlasting sin" (Mk 3:28– 29), for a *finite* response to *faith*, either via a discrete *dx* or a non-limiting positive wire that does not reach infinity, would certainly miss the heavenly destination. This is clearly very sad, but the choice of faith is evidently between the *infinite* and the *finite*, as reflected by the words of Jesus when He said, surely reflecting on His union with God the Father, "if you do not believe that I AM, you will die on your sins" (Jn 8:24).

How not to contemplate the ever positive and unitive cloud and implore, "O Lord increase our *faith* (Lk 17:5), turn away your face from our sins, blot out all our guilt, a clean heart for us, God, renew in us a *steadfast* Spirit"? (Ps 51:11–12), or, as it is said in the rite of peace during the Catholic liturgy, "Lord Jesus Christ, look not on our *sins*, but on the

*faith* of your Church, and grant us the peace and unity of your kingdom where you live forever and ever"?

As the symbol +, representing the *cross*, permeates the construction of the artful diagram and as Jesus' *resurrection*, from a dissipated multifractal into the uniform condition and later on to the Father at infinity, is also potently reflected in the limiting diagram, we may see pictorially that only through Jesus we may achieve salvation. This is, of course, consistent with a host of Biblical citations, but in particular with the fact that, by descending into a dissipative multifractal state and picking up our due chastisement (Is 53:5), He rescued us from our slavery of sin (Rom 6:6).

Of course, the notions in this section do not imply that sinfulness combined with faith should be our response, for Jesus clearly calls us to the whole triune diagram when He commands us to be *perfect* as the Father in heaven is *perfect* (Mt 5:48). That this is the case may also be seen in Jesus' reminders "beware that your hearts do not become drowsy from carousing and drunkenness and the *anxieties* of daily life" (Lk 21:34) and "do not worry about tomorrow, tomorrow will take care of itself, sufficient for *a day* it its own evil" (Mt 6:34), that call us to practice faith and deeds (Jas 1:22–25), leading to the fulfillment of the associated central limit theorem "adding" a loving and virtuous (dependent) life that reflects our ultimate *freedom*. For in the very spirit of the diagrams based on the limiting cloud, we ought to listen to the words of the apostle Paul when he explained "do not conform yourself to this age but be *transformed* by the renewal of your mind, that you may discern what is the will of God" (Rom 12:2, Eph 4:22–24).

### **Clouds vs. Mountains**

As the space-filling wires of the + + and + - cases define limiting bells that have either *infinite* or *finite* means:



such diagrams may be used to speak about our "spiritual moods" and our possible ultimate destinations.

As described in the book of Genesis, "In the beginning, when God created heaven and earth, the earth was a formless wasteland, and darkness covered the abyss, while a mighty *wind* swept over the waters" (Gn 1:1–2). Then, it is sensible to portray the initial creation event, and that of other components, such as light (Gn 1:3), the sky (Gn 1:7), vegetation (Gn 1:11), the stars (Gn 1:16), animals (Gn 1:20), and man (Gn 1:26), as transitions from *dissipated* "multi-fractal" states into other far more *organized* states, via suitable and rather artful *space-filling* wires reflecting God's mighty infinite power. In this spirit, it is also sensible to suggest that while the *finite* + - and - - cases for all parameters and the *finite* + + case for all parameters before the limit may be used to denote the "*creation of the perishable*," the limiting + + case may be employed to represent the "*creation of the eternal*."

Given that "God created man in his image, in the *divine* image He created him" (Gn 1:27), our appearance and destiny is undeniably associated with the limiting singular bell, and, as such, we are *normally* drawn to the awesome triune diagram and the infinite. Unfortunately, however, when we pay attention to the whispers of the enemy, the Devil (Rv 12:9), we experience far from optimal spiritual states that in their intrinsic *nega*-*tivity* and lack of due *faith* generate the obstacles we face in "mountains" (Zec 4:6–7) or in somber "clouds." These are rather sad states that potently remind us, in the associated *ups* and *downs* of our mood and in the

inefficiency of our yielding to God, why "the one who *doubts* is like a *wave* of the sea that is driven and tossed away by the *wind*" (Jas 1:6), why "unless our *faith* is *firm*, we shall not be *firm*" (Is 7:9), and why we ought to always return to our *positive* and *unitive* essence so that our hearts may be fully filled up. For, even if it may appear otherwise, the negation of the + + cloud is not another cloud upside down, but rather a totally defeated finite condition, that may or may not converge at all as in the - - case, and such states, by missing the heavenly invitation, must be avoided at all costs.<sup>36</sup>

Based on the impartial vantage point of science, we may see that our best decision is to yield to the inimitable love of *heaven*, for it is indeed tragic not to reach the ultimate loving state. These notions are reaffirmed in a variety of Biblical quotations that contrast the infinite and the finite in rather graphical ways, such as "the man who strays from the way of good sense, who will abide in the assembly of the *shades*" (Prv 21:16), in "*shades* that cannot rise" (Is 26:14) vs. "your dead shall live, their corpses *shall rise*, awake and sing, you who lie in the *dust*, for your dew is a dew of *light*, and the land of shades gives *birth*" (Is 26:19).

