MORE LESSONS FROM COMPLEXITY

The Origin: The Root of Peace

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The last few decades have witnessed the development of a host of ideas aimed at understanding and predicting nature's ever present complexity.[1] It is shown that such a work provides, through its detailed study of order and disorder, a suitable framework for visualizing the dynamics and consequences of mankind's ever present divisive traits. Specifically, this work explains how recent universal results pertaining to the transition from order to chaos via a cascade of bifurcations point us to a serene state, symbolized by the convergence to the origin in the root of a Feigenbaum's tree, in which we all may achieve peace.

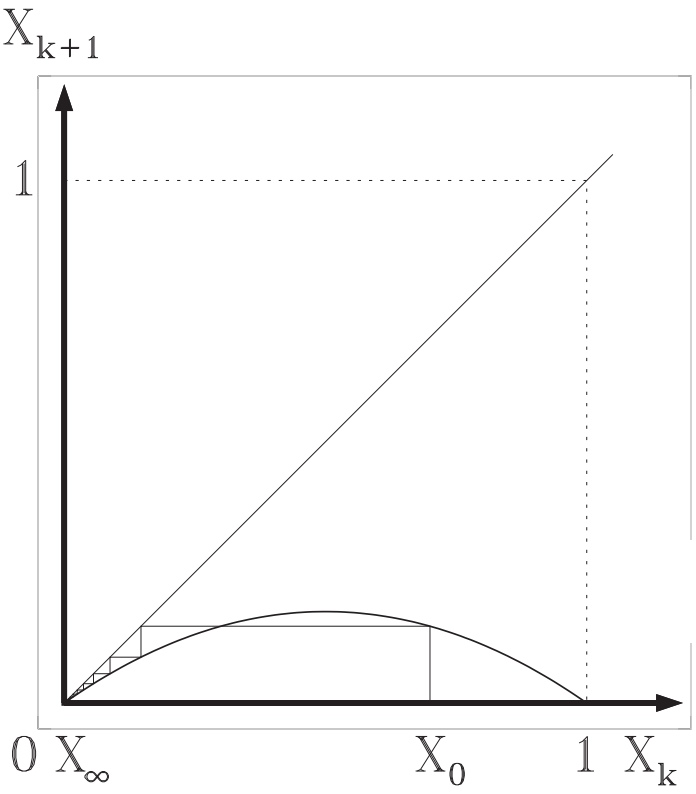
The Logistic Map

Recall the exotic dynamics of the *logistic map*,[2]

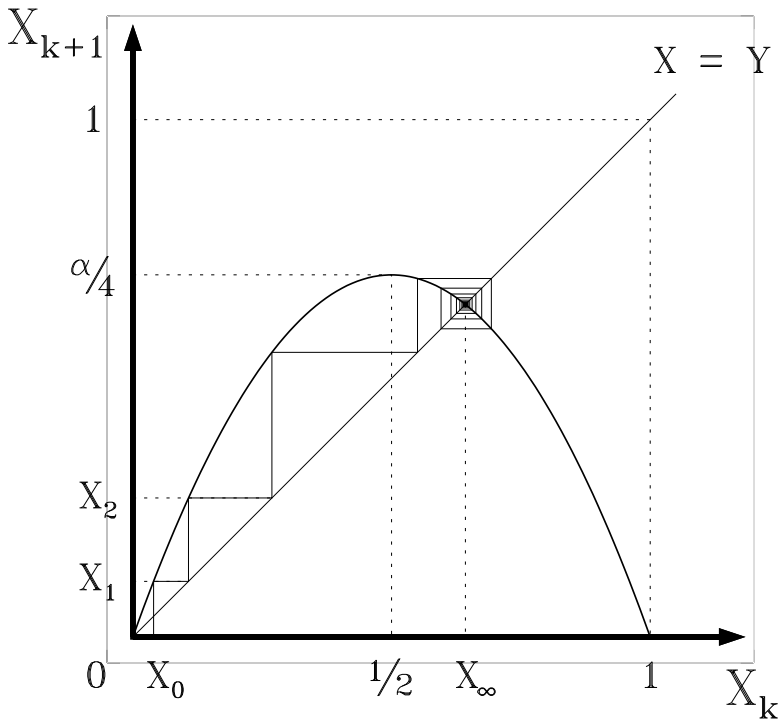
$$X_{k+1} = \alpha X_k \cdot (1 - X_k),$$

that is, the chain of stable and unstable $X_\infty(\alpha)$, where X_k denotes the normalized size of a population at generation k and α is a free parameter having values between 0 and 4:

