Reflect on complexity and take it easy

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Cover. It is a great joy for me to share this presentation honoring Rafael, who, by giving me the opportunity to work with him at MIT, changed my destiny. This is a particularly joyous occasion for me, for this year also corresponds to the 25th anniversary of my doctorate.

This talk presents an overview of how my efforts to understand hydrologic complexity led me to other musing regarding complexity in general. As you shall see, it is my believe that universal traits of complexity provide relevant hints for us as human beings, reminders that invite us to live in peace and guided by the phrase "**take it easy**," or as Rafael has consistently told me in Spanish when discussing these matters: "**tómalo suave**."

I wish to thank Fatih and Gayle for organizing this event and for the opportunity I have been given to speak. I hope that all of you will find the talk interesting and that Rafael will enjoy it very much.

Page 2. It all started on my first few years at UC Davis when I learned about the lovely work on iterated systems. Here you see, on the left, the famous Sierpinski triangle built iterating three maps that travel to the middle of the vertex (x_n, y_n), and, on the right, a fractal interpolating function generated via the two simple affine mappings shown.

As the pattern on the right resembles the profile of a mountain due to the positive-negative sign on the parameter z, it occurred to me that one could perhaps generalize the ideas in order to get suitable surfaces from which to study the evolution of river networks. Unfortunately, such was not the case as plausible extensions yield surfaces with infinite unnatural ridges.

Page 3. By about the same time, a lovely little paper came my way, one that describes the organization of energies in fully developed turbulence.

Here you see the breaking of equilibrium into a cascade of negative spirals and the universal partitioning of energies in the surprisingly simple proportion 70-30, yielding a skewed multifractal organization of thorns supported by Cantor dust that leads to dissipation, as found in various types of turbulence.

As the multifractal texture may be obtained finding a histogram of points in x while iterating the two maps below using a 70-30 coin, and as such correspond precisely to the first components of the ones in the previous page

giving the mountain profile, one good day, no doubt inspired by my MIT training and by my own work with Rafael, I got an idea.

Page 4. It occurred to me to combine fractal interpolating functions and multifractals to study their implied derived distributions, thinking that in such a way one may obtain a description of rainfall.

Here you see an example of such a combination when using a 70-30 coin. While dx is the multifractal histogram the iterated maps generate over x, the implied histogram dy is a complex-looking set whose geometry is found by weighing the crossings of the fractal function f via the illumination dx. Notice how the notions are Platonic as both histograms are shadows, just as Plato argued about reality in his allegory of the caveman.

As you can see, my original idea regarding rainfall may very well be true, but what makes the construction special are the facts that the seemingly-random set dy is at the end entirely deterministic and that it may be fully defined based on the few parameters that determine the building blocks dx and f.

The construction no doubt represents also a romantic Platonic idea in this day an age, that is, in the twenty first century and after quantum mechanics, but we have been playing with these notions throughout the years and we can indeed generate interesting types of complexity without invoking chance.

Page 5. For instance, by now we know that the notions of shadows, projections to be technically accurate, produce, by varying the underlying parameters, a host of sets that resemble natural time series, including the observed statistical features such as autocorrelations and power-law power spectra.

Page 6. And the Platonic ideas may be used to approximate specific data sets, as shown on the right hand side for a storm gathered in Boston. As you can see, the real and the projected sets, this latter one in the middle via the iteration of four maps, although not identical, clearly belong to the same family, and such happens to be the case as they share similar statistical, multifractal and chaotic features.

Page 7. It happens that the ideas may be extended to higher dimensions, iterating affine maps that contain more components and such that they define fractal interpolating functions from one to two or from one to three dimensions. As illustrated here, shadows over two and three dimensions may now be defined based on a parent multifractal illumination and such derived distributions resemble a host of natural sets, complex patterns that may be fully understood via a few number of parameters.

Page 8. Besides attempting to study specific patterns in space, one may use the notions in order to study their dynamics as illustrated here.