How not to share in the ultimate ++ diagram the love of the Psalmist when he exclaims, "Lord, your love reaches to *heaven*, your fidelity, to the *clouds*" (Ps 36:6), "how precious is your love, O God!, we take refuge in the *shadow of your wings*" (Ps 36:8), in the same angel wings leading to the limiting bell! For the only way to move away from the deceptive mountains is to *cross* the negative to make it positive and to fully increase the faith, so that we may fulfill Jesus' words when He said, "Amen, I say to you, if you have *faith* and *do not waver*, not only will you do what has been done to the fig tree,<sup>37</sup> but even if you say to this *mountain*, 'be lifted up and thrown into the sea,' it will be done, whatever you ask for in *prayer* with *faith*, you will receive" (Mt 21:21–22). For as is appreciated in the fullness of faith and dimension, the Devil, who forever eats the *dust* appointed to his falseness (Gn 3:14), can only strike at our heels while we bruise him at his head (Gn 3:15).

#### **Triune Numerical Representations**

The beautiful Trinitarian diagram, seen here yet one more time:



contains the numbers 0, 1 and  $\infty$ , and also the three famous irrational numbers  $\pi$ ,  $\sqrt{2}$  and *e* included in the formula of the bell, as follows.

First 0, 1 and  $\infty$ . Such happen in the bell at *infinity*, as all the mass there is concentrated in one *infinite* spike exhibiting *zero* variation and *zero* entropy. These three numbers are found in the uniform histogram, as such a portion of the diagram ought to be interpreted not as a static entity but rather as a dynamic representation of what Jesus fulfilled and fulfills, that is, *zero* sinfulness while maintaining *unity* in the *uniform* histogram and with the Father *forever*. The three numbers also happen on the limiting + + cloud, as such is built by *infinitely* many midpoint additions of *unity*, giving an object that becomes concentrated at infinity with all likelihood, that is, with points excluded having a probability of *zero*. Altogether, the *united* diagram represents a remarkable symphony of *unity*, which, no doubt, points us to reflect on the sacred.

Now the three famous irrational numbers, which turn out to symbolize the members of the Holy Trinity, are as follows. If the uniform

condition, denoting the perfection of Jesus' actions, is accumulated from left to right, the amount of mass up to a given location always equals such a location, and a graph of such an operation gives a *straight* ramp:<sup>38</sup>



that further represents the call to straightness and conversion that Jesus always fulfilled (Jn 1:23).

As the resulting *hypotenuse*,<sup>39</sup> with the symbolically perfect equation X = Y showing the cross and the silhouette of Jesus crucified on it,<sup>40</sup> represents the most efficient state of love, and as its length equals $\sqrt{2}$ , in virtue of the Pythagorean theorem, we may see how such a number symbolizes God the Son, as He is the *Messiah* (Jn 1:41) who is the precious *root* to *peace*.<sup>41</sup>

As seen on the same diagram, when one "lands" on the hypotenuse, one slides into the point (0,0) which is the *origin*.<sup>42</sup> Given that such an operation symbolically fulfills Jesus' words when He said, "I am the way and the truth and the life and no one comes to the Father except through me" (Jn 14:6), we may see God the Father in the origin and we may appreciate why  $\pi$ , the most celebrated number in the history of mankind, in the shape of a *zero*, symbolizes the one who is the *Origin* of all, as reflected by the encompassing bell at infinity having *zero* variation and representing the one who is (Ex 3:14).

The other irrational number e turns out to symbolize God the Holy Spirit. This may be seen geometrically in the outer and hence *loving* movement of the letter e that opposes the inner movement of the negative spiral present in the selfish eddies shaped as a 6,<sup>43</sup> and also in the fact that when we translate Jesus' command for us to love one another (Jn 13:34–35) including our *enemies* (Mt 5:44) into the language of *calculus* as "integration without differentiation," we find the positive *exponential* function as the unique solution:

$$\int e^x dx = \frac{de^x}{dx} = e^x.$$

This is quite lovely and perhaps abrupt, but there is more. If we consider the specific words of Jesus in His famous discourse about the *vine* and the *branches* (Jn 15:1–10) and translate it mathematically, along the way we find additional hints of the *Spirit*, as follows.