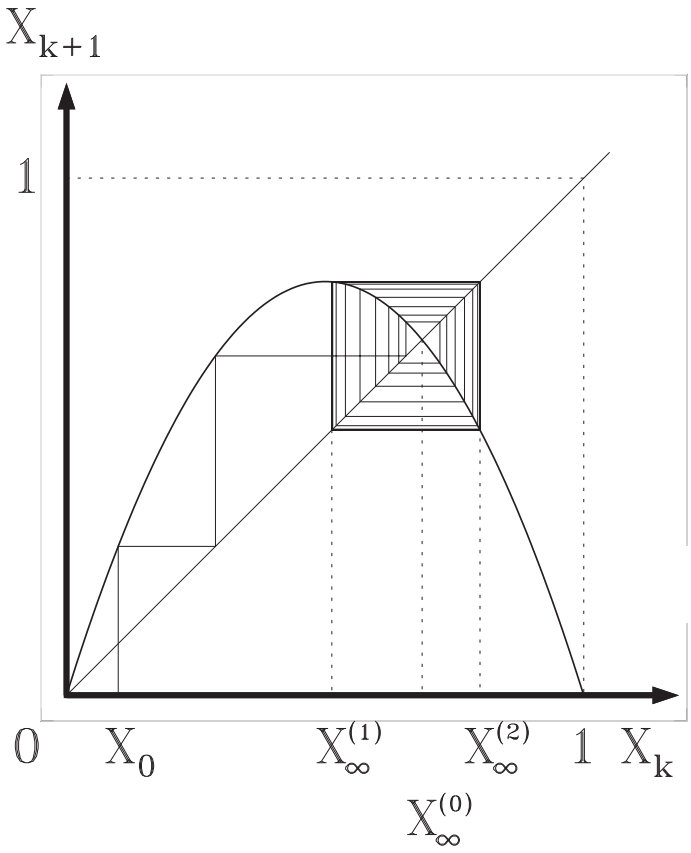
- $\alpha \leq 1$, parabola below $X = Y$, $\Rightarrow X_\infty = 0$,



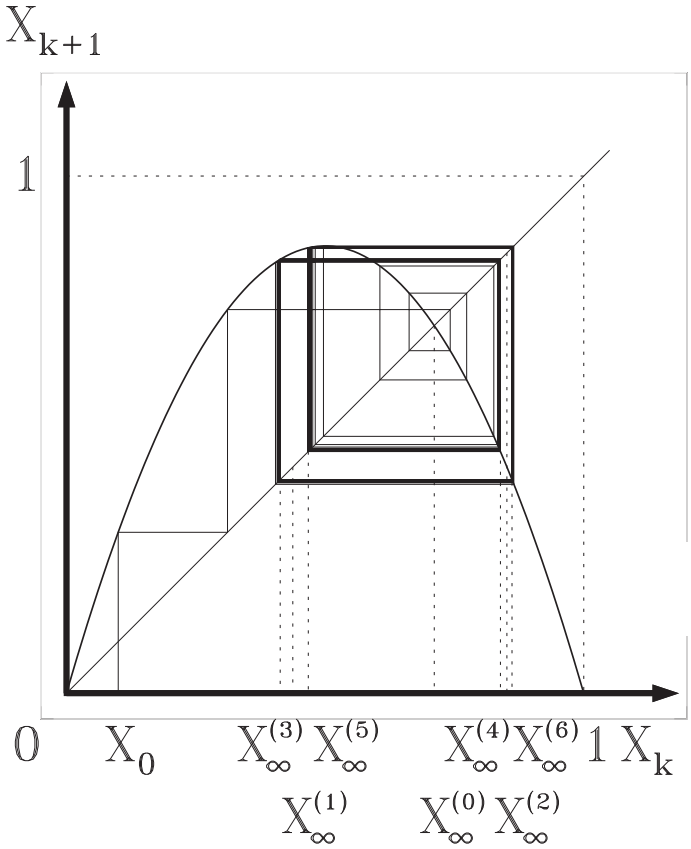
- $1 < \alpha \leq 3$, parabola above, $\Rightarrow X_\infty = (\alpha - 1)/\alpha$,



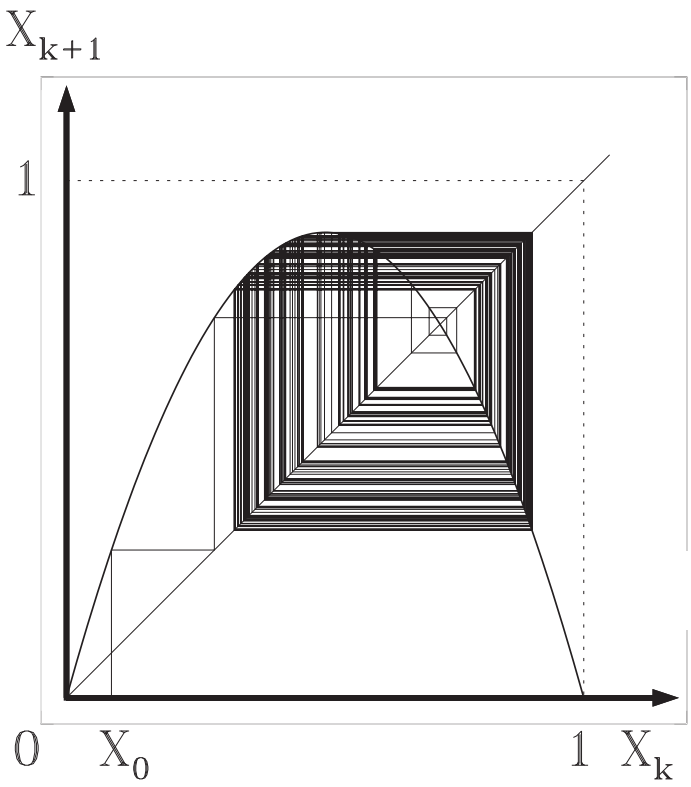
- $3 < \alpha \leq 3.449\ldots \Rightarrow X_\infty = \{X_\infty^{(1)}, X_\infty^{(2)}\}$, *period 2*,



