NOTHING ELSE BUT SHADOWS

From Turbulence to Hydrology and Beyond

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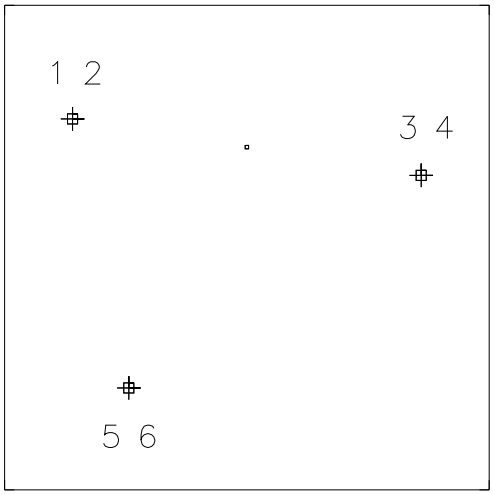
Washington, D.C., March 21, 2012

The Chaos Game

(Barnsley, 1988)



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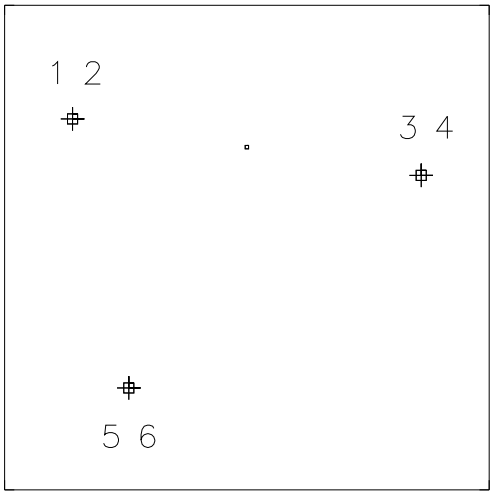

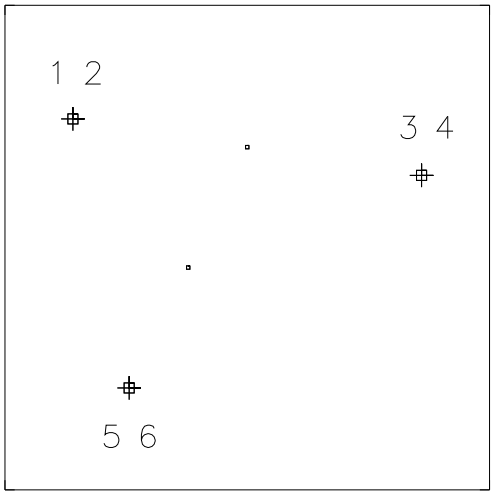

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