At the beginning of the passage Jesus says, "*I am the true vine*, and *my Father is the vine grower*. He takes away every branch in me that does not bear fruit, and everyone that does he *prunes* so that it bears more *fruit*. You are already pruned because of the word that I spoke to you. *Remain in me, as I remain in you*. Just as a branch cannot bear fruit on its own unless it remains on the vine, so neither can you unless you remain in me" (Jn 15:4). Then, the key elements may be summarized as

$$1 + 1/x$$

where the first 1 denotes Him, the *one* and only who did not sin (1 Pt 2:22), the *one* who died for our transgressions (1 Pt 2:24), and also the *only* divine son of God (Jn 1:18); the quotient 1/x represents *us*, the disciples, as the branches picking up our *crosses x* (Mk 8:34–35) and being pruned to the *zero* so that we may find the *origin*; and the + sign symbolizes our required union with Jesus as the vine.

In the remainder of the passage Jesus concludes, "*I am the vine*, you are the branches. Whoever remains in me and I in him will bear much fruit, because without me you can do nothing. Anyone who does not remain in me will be thrown out like a branch and wither; people will gather them and throw them into a *fire* and they will be burned. *If you remain in me and my words remain in you, ask for whatever you want and it will be done for you.* By this is my Father glorified, that you bear *much fruit* and become my disciples. As the Father loves me, so I also love you. *Remain in my love. If you keep my commandments, you will remain in my love*, just as I have kept my Father's commandments and remain in his love" (Jn 15:5–10). Then the whole passage may be further translated as

$$(1+1/x)^x$$

where the exponent reflects the disciple's increased *power* in asking for anything in His name.

As in the limit, when *x* grows to *infinity*, such an expression gives precisely the exponential number *e*, it may be concluded that such symbolizes our ultimate gift of *consciousness*, the *Holy Spirit*, which empowers us to love everyone as God does.

These curious associations between the members of the Holy Trinity, the Father, the Son and the Holy Spirit, and the three most famous irrational numbers  $\pi$ ,  $\sqrt{2}$  and *e* related to the geometry of circles, squares and spirals, certainly add to the reverence we ought to have before the triune God. This is due to the lovely explanations herein and the consistent facts that  $e \approx 2.71$  is located in the middle of the other two numbers and that  $\sqrt{2} \approx 1.41$  is located to the right of  $\pi \approx 3.14$  (as seen from within the real line) (Lk 22:69).

#### Wires in Higher Dimensions and their Projections

The notions leading to wires and projections may be extended to *higher dimensions*, iterating simple linear rules that maintain the *x*-component decoupled from the others,  $^{44,45}$ 

$$w_1(x, y, z) = (x/2, x + d_1y + h_1z, x + l_1y + m_1z)$$
  

$$w_2(x, y, z) = (x/2 + 1/2, 1 - x + d_2y + h_2z, 1 - x + l_2y + m_2z)$$

and with the larger set of new parameters expressed in polar coordinates,

$$D_{n} = \begin{pmatrix} d_{n} & h_{n} \\ l_{n} & m_{n} \end{pmatrix} = \begin{pmatrix} r_{n}^{(1)} \cdot \cos \theta_{n}^{(1)} & -r_{n}^{(2)} \cdot \sin \theta_{n}^{(2)} \\ r_{n}^{(1)} \cdot \sin \theta_{n}^{(1)} & r_{n}^{(2)} \cdot \cos \theta_{n}^{(2)} \end{pmatrix}, \qquad n = 1, 2$$

As before, using the two rules guided by coin tosses generates *wires*, now defined from *x* into the plane (*y*, *z*), that, while interpolating the points  $\{(0,0,0), (1/2,1,1), (1,0,0)\}$  turn out to have *fractal dimensions* that now range from 1 to 3.