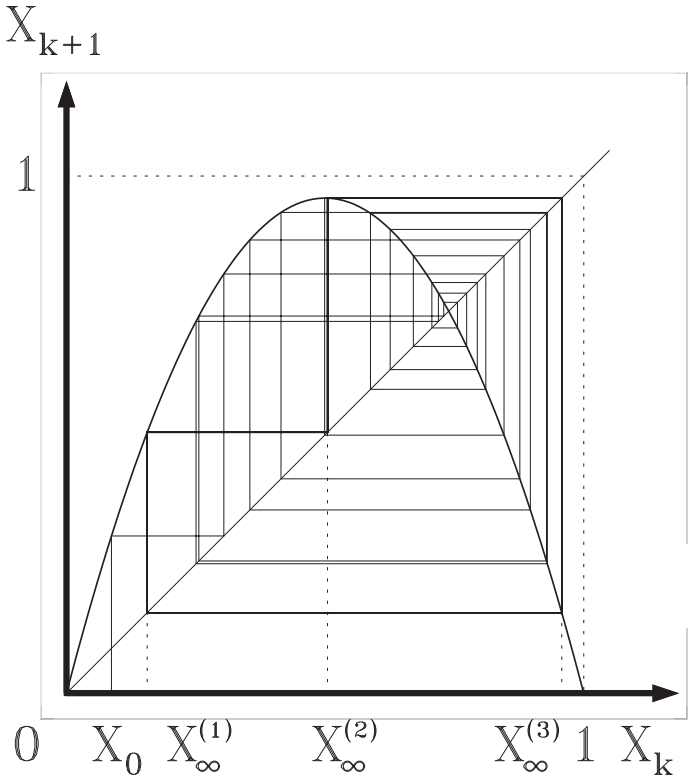
- $3.449\ldots < \alpha \leq 3.544\ldots \Rightarrow X_\infty = \{X_\infty^{(3)}, X_\infty^{(4)}, X_\infty^{(5)}, X_\infty^{(6)}\}$,



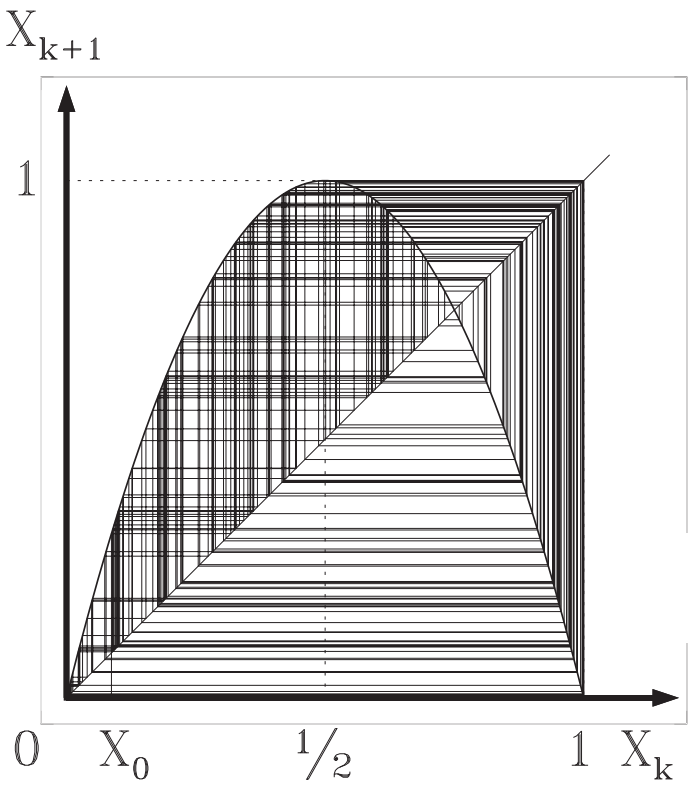
- $\alpha_\infty = 3.5699\ldots < \alpha = 3.6 \Rightarrow X_\infty = \textit{strange attractor}$,



