

More lessons from complexity - The origin: The root of peace

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The last few decades have witnessed the development of a host of ideas aimed at understanding and predicting nature's ever present complexity (see for instance, Mandlebrot, 1982; Bak, 1996; Wolfram, 2002). It is shown that such a work provides, through its detailed study of order and disorder, a suitable framework for visualizing the dynamics and consequences of mankind's ever present divisive traits. Specifically, this work explains how recent universal results pertaining to the transition from order to chaos via a cascade of bifurcations point us to a serene state, symbolized by the convergence to the origin in the root of a Feigenbaum's tree, in which we all may achieve peace.

The Logistic Map

Recall the exotic dynamics of the *logistic map* (see for example, Rasband, 1990; Peitgen, et al., 1992; Beck & Schlögl, 1993):

$$X_{k+1} = \alpha X_k \cdot (1 - X_k)$$

that is, the chain of ultimately stable (and unstable) values $X_{\mu}(\alpha)$ found iterating the map, where X_{μ} denotes the normalized size of a *population* at generation k and α is a free parameter having values between 0 and 4:

- When $\alpha \leq 1$, the logistic *parabola* is below the one to one line (added to aid in the calculations), and then $X_{m} = 0$ (Figure 1);
- When $1 < \alpha \le 3$, the parabola is above the line $\mathbf{X} = \mathbf{Y}$ and $X_{\alpha} = (\alpha - 1)/\alpha$, the non-zero intersection between the curve and the straight line (Figure 2);
- When $3 < \alpha \le 3.449..., X_{\infty} = \{X_{\infty}^{(1)}, X_{\infty}^{(2)}\}$ and the population settles into an oscillation repeating every two generations (Figure 3);
- When 3.449... < $\alpha \le 3.544..., X_{m} = \{X_{m}^{(3)}, \}$ $X_{m}^{(4)}, X_{m}^{(5)}, X_{m}^{(6)}$. The population ultimately repeats every four generations, and the dynamics have experienced a *bifurcation* (Figure 4);

- When α is increased up to a value α_{i} = 3.5699..., successive bifurcations in powers of two happen quickly, that is, the dynamics repeat exactly every 2^n generations, for any value of *n*;
- When $\alpha_{m} < \alpha \le 4$, behavior is found either periodic or non periodic. For instance, for $\alpha =$ 3.6 an infinite strange attractor with a whole in the middle is found (Figure 5);
- When $\alpha = 3.83$, $X_{\infty} = \{X_{\infty}^{(1)}, X_{\infty}^{(2)}, X_{\infty}^{(3)}\}$ and the dynamics oscillate every 3 generations (Figure 6);
- When $\alpha = 4$, the most common behavior is non periodic and a dense strange attractor over the interval [0, 1] is found (Figure 7).





Figure 2 Convergence to a fixed point



Figure 3 Convergence to a 2-cycle.



Figure 4 Convergence to a 4-cycle



Figure 5 *Convergence to a "dusty" non-repetitive attractor*



Figure 6 Convergence to a 3-cycle



Figure 7 *Convergence to a maximal non-repetitive set*

At the end, the cascade of stable *period-doubling bifurcations* (before α_{∞}) and the emergence of **chaos** (strange attractors) intertwined with periodic behavior (including any period greater than two) is summarized via the celebrated **Feigenbaum's diagram** (Figure 8).

This is so named after Mitchell Feigenbaum who showed that the bifurcation openings and their durations happen **universally** for a class of unimodal maps according to two universal constants F_1 and F_2 , as follows (Feigenbaum, 1978) (refer to Figure 9):

$$d_n/d_{n+1} \to F_1 = -2.5029..., \Delta_n/\Delta_{n+1} \to F_2 = 4.6692..$$



Figure 9 Successive bifurcations. \overline{X} corresponds to supercycles, i.e., $\frac{1}{2}$ for the logistic map.

For example, other "*fig trees*" guided by F_1 and F_2 and for the two simple mappings $f(X) = \alpha X \cdot (1 - X^3)$ and $f(X) = \alpha X \cdot (1 - X)^3$ are shown below[1]. Notice how such contain: a straight "*root*," a bent "*branch*," bifurcation branches, and then, in an orderly intertwined fashion, following Sharkovskii's order (see for example, Rasband, 1990; Peitgen, *et al.*, 1992; Beck & Schlögl, 1993), periodic branches, and the ever dusty "*foliage of chaos*," where the unforgiving condition of sensitivity to initial conditions rules.

Chaos theory and our quest for peace

s the dynamics of the logistic map describe several physical processes (see for instance, Cvitanović, 1989; Bai-Lin. 1984), including fluid *turbulence* as induced by heating, that is, **convection**, it is pertinent to consider such a *simple* and *universal* mechanism to study how "chaos" and its related condition of "violence" may arise in the world.