Based on such new wires and on either uniform or multifractal illuminations, histograms over the plane (y, z) may be obtained that generalize the *Platonic* notions about *natural complexity* explained earlier:<sup>46</sup>



As varying the parameters results in a host of interesting complex projections whose peculiar shapes encompass typical natural spatial patterns, these generalized notions further suggest that what appears complex and random in nature may perhaps be understood, in a Platonic way, as a projection that ultimately does not depend on the concept of chance.<sup>47</sup>

In the limit, when the sixteen sign combinations on the new *scaling* parameters  $r_n^{(j)}$  have magnitudes that tend to 1 and when the angles on

each rule are synchronized such that  $\theta_n^{(1)} = \theta_n^{(2)} + k_n \pi$  for an integer  $k_n$ , the resulting wires fill up three-dimensional space and such results in projections that generalize what was reported earlier, as follows.<sup>48</sup>

Analogous to the + - case, there are cases that define *circular*, and to a lesser extent *elliptical*, bells with *finite* means and with variances that grow to infinity, as the magnitude of the scalings tends to 1. As before, such bells happen universally for arbitrary non-discrete inputs. An example of such a behavior is the - + + + case, with parameters  $-r_1^{(1)} = r_1^{(2)} = r_2^{(1)} = r_2^{(2)} = 0.9995$ , and all angles equal to 60 degrees, that gives:<sup>49</sup>



where x vs. y and x vs. z portray the wire, dx denotes a multifractal input, dy and dz are the outputs as seen from the y and z directions, and dyz is the projection over the entire (y, z) plane, as seen from above.

Analogous to the - - case, there exist *oscillatory* limits when scalings are either all *positive* or all *negative* and when the angles are such that they are all equal. In those cases, oscillations amongs $u = 2\pi/\theta$  bells are found and the corresponding means strikingly draw either regular polygons or stars inscribed on a circle.<sup>50</sup>

Finally, and analogous to the + + case, although not with all the new scalings positive, there are cases that yield bells that *concentrate* their masses at "infinity," at the extremes of rays emanating at the origin and with orientations that depend on the angles  $\theta_n^{(j)}$ . These instances pro-

vide other remarkable singular representations of the Holy Trinity that ultimately hold an arbitrary non-discrete "ocean," uniform or not, in a single point at infinity.

### **Exotic Beauty in Two-Dimensional Bells**

As the histograms of the chaos game summarize the iteration of the rules many times, say ten million dots, it becomes relevant to study how the circles or ellipses of bells having finite means are formed, say in successive groups of 20,000 points.

As such, iterating the same two rules giving the circular bell just defined via the - + + + case, but adding one more 9 in the scalings' magnitude so that it is 0.99995, yields a succession of patterns (from top to bottom and from left to right) that surprisingly define exquisite decompositions of the limiting bell:<sup>51,52</sup>

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These beautiful images are just examples of the infinitely many patterns that remarkably form, by superimposition, perfect circles and two-dimensional bells. The specific shapes of such sets turn out to be intimately related to the precise sequence of iterations used and their overall symmetries depend on the angles  $\theta_n^{(j)}$  and on the signs of the scalings  $r_n^{(j)}$ . In the implied central limit of this lovely chaos game, and for any type of coin, there is *hidden order* in chance, and such expresses vital *beauty*, as from "glory to glory" (2 Cor 3:18), even if the bell has finite mean and the patterns are hence perishable.<sup>53</sup>

Just by playing with the simple rules, it may be shown that the geometric structures of nature's *ice crystals* are found inside two-dimensional bells as mathematical *designs*:



These examples were found filling up templates of known ice crystals step by step, using two suitable rules having angles equal to 60 degrees and building sequences of iterations based on groups defined via fair coins until the patterns arise.<sup>54</sup>

As it may be imagined by now, the bell contains, just by varying the angles, a host of rosettes that resemble, among others, the two-dimensional geometric structure of biochemical compounds.

Strikingly, the iteration of two suitable maps with angles equal to 36 degrees and guided by the first 40,000 bits in the binary expansion of  $\pi$ , generates, in the second set made of 20,000 dots, a rosette that very improbably resembles the one of life's DNA, with the actual pattern shown on the left and the bell design on the right:<sup>55</sup>



This is, of course, a rather suggestive result, especially given the symbolic relation between  $\pi$  and God the Father explained earlier, but it ought to be recognized that the patterns herein are just inanimate images that, on the other hand, our mighty God can bring to life.

As a single space-filling infinite wire may encode infinitely many rosettes that are made explicit via binary expansions of a host of irrational numbers and as such should be the case also in yet higher dimensions, we realize that the concepts of wires and projections boggle our minds and invite us to further praise the majesty of God.

For how not to admire the beauty and variety of patterns inside circular bells and exult, "how *precious* to me are your *designs*, O God, how *vast* the *sum* of them" (Ps 139:17), or just repeat with the Psalmist, "*great* are the works of the Lord, to be treasured for all their delights, *majestic* and *glorious* is your work, your wise *design* endures *forever*" (Ps 111:2–3).