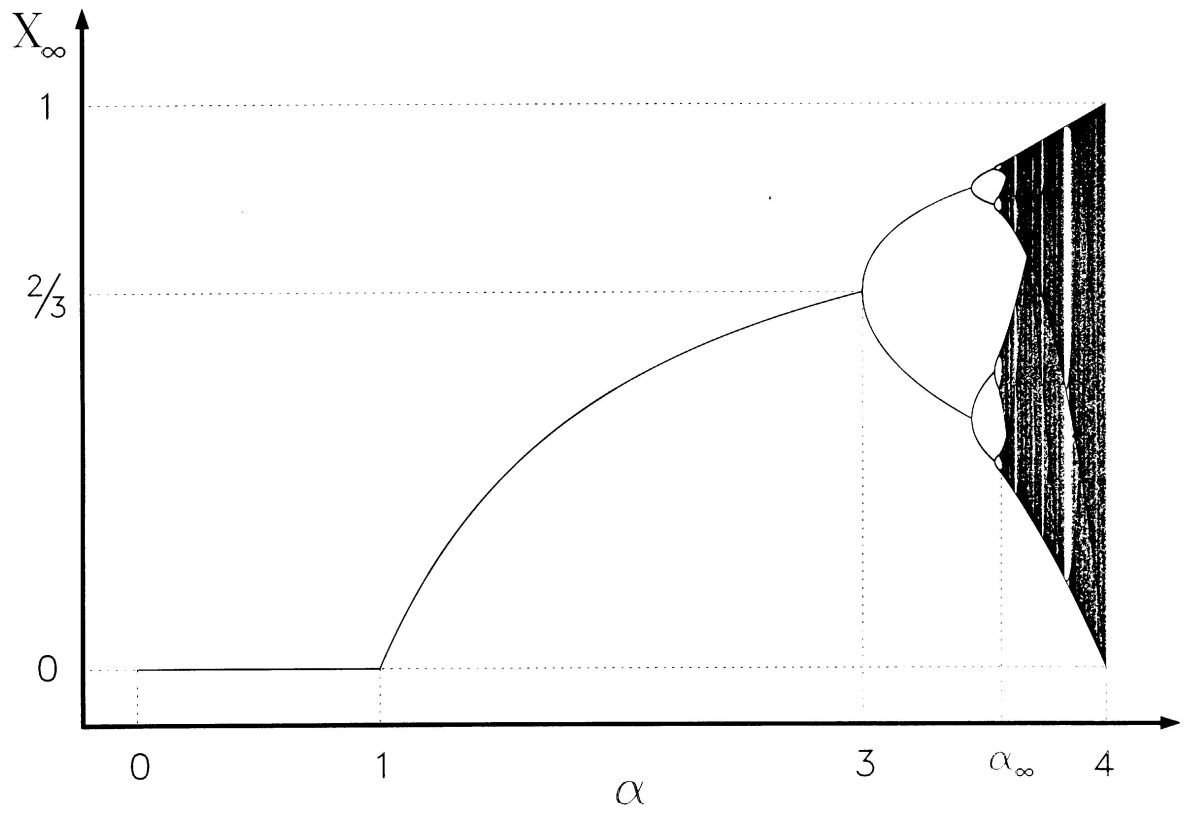
- $\alpha = 3.83 \Rightarrow X_\infty = \{X_\infty^{(1)}, X_\infty^{(2)}, X_\infty^{(3)}\}$, *period 3*,



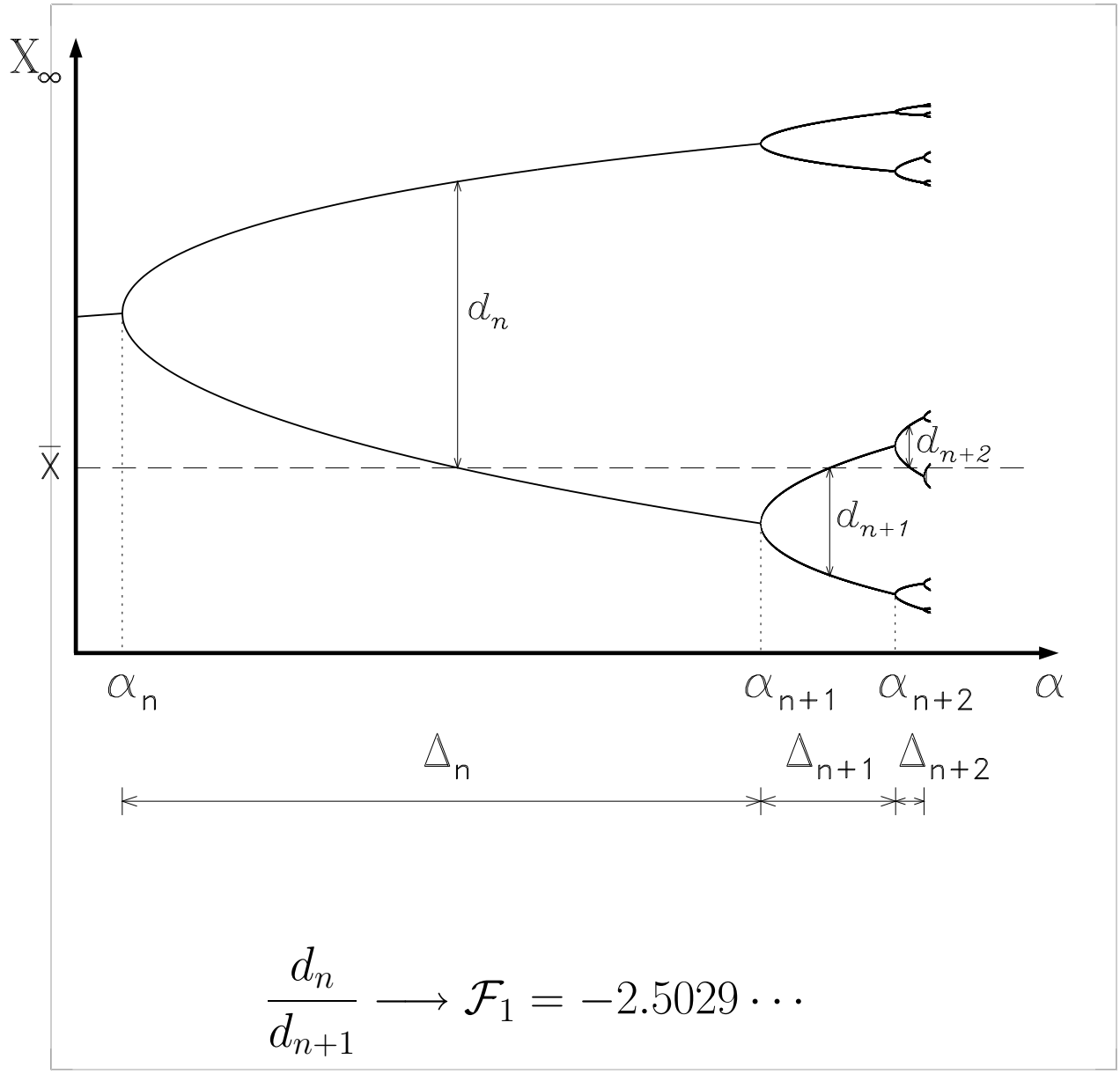
- $\alpha = 4 \Rightarrow X_\infty =$ *strange attractor*,