Shown on the left are the observed vertically-averaged bromide patterns as gathered in the Borden site and, adjacent to them, fitted patterns as found via projections of fractal functions. As seen, the fractal-based sets do resemble the natural ones in shape and location and one may visualize the evolution of the plume via the underlying fractal-multifractal parameters shown on the right. As in this example there are noticeable trends in parameters as a function of time, one may extrapolate such tendencies to find predictions that turn out to be close renderings for the site.

These results support the yet more romantic notion that geometry may one day provide a dual representation of the physics of the patterns, one that will not rely explicitly on differential equations but on geometric trends. I envision tackling this interesting problem in years to come, for I sense here relevant opportunities for mining complex data sets in one and higher dimensions.

Page 9. At the onset of all these notions, on another good day we decided to study what happened when the original fractal function filled up the plane, that is, when the parameter z defined earlier tends to its maximal value of 1.

As you can see, this case leads to surprises, for such a plane-filling function transmutes the thorns over dust in the input multifractal into the harmonic and smooth Gaussian distribution. To our delight, the result is also universal as it is valid for any non-discrete input dx and also for higher dimensional space-filling limits.

As the results imply a non-intuitive transformation of dissipation into conduction, and not the other way around, such ideas urged me to meditate on the ultimate meaning of such limiting functions. Resisting the un-scientific thought at first, I finally **took it easy** while concluding that such transforming functions were related somehow to love. For, what else could transform dust and dissipation into something harmonic and conducting but love?

With the passage of time, we were able to prove the truthfulness of the Gaussian limit in the one-dimensional case, but the two-dimensional case remained elusive. On yet another good day it occurred to us to study how the circles were formed on bivariate independent bells, drawing not the final summary of say 15 million iterations, but rather plotting successive groups of iterations, say every 2,000 points.

Page 10. What we found is surprising to this day and is shown here. The

iteration of simple affine maps defines in the limit exotic decompositions of the two-dimensional circular bell. The two images here are just examples of an infinitude of patterns that, as they magically interlock with each other and with many more not shown, form perfect circles and truthful bells.

The specific and unexpected geometries shown depend on the precise sequence used to perform the iterations and on other affine function parameters that dictate the number of tips the patterns will have. As you can see, in this central limit there is hidden order in chance.

Page 11. We know by now that all ice crystals and several biochemical rosettes, including the one of DNA, are mathematical designs found inside the bell. Here above on the right is the DNA rosette as seen in biochemistry books, with the double helix itself perpendicular to the shown plane and giving the rosette as a shadow, and below a representation found iterating two suitable affine maps guided by the binary expansion of π.

Page 12. Although all this is quite beautiful and intriguing, there is yet a truly special case that ended up changing my life.

When the parameter z has an ever positive effect on both maps, remember that before it was negative on the second map, one gets a fractal function shaped as a cloud and not as a mountain, and now the limit, when z tends to 1, defines also a bell but now centered at infinity.

What you are looking at here is not the final limit but what is obtained when $z = 0.99$. As z tends to one, the bulk of the cloud goes to infinity, the mean and the variance of the derived distribution tend to infinity, but the coefficient of variation tends to zero, yielding a spike at infinity with probability one. With a bit of imagination we could see how, in a mystical manner, this powerful transformation, maximally positive, raises it all to the clouds, filtering any kind of disorder and dust, including mine, and into an improbable condition of plenitude.

I naturally **took it easy** while pondering this result and normally immersed myself in the special warmth of such a transformation that annuls entropy and yields the prescribed freedom from thorns and dust. I assure you that such was quite a treat, one that, despite my faults, endures to this day.

Page 13. As time passed and inspired by the lack of peace in my home country Colombia and no doubt aided by the pain of my own experiences with division, I started to realize that other complex processes, such as cascades, were also useful to study how we humans create our own turbulence. After all,

we are all confronted by inertial forces that break our internal cohesion and, when such happens, crossing the threshold of our own Reynolds' numbers, produces the rage of our turbulent and seemingly-random behaviors.