#### **Bells vs. Power Laws**

As recent studies of social networks suggest that *friendship*, even if loosely defined, is nicely described by a Gaussian distribution,<sup>56</sup> but webs of sexual promiscuity do not follow the *normal* bell curve but are rather represented by so-called *power laws*,<sup>57</sup> it becomes relevant to study in some detail the latter in order to further appreciate the loving bell.

During the past few years it has been found the ubiquitous presence of power laws in a variety of processes associated with natural complexity.<sup>58</sup> Perhaps the most famous of them is the one that defines the Gutenberg-Richter scale for earthquakes:<sup>59</sup>



As may be noticed, such a graph exhibits, in its tail, a close to linear behavior on double-logarithmic scales, that reflects the *power-law* relation  $P[X \ge x] \sim x^{-c}$ , with the exponent  $c \approx 1$  As may be expected, the events of a lower magnitude have a larger exceeding probability and those of larger magnitude have a lower exceeding probability, but the transition between scales happens on a "heavy tail" without a notorious peak, hence indicating that there is no typical earthquake having a "characteristic" size.

Power laws are also encountered on other manifestations of natural *violence* such as floods, avalanches, volcanic eruptions and forest fires.<sup>60</sup> Although the exponent c is not equal for all processes, the fact that the distributions align in particularly simple ways expresses the presence of *universal* traits in such phenomena, as in geometric fractal sets,<sup>61,62</sup> and hints that such processes are, at the end, not as complex as they may possibly be.

In addition to natural fury, power laws also appear prominently in various situations associated with human behavior. As first observed by V. Pareto, such laws are found in the biased income and wealth inequalities in the world, within each one of the nations and also in the world at

large. Such notions are illustrated below in the painful widening of world income distributions from 1960 to 1997, in two "lines" with negative slopes in double-logarithmic scales and corresponding to the percentiles 30<sup>th</sup> to 85<sup>th</sup>.<sup>63</sup>



Another fundamental example of power laws pertains to the distribution of human conflicts and war. As first noted by L. F. Richardson, the distribution of the number of people killed in a conflict (the severity of the event), from those who die due to gang activity all the way to the world wars, align in an approximate law in a double-logarithmic plot:<sup>64</sup>



Surprisingly, or perhaps not, mankind's perverse *violence* in wars and inequalities is fitted by rather simple "lines" in double-logarithmic scales, and such has relevant implications, as follows. If we like to assure that a big conflagration will not happen, there is no solution but to abolish the power law, not only at the large scales, but in particular at the smaller scales from which such emanates. If we like to live in a just word, one that does not contain the rather symbolic *two thirds* of its citizens, 2/3 = 0.666..., in poverty,<sup>65</sup> and one that does not reflect the negativity of our evil degrees of separation,<sup>66</sup> there is no solution but to abolish all power laws reflecting the greed that reflect the love of money as the root of all evil (1 Tm 6:10). For even though it appears that there are better nations than others not only in their gross domestic products but also in their wealth distributions, the latter may be fitted via multiplicative cascades that capture their equally inefficient states relative to the most economic *uniform* histogram.<sup>67</sup>

There is no other solution. If we like to achieve the desired condition of *peace* in the world, we need to stop measuring human beings by finite numbers and we need to abolish power laws.

As recently it has been found that power laws happen due to a handful of mechanisms,<sup>68</sup> it hence becomes sensible to learn about them to see how we may, by avoiding them, achieve a better world. Power laws appear via *multiplicative cascades* leading to multifractal histograms; via *preferential connections* in networks; via the so-called *highly-optimized tolerance*, *hot*, a procedure that uses a global principle to determine its outputs; and, prominently, via the celebrated *self-organized criticality* that, as an example, grows a sand pile until its excessive and critical slope gives rise to predictable avalanches of unpredictable sizes but yielding a power-law distribution:<sup>69</sup>



As may be hinted, to construct *love* we ought to counter such mechanisms. This means *rectifying* the biased histograms via "cutting the mountains and filling the valleys" and establishing the *uniform* condition (Lk 3:5); *leveling* the playing field defining a truly inclusive "network" that reflects our impartial love (Jas 5:8–9); exchanging the global optimization notions in *hot* by *local optimization* reflecting love to those around us (Mt 25:35–36); and avoiding energy accumulations that clearly destroy unity (Mt 16:24), even if we play unloving and selfish "games" at the beach and even if we do not recognize that the "invisible hand" there is our own.