At the end, the chain of stable *period-doubling bifurcations* (before α_∞) and the emergence of *chaos* (strange attractors) intertwined with periodic behavior (for any period greater than two) is summarized via the celebrated *Feigenbaum's diagram*,



so named after Mitchell Feigenbaum who showed that bifurcations and their openings happen **universally** for a class of unimodal maps according to two universal constants $\mathcal{F}_1$ and $\mathcal{F}_2$, as follows:

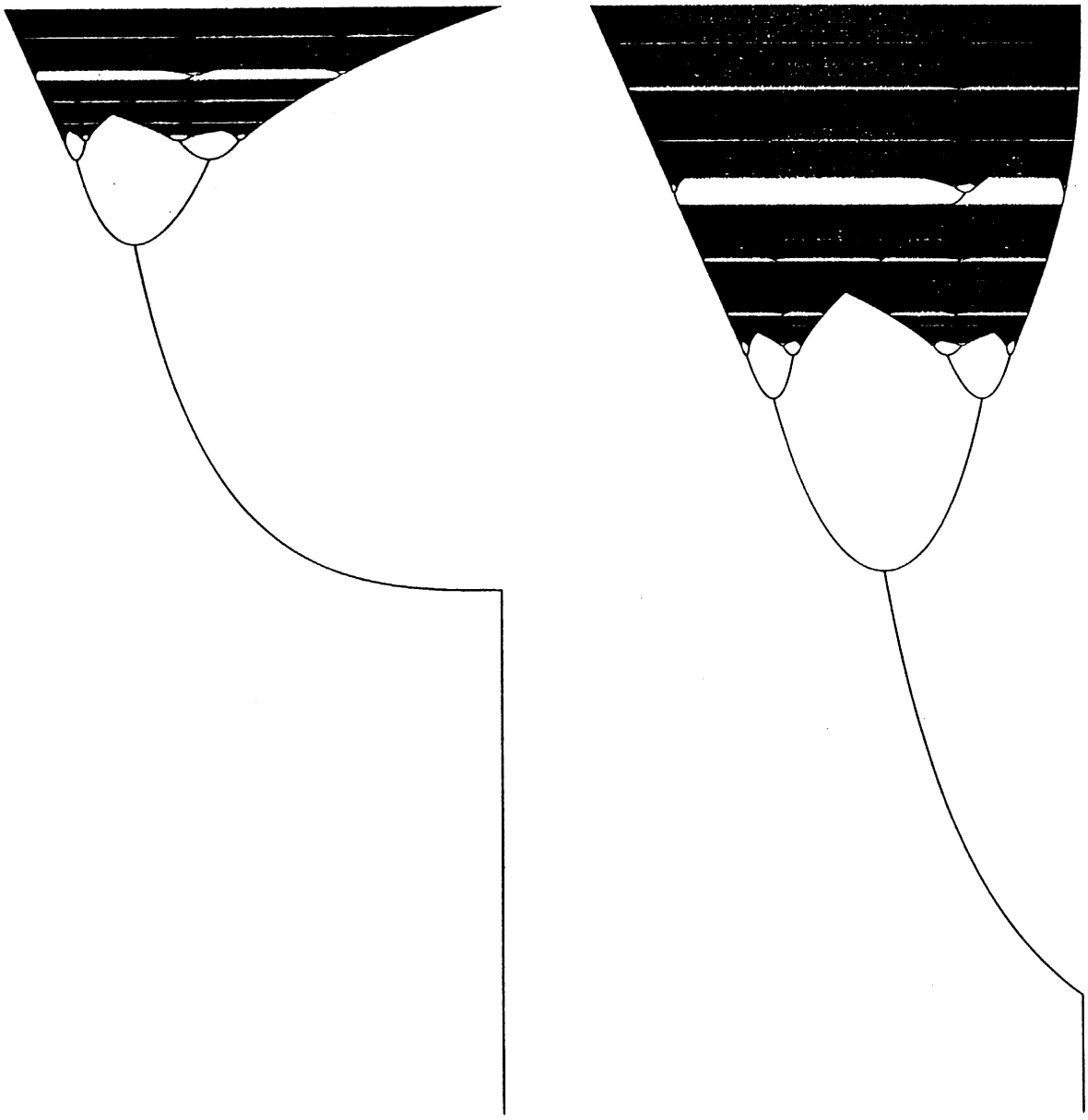


$$\frac{d_n}{d_{n+1}} \longrightarrow \mathcal{F}_1 = -2.5029\cdots$$

$$\frac{\Delta_n}{\Delta_{n+1}} \longrightarrow \mathcal{F}_2 = 4.6692\cdots$$

$$\bar{X} = 1/2 \Leftrightarrow supercycles$$