Here you see the prototypical cascades propagating imbalances and holes and their associated systems. One, on the left, if even it should be on the right, based on the proliferation of inequalities driven by competition and resulting in the cynicism of toxic assets and bailouts in our days. And the other, no doubt historically defeated but alive, based on discriminations and their related distrust and fear that result while imposing equality by force.

Note how these simple ideas not only reflect the state of affairs of our fractal world, but also remind us of our own selfish postures, negative spirals as depicted on a divisive cascade that coincidentally express that 2/3 of the world population, that is 0.666... of all, live under conditions of poverty.

Observe how out of these not so silly ideas we may appreciate why 6,000 kids die a day because of lack of water, for me this is the disgrace of hydrology, and why we, in truth, have been living for a long time in an era of violence and terror characterized by excessive thorns and ample dust. For the wealth distribution of our United States, circa 1998, with its skewed concentration of wealth, is nicely fitted via the turbulent cascade using precisely the same parameter prescribed to nature, that is, a biased 70-30 coin, a not so nice result that hints at dissipation, as some experience in these days of crisis.

At the end, I think these results reiterate the ancient call for us to "cut the mountains and fill the valleys" in order to achieve the uniform and just condition in which we would like our sons and daughters to reside.

Page 14. For when looking at cumulative distributions we may recognize the essential crossroads that we face, either to miss the origin landing on a devil's staircase associated with a cascade or to converge to the origin via the lovely symbol of the root and the straight hypotenuse, which by not being associated with thorns or dust is hence the true pathway of peace.

What a simple equation reflects what it means to **take it easy**! Only $Y = X$! But this is not trivial at all for it entails living at low Reynolds numbers, having always a fair and positive slope and exercising an unnatural positive spiral to achieve the loving unity we experience when in cloud 9.

Page 15. As you know, the last few decades have seen the advent of power laws in a variety of fields. These are log-log relations with negative slopes that reflect surprising order in the distribution of natural violence, as in

earthquakes above left, but also in other processes that lack characteristic scales such as avalanches, hurricanes, volcanic eruptions and forest fires.

As such an ubiquitous behavior about complexity is also seen in the Pareto distributions of wealth inequalities, within nations and in the world at large as shown here in the middle, and also in the distribution of conflicts and wars, although not perfectly thanks God in regards to WWII, shown on the right, I wondered if we could learn from such laws in order to achieve peace.

In this regard, common sense suggests that the answer is yes and that we ought to avoid the mechanisms that we know produce power laws such as multiplicative cascades, preferential connexions on networks, highly optimized tolerance and the celebrated self-organized criticality, which quite predictably yields avalanches of unpredictable sizes when energies get accumulated.

We indeed need to **take it easy**, for "humans without scale" abuse their essential power and by their selfishness and greed yield power-laws. I say it is best to say no to power laws and yes to powerlessness, for there is no invisible hand growing a sand pile or guiding a rotten market but our own. For ultimately peace is possible first in our own hearts and then in the world, as there is no way to avoid a large war if we do not extinguish the little ones.

Page 16. Now to finish up these reflections, I like to add that I believe chaos theory also provides compelling hints for us to **take it easy** and be good, or if you allow me, urgent hints so that we converge and love one another.

Here on the right you see one of the most important icons of modern science in the bifurcations diagram generated via the logistic parabola, on the left, that yields a plethora of behaviors, from convergence to the origin, below, to a hellish unrepeated traveling in high heat, above, well described by the word chaos. This is a fig tree, that is what feigenbaum means, a symbolic fruitless tree with periodic tender branches and infinite sprouting of buds into thorns over dust, that I think was properly cursed and that provides an improbable but logical fulfillment of an ancient parabola about a second visitation.

For in consonance with the reflections herein and other ancient ones, it is best to come down the tree to **take it easy** at the root. For our optimal logistic is to remain below the ever precious line, so that by converging to the origin we may all avoid biting the dust, which is death.

Page 17. To end, I like to share with you a song I wrote for Rafael some years ago, one that summarizes this work and which I hope you will enjoy. It is called, not surprisingly, "**tómalo suave**."