Clearly, all these ideas point us to the non-preferential organization of love and, hence, to the unique and positive *exponential* function as the solution we all seek. Given that the *natural* logarithmic function:

$$\ln(x) = \int \frac{1}{x} dx,$$

is the *inverse* of the exponential and as such has a *negative* connotation in our inability to do anything by ourselves, as expressed in Jesus' discourse about the *vine* and the *branches* (Jn 15:1–10), we may observe, in an ingenious manner, how the *minimization of energy* always present in our untamed *selfishness*, shaped as the *negative* spiral 6 of natural *eddies*, generates power laws and how the inherent lack of union produced by violence is in fact a manifestation of *evil* that portrays three symbolic negations (Mk 14:66–72), in the two *logarithmic* scales and in the approximate line with a *negative* slope.

For the word *log* comes from cutting a number according to a base (in this case 10), but it also means to cut down trees to produce cylindrical logs that, quite accurately, express what we commonly have in our eyes that prevent us from helping one another (Mt 7:3–5). After all, all power laws herein help us appreciate what happens when we lose our intrinsic scale and abuse our powers. For, as may be further appreciated, such natural

laws with "heavy tails," despite their high "determination coefficients,"  $r^2$ , and here language is subtle indeed, turn out to be just false and incomplete illusions of straightness, as they represent grotesque "regressions" to dissipation and terror that in no way resemble the immaculate and real rectitude to which we are all called.<sup>70</sup>

These lessons based on power laws are certainly simple and they remind us that if we abandon our human consciousness we end up paying the just and *ubiquitous* consequences of our avalanches and cascades.<sup>71</sup> For in order to solve the problems we face, we ought to stop blaming others, including the Devil himself. These notions, perhaps ancient but certainly true, are pertinent in this day and age when we have come to accept the indifference of the divisive winds as something "normal," when we believe and are guided by the false dogmas of "competition," "domination," and the "survival of the fittest," and when the natural stresses and superfluous distractions in our busy modern lives have obscured the very reason of our existence in glorifying God (Ps 29:2). Aren't we playing with fire not paying attention to the tried metaphor, explicit in power laws, that conflagrations and economic inequalities do not know how big they will become before they start?

There is one and only one solution. We have no option but to earnestly become "childlike" so that by growing our true *determination* to unity, 1=0.999..., that is, by *maximizing* our energies, we all may become good friends and play together as brothers and sisters. For only in accepting the power of *powerlessness* we may fulfill a rather simple and beautiful universal equation:

$$\vec{i} = 0.999... = 1$$

that includes the three numbers 0, 1 and  $\infty$  ever-present in the lovely diagram of the Holy Trinity, and that add the positive spiral 9 as a reminder that in order to achieve *unity* we ought to *love* forever.<sup>72</sup>

### A Bit More on the Kingdom of Love

When Jesus died for all of us, there was an appointed eclipse of the sun (Am 8:9) that happened between noon and three in the afternoon, or, as it is recorded from the beginning of daylight, between the 6<sup>th</sup> and the 9<sup>th</sup> hours (Mk 15:33). *Darkness* indeed came into the whole land and based on the associations herein and also on previous works,<sup>73,74</sup> *darkness* indeed remains in us between the *negative* spirals that reflect our sinfulness and the positive ones that represent our love.

How soothing it is to realize that Jesus died out of *love* for us precisely at the 9th hour (Mt 27:46–50) and that the veil of the temple was thorn then precisely by the middle (Lk 23:45), for such are the conditions required to mend and maintain *equilibrium*.<sup>75</sup> How reassuring it is to contemplate the *united* and yet *triune* diagram just one more time:



and recognize in it, via the ever present and symbolic *ones* and the associated *powers of zero*, an absolutely fantastic symphony of *saintly love* that reflects the *positive* spiral associated with Jesus' life and His *cross*.

For in a rather consistent manner, "the *fruit* of the *Spirit* is love, joy, peace, patience, kindness, generosity, faithfulness, gentleness, self-control" (Gal 5:22–23), *nine* components; the *beatitudes* define *blessedness* to: the poor in spirit, those who mourn, the ones who are meek, those who hunger and thirst for righteousness, the ones that are the merciful, those clean of heart, the peacemakers, the ones who are persecuted for the sake of righteousness, and those insulted persecuted falsely because of Jesus (Mt 5:3–11), again *nine* elements; and we celebrate a *novena* to commemorate the days the disciples prayed between Jesus' ascension into heaven (Acts 1:2) and the coming of the Holy Spirit (Acts 2:1–4).

Knowing from experience that our faith encounters with the living God are beyond our understanding and fully realizing that the incompleteness of the reflections from science to faith presented here support the wisdom of St. Jerome when he affirmed that "the true profession of the mystery of the trinity is to own that we don't comprehend it," this work ends, however, with a poem-song that reiterates God's ever present invitation for us to **Love**.