Other *"fig trees"* for the mappings $f(X) = \alpha X(1 - X^3)$ and $f(X) = \alpha X(1 - X)^3$ are shown below.[3] Notice that they contain: a straight *"root,"* a bent *"branch,"* bifurcation branches, and then, in an orderly intertwined fashion, following Sharkovskii's order,[2] periodic branches, and the ever dusty *"foliage of chaos,"* where the unforgiven condition of sensitivity to initial conditions rules:



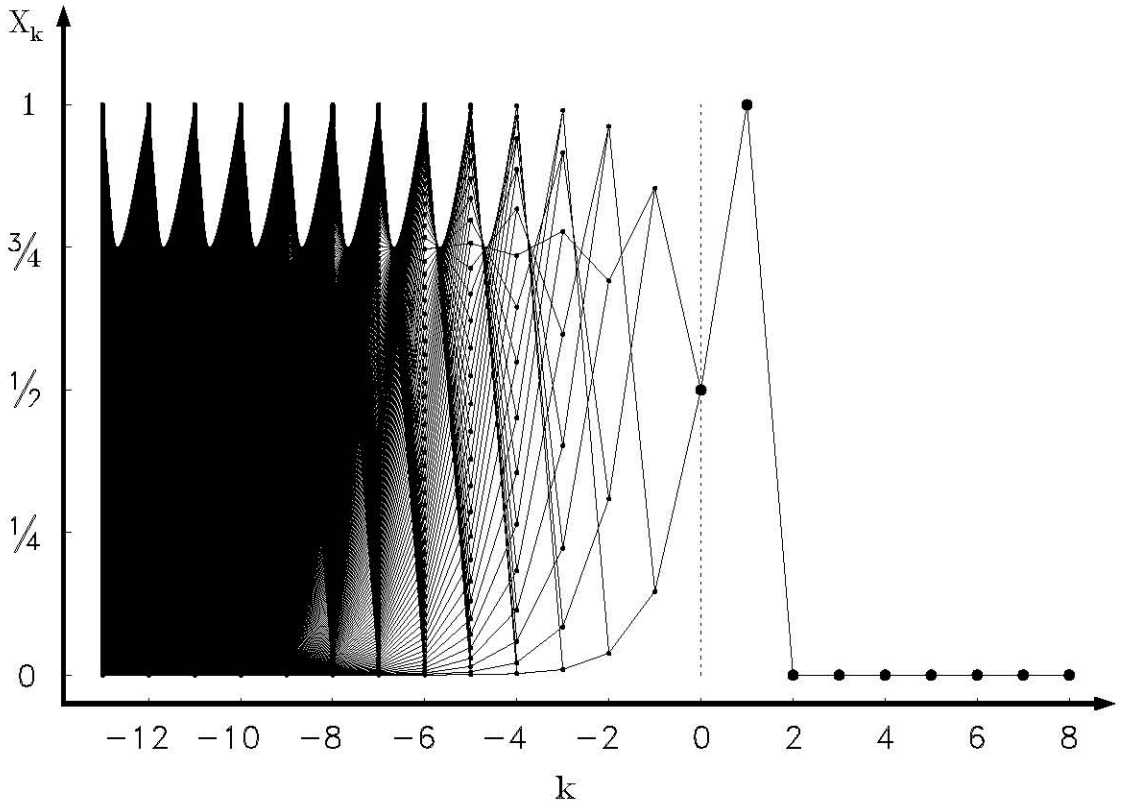
Chaos Theory and Our Quest for Peace

As the dynamics of the logistic map describe several physical processes,[4] including fluid turbulence as induced by heating, that is, **convection**, it is pertinent to consider such a *simple* and *universal* mechanism to study how "chaos" and its related condition of "violence" may arise in the world.