Given that the key parameter α , associated with the amount of heat (Libchaber & Maurer, 1978), dictates the ultimate organization of the fluid, we may see that it is wise to keep it small (in the world, and within each one of us) in order to avoid undesirable "nonlinearities." For although the allegorical fig trees exhibit clear order in their pathway towards disorder, we may appreciate in the uneasy jumping on strange attractors (and also on periodic ones), the anxious and foolish frustration we often experience (so many times deterministically!) when we, by choosing to live in a hurry, travel from place to place to place in "*high heat*" without finding our "*root*."

In this spirit, the best solution for each one of us is to slow down altogether the pace of life, coming down the tree, so that by not crossing the main *threshold* $\mathbf{X} = \mathbf{Y}$, that is, by choosing $\alpha \le 1$, we may surely live without turbulence and chaos in the



Figure 10 Bifurcation diagrams for two simple nonlinear maps



Figure 11 Orbits of the logistic map ($\alpha = 4$) ending up at zero

robust state symbolized by $X_{\infty} = 0[2,3]$. For there is a marked difference between a seemingly laminar condition as it happens through *tangent bifurcations* (see for example, Rasband, 1990; Peitgen, *et al.*, 1992; Beck & Schlögl, 1993) and being truly at **peace**, for the former invariably contains dramatic bursts of chaos and ample intermittency (see for example, Rasband, 1990; Peitgen, *et al.*, 1992; Beck & Schlögl, 1993).

As **zero**, that is, converging to the **origin**, is identified as the desired state, it is sensible to realize that such an organization, a *trivial* solution for $X_{\infty}(\alpha)$, even if unstable, may be reached even when the worst chaos engulfs us ($\alpha = 4$). For the precise dynamics of the *pre-images of zero* do not wander for ever in high heat, but rather find the way to the **root** through a delicate hopscotch by the *middle*[4] (Figure 11). For it is tragic indeed to "oscillate for ever" (Figure 12). And more tragic yet to be close to "*the point*" and miss it altogether forever[5] (Figure 13). For the *butterfly effect*, with all probability and contrary to the illusion that it provides us with options, leaves us irremediably trapped in dust.

At the end, the emergence of the modern science of complexity helps us visualize our ancient

choices. It is indeed best for us to live in serenity and in a **simple** manner, not amplifying and hence heeding the voice. For only the conscious order of **Love** does not suffer the destiny of arrogant stubbornness that justly receives the same "bad luck" of a parabolic tree that did not have any fruit, the same one that with its tender branch(es) and budding leaves, also announces horrendous times, but also very good ones, times of **joy** and of **friendship**.

Acknowledgments

This piece is dedicated, in memoriam, to Fr. Rafael García Herreros, a humble man with a clear vision, whose special touch instilled in me the need to dream, in order to contribute to mend our chaotic world.



Figure 13 *Some orbits of the logistic map* ($\alpha = 4$) *ending in a strange attractor*

FEIGENBAUM'S PARABEL

(Carlos E. Puente)

In the confines of science majestically stands a tree, with all numerals in dance in emergent chaos to see.

In the instance of a trance a good day I drew a link, and here it is, at a glance, the wisdom that I received.

Foliage of disorder trapped in empty dust, jumps astir forever enduring subtle thrust.

Crossing of the outset leaving faithful root, looming tender offset failing to yield fruit.

Cascade of bifurcations, increasing heat within, inescapable succession of branches bent by wind.

Sprouting of dynamics attracted to the strange, oh infinity reminding at the origin: the flame.

In the midst of chaos there is a small gate leading to fine rest.

In the midst of chaos there are loyal paths inviting to a dance.

On top of the fig tree there is a key point that runs to the core.

On top of the fig tree there is a clear light that averts a fright. **In the midst of chaos** there is leaping game discerning the way.

In the midst of chaos there is a fine well watering the brain.

On top of the fig tree there is a clean frame that cancels the blame.

On top of the fig tree there is mighty help that shelters from hell.

In the midst of chaos, look it is there, in the midst of chaos, logistics in truth, in the midst of chaos, a clear faithful route, in the midst of chaos, leading to the root.

On top of the fig tree, this is no delusion, on top of the fig tree, a sought needle's eye, on top of the fig tree, the symbol of wheat, on top of the fig tree, surrounded by weeds.

Could it be, oh my friends, that science provides a rhyme?, for a rotten tree foretells the very advent of time.

Could it be, oh how plain, that nature extends a call?, for old parable proclaims the crux in growing small.



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Notes

[1] Feigenbaum means fig tree in German.

[2] Of course, losing all our "rabbits" is quite undesirable, but "surrendering" to live slowly is a rather good investment.

[3] Although it may be argued that the implied laminar flow is "boring," this is the "common ground" that ensures proper communication amongst us. For $\alpha > 1$, in its relentless amplification from generation to generation, corresponds to a "proud" state that prevents us from seeing with the eyes of somebody else.

[4] The symbol of the (square) *root* may also be seen after two twists (left and up) of the graph! For additional insight on this concept consider Puente (2006a,b).

[5] If tragedy may be quantified...

Choices

Resting	
Surrendering	
Simplicity	
Order	
Serenity	
Peace	
To decrease	
Below $\mathbf{X} = \mathbf{Y}$	

Wandering Drifting Complexity Disorder Turbulence Chaos To increase Above **X** = **Y**