#### **OH KINGDOM OF 9's**

Unity is made of many 9's...

Oh kingdom of 9's longing to arrive, oh plentiful love oh glorious might.

Oh kingdom of 9's yearning to revive, filling up space oh unitive art.

Oh fine convergence in beloved encounters, oh supreme coincidence in everlasting symbols.

Oh great convergence that heals all wounds, oh singular coincidence that embellishes life.

Oh kingdom of 9's longing to arrive, oh plentiful love oh glorious might.

Oh kingdom of 9's yearning to revive, filling up space oh unitive art. Oh fine convergence in the star and the rose, oh supreme coincidence in the generous spiral.

Oh great convergence that grows saintly faith, oh singular coincidence that satisfies all thirst.

Oh kingdom of 9's longing to arrive, oh plentiful love oh glorious might.

Oh kingdom of 9's yearning to revive, filling up space oh unitive art.

The triune is calling with powerful voice.

# Always come the 9's helping us to grow.

Oh heaven spreads its truth all along.

June 2011

#### Carlos E. Puente

# Always come the 9's weaving a whole song.

Oh choose equilibrium, it is the best choice.

# Always come the 9's helping us to grow.

The plus is solution, the bell gives a gong.

# Always come the 9's weaving a whole song.

In faithful dimension, in love let's rejoice.

# Always come the 9's helping us to grow.

Oh death is defeated in Him we all belong.

# Always come the 9's weaving a whole song.

Unity, one, two, three, unity for you and me.

Oh kingdom of 9's longing to arrive, oh plentiful love oh glorious might. Oh kingdom of 9's yearning to revive, filling up space oh unitive art.

So that the mind gets it to  $\heartsuit$  and  $\heartsuit$  and  $\heartsuit$ , to become really positive to  $\heartsuit$  and  $\heartsuit$  and  $\heartsuit$ , so that the mouth sings it to  $\heartsuit$  and  $\heartsuit$  and  $\heartsuit$ , to walk the simple way to  $\heartsuit$  and  $\heartsuit$  and  $\heartsuit$ , so that the heart feels it to  $\heartsuit$  and  $\heartsuit$  and  $\heartsuit$ . to dream a good day ahead to  $\heartsuit$  and  $\heartsuit$  and  $\heartsuit$ , so that goodness reigns to  $\heartsuit$  and  $\heartsuit$  and  $\heartsuit$ , to arrive to a mansion yes, to  $\heartsuit$  and  $\heartsuit$  and  $\heartsuit$ .

Unity, one, two, three, unity for you and me.

Unity contains great many 9's: 1=0.999..., unity includes infinite ♡.

#### Notes

- 1. With due reverence and love, and mindful of my sinfulness, this work is dedicated to the Most Holy Trinity: to God be all glory, honor and praise.
- 2. Carlos E. Puente lectures at the Department of Land, Air and Water Resources, University of California, Davis.
- 3. Catechism of the Catholic Church: Second Edition (Doubleday Religion, 2003), p. 825.
- C. E. Puente, "On the nature of equilibrium," in *Omega*, 6.2 (2007), pp. 85– 105.
- C. E. Puente, "Faith lessons from chaotic fig trees," in *Omega*, 8.2 (2009), pp. 157–194.
- 6. M. F. Barnsley, Fractals Everywhere (Academic Press, 1988), p. 394.
- 7. Ibid.
- 8. B. B. Mandelbrot, The Fractal Geometry of Nature (Freeman, 1982), p. 468.
- 9. Barnsley, Fractals Everywhere, p. 394.
- 10. Ibid.
- 11. Ibid.
- 12. C. E. Puente, Treasures Inside the Bell (World Scientific, 2003), p. 97.
- 13. J. Feder, Fractals (Plenum Press, 1988), p. 283.
- 14. Puente, "On the nature of equilibrium," pp. 85–105.
- 15. Puente, Treasures Inside the Bell, p. 97.
- 16. Puente, "On the nature of equilibrium," pp. 85–105.
- 17. C. E. Puente, "A universe of projections: may Plato be right?," in *Chaos, Solitons & Fractals*, 19 (2004), pp. 241–253.

18. Ibid.

19. Ibid.

- 20. Puente, Treasures Inside the Bell, p. 97.
- C. E. Puente, M. M. López, J. E. Pinzón and J. M. Angulo, "The Gaussian distribution revisited," in *Advances in Applied Probability*, 28(2) (1996), pp. 500–524.
- 22. Ibid.
- 23. Ibid.
- 24. Puente, Treasures Inside the Bell, p. 97.
- 25. Ibid.
- 26. As explained in note 12, the standardized moments for dy based on the limiting cloud and a uniform dx do converge to those of the standard bell having mean zero and variance one. The following table shows that such a convergence happens in an orderly manner as z increases towards the limit of 1.