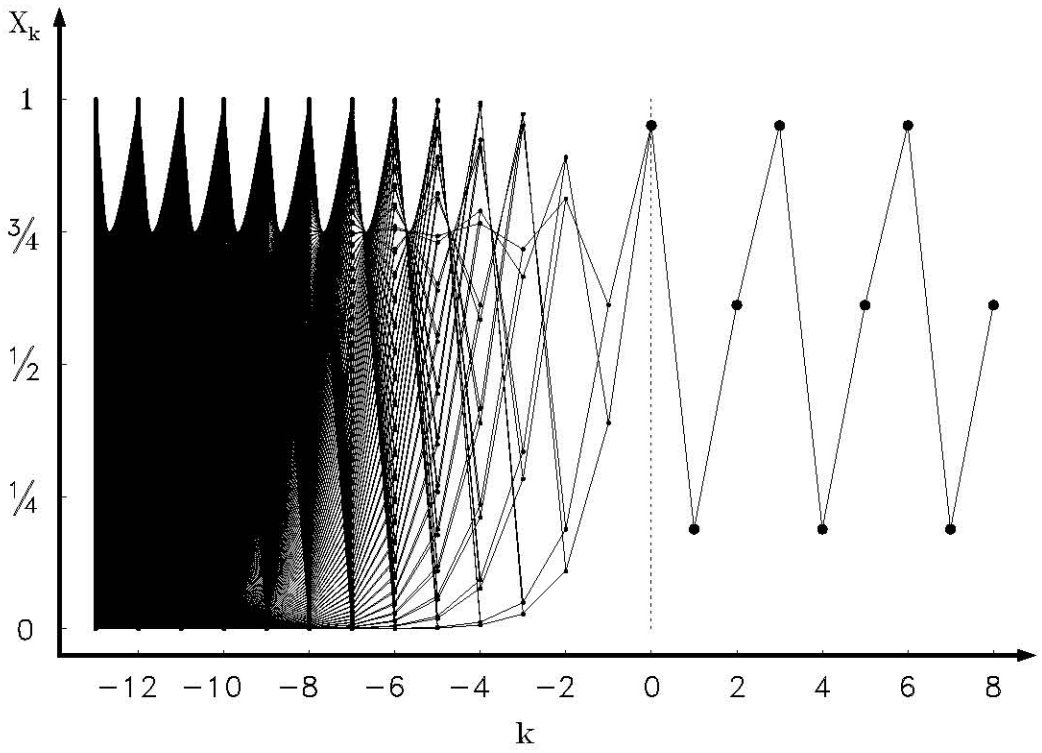
Given that the key parameter α, associated with the amount of heat, dictates the ultimate organization of the fluid, we may see that it is wise to keep it small (in the world, and within each one of us) in order to avoid undesirable "nonlinearities." For although the allegorical fig trees exhibit clear order in their pathway towards disorder, we may appreciate in the uneasy jumping on strange attractors (and also on periodic ones), the anxious and foolish frustration we often experience (so many times deterministically!) when we, by choosing to live in a hurry, travel from place to place to place in "high heat" without finding our "root."

In this spirit, the best solution for each one of us is to slow down altogether the pace of life, coming down the tree, so that by not crossing the *threshold*, that is, $\alpha \leq 1$, we may surely live without turbulence and chaos in the robust state symbolized by $X_\infty = 0$.[5,6] For there is a marked difference between a seemingly laminar condition as it happens through *tangent bifurcations*[2] and being truly at **peace**, for the former invariably contains dramatic bursts of chaos and ample intermittency.[2]

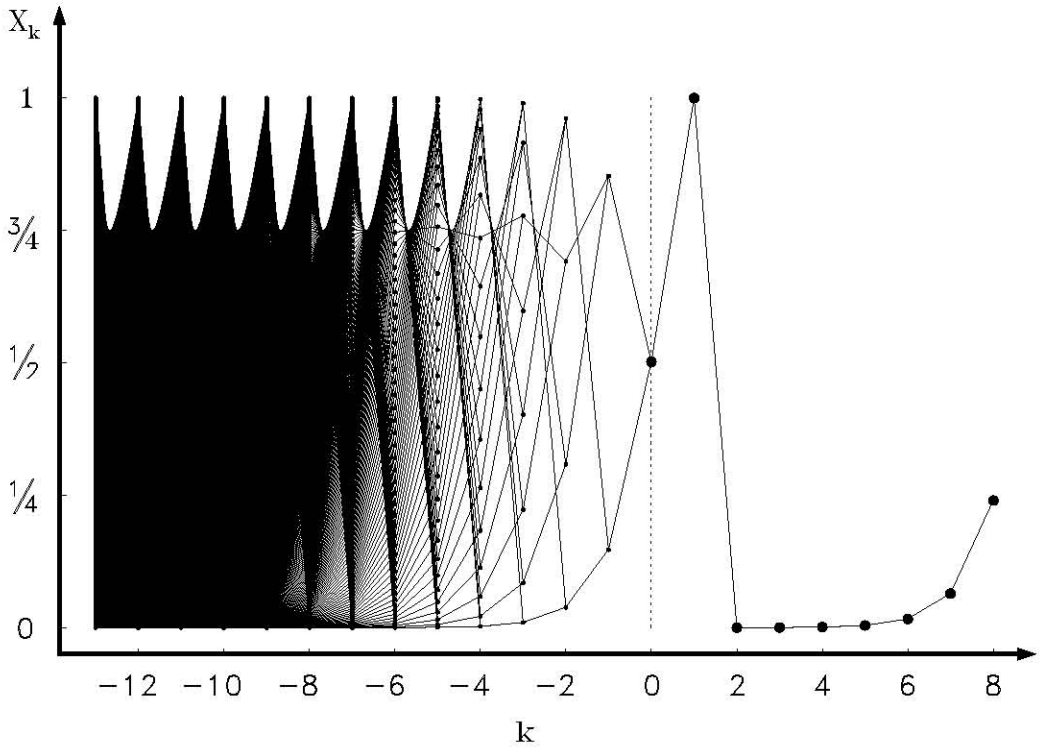
As **zero**, that is, converging to the **origin**, is identified as the desired state, it is good news to realize that such a condition, a *trivial* solution for X_∞ (even if unstable), may be reached even when the worst chaos engulfs us ($\alpha = 4$). For the precise dynamics of the *pre-images of zero* do not wander for ever in high heat, but rather find the way to the **root** through a delicate hopscotch by the *middle*:[7]



For it is tragic indeed to "oscillate for ever:"



And more tragic yet to be close to "the point" and miss it:[8]



For the *butterfly effect*, with all probability and contrary to the illusion that it provides us with options, leaves us irremediably trapped in dust.

At the end, the modern science of complexity help us visualize our ancient choices. It is indeed best for us to live in serenity and in a **simple** manner, not amplyfying and hence heeding the voice. For only the conscious order of **Love** does not suffer the destiny of arrogant stubbornness that justly receives the same "bad luck" of a parabolic tree that did not have any fruit, the same one that with its tender branch(es) and budding leaves, also announces horrendous times, but also very good ones, times of **joy** and of **friendship**.

Choices

Resting	*Wandering*
Surrendering	*Drifting*
Simplicity	*Complexity*
Order	*Disorder*
Serenity	*Turbulence*
Peace	*Chaos*
To decrease	*To increase*
Below $Y = X$	*Above* $Y = X$

REFERENCES AND NOTES

1. For instance, B. B. Mandelbrot, *The Fractal Geometry of Nature*, (Freeman, 1982); P. Bak, *How Nature Works*, (Copernicus, 1996); and S. Wolfram, *A New Kind of Science*, (Wolfram Media Inc., 2002).
2. See for example, S. N. Rasband, *Chaotic Dynamics of Nonlinear Systems*, (John Wiley & Sons, 1990); H.-O. Peitgen, H. Jürgens, D. Saupe, *Chaos and Fractals. New Frontiers of Science*, (Springer-Verlag, 1992); C. Beck and F. Schlögl, *Thermodynamics of Chaotic Systems. An Introduction*, (Cambridge University Press, 1993).
3. Feigenbaum means fig tree in German.
4. See for instance, P. Cvitanović (Ed.), *Universality in Chaos*, (Adam Hilger, 1989).
5. Of course, losing all our "rabbits" is quite undesirable, but "surrendering" to live slowly is a rather good investment.
6. Although it may be argued that the implied laminar flow is "boring," this is the "common ground" that ensures proper communication amongst us. For $\alpha > 1$, in its relentless amplification from generation to generation, corresponds to a "proud" state that prevents us from seeing with the eyes of somebody else.
7. The symbol of the square root may also be seen here after two twists of the graph!
8. If tragedy may be quantified...

FEIGENBAUM'S PARABEL

(Carlos E. Puente)

In the confines of science
majestically stands a tree,
with all numerals in dance
in emergent chaos to see.

In the instance of a trance
a good day I drew a link,
and here it is, at a glance,
the wisdom that I received.

Foliage of disorder
trapped in empty dust,
jumps astir forever
enduring subtle thrust.

Crossing of the outset
leaving faithful root,
looming tender offset
failing to yield fruit.

Cascade of bifurcations,
increasing heat within,
inescapable succession
of branches bent by wind.

Sprouting of dynamics
attracted to the strange,
oh infinity reminding
at the origin: the flame.

In the midst of chaos
there is a small gate
leading to fine rest.

In the midst of chaos
there are loyal paths
inviting to a dance.

On top of the fig tree
there is a key point
that runs to the core.

On top of the fig tree
there is a clear light
that averts a fright.

In the midst of chaos
there is leaping game
discerning the way.

In the midst of chaos
there is a fine well
watering the brain.

On top of the fig tree
there is a clean frame
that cancels the blame.

On top of the fig tree
there is mighty help
that shelters from hell.

In the midst of chaos,
look it is there,
in the midst of chaos,
logistics in truth,
in the midst of chaos,
a clear faithful route,
in the midst of chaos,
leading to the root.

On top of the fig tree,
this is no delusion,
on top of the fig tree,
a sought needle's eye,
on top of the fig tree,
the symbol of wheat,
on top of the fig tree,
surrounded by weeds.

Could it be, oh my friends,
that science provides a rhyme?,
for a rotten tree foretells
the very advent of time.

Could it be, oh how plain,
that nature extends a call?,
for old parable proclaims
the crux in growing small.