Order	z = 0.9999	z = 0.999999	z = 0.99999999	$\mathcal{N}(0,1)$
3	-0.032656325	-0.003265983	-0.000326599	0.0
4	3.001639399	3.000016400	3.000000164	3.0
5	-0.326658273	-0.032659923	-0.003265986	0.0
6	15.035259628	15.000352666	15.000003527	15.0
7	-3.432783098	-0.342932064	-0.034292860	0.0
8	105.643263393	105.006430686	105.000064307	105.0
9	-41.260116637	-4.115251502	-0.411514386	0.0
10	956.901068056	945.118861509	945.001188600	945.0
11	-568.823119975	-56.586203271	-5.658324296	0.0
12	10628.732737598	10397.330855430	10395.023307907	10395.0

- 27. C. E. Puente, "Yet more lessons from complexity. Unity: the key of peace," in *E:CO, Emergence, Complexity and Organization*, 8.4 (2006), pp. 104–111.
- 28. All Biblical quotations come from *New American Bible, Saint Joseph Edition* (Catholic Book Publishing Co., 1992).
- 29. Puente, "On the nature of equilibrium," pp. 85-105.
- 30. Ibid.
- 31. C. E. Puente, The Hypotenuse (Author House, 2006), p. 144.
- 32. Puente, "On the nature of equilibrium," pp. 85–105.
- 33. Catechism of the Catholic Church, p. 825.
- 34. Puente, "On the nature of equilibrium," pp. 85–105.
- 35. Ibid.
- 36. Puente, "Faith lessons from chaotic fig trees," pp. 157-194.
- 37. Ibid.
- 38. Puente, The Hypotenuse, p. 144.
- 39. Ibid.
- 40. Puente, "Faith lessons from chaotic fig trees," pp. 157-194.
- 41. Puente, "On the nature of equilibrium," pp. 85–105.
- 42. Puente, The Hypotenuse, p. 144.
- 43. Puente, "On the nature of equilibrium," pp. 85–105.
- 44. Puente, Treasures Inside the Bell, p. 97.
- 45. Barnsley, Fractals Everywhere, p. 394.
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- 47. Puente, "A universe of projections," pp. 241-253.
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- 49. C. E. Puente and M. G. Puente, "Ice crystals inside the bell," in *Visual Mathematics*, 6(1) (2004).
- 50. Puente, et al., "Bells galore," pp. 367-378.
- 51. Puente, Treasures Inside the Bell, p. 97.
- 52. Puente and Puente, "Ice crystals inside the bell."
- 53. Puente, Treasures Inside the Bell, p. 97.
- 54. Puente and Puente, "Ice crystals inside the bell."
- 55. C. E. Puente, "DNA,  $\pi$  and the bell," in *Complexity*, 6(2)(2000), pp. 16-22.
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- 59. Puente, "Yet more lessons from complexity," pp. 104-111.
- 60. B. Malamud, "Tail of natural hazards," in *Physics World*, 17(8)(2004), pp. 31–35.
- 61. Mandelbrot, The Fractal Geometry of Nature, p. 468.
- 62. Feder, Fractals, p. 283.
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- 65. H. De Soto, The Mystery of Capital (Basic Books, 2000), p. 275.
- 66. D. J. Watts, *Six Degrees: The Science of a Connected Age* (W. W. Norton, 2003), p. 384.
- 67. Puente, The Hypotenuse, p. 144.

- 68. D. Sornette, *Critical Phenomena in Natural Sciences* (Springer, 2009), p. 528.
- 69. P. Bak, How Nature Works (Copernicus, 1996), p. 212.
- 70. Puente, "Yet more lessons from complexity," pp. 104–111.
- 71. M. Buchanan, Ubiquity (Crown Publishers, 2001), p. 288.
- 72. As expressed by the limiting + + case and the limiting higher-dimensional cases that concentrate at infinity, there is more than one representation that is consistent with was has been said here about the Holy Trinity. In fact, one can modify the coordinates of the points by which the wires pass to define infinitely many congruent representations, all of which point, in the same spirit, to the plenitude of love.
- 73. Puente, "On the nature of equilibrium," pp. 85–105.
- 74. Puente, "Faith lessons from chaotic fig trees," pp. 157-194.
- 75. Puente, The Hypotenuse, p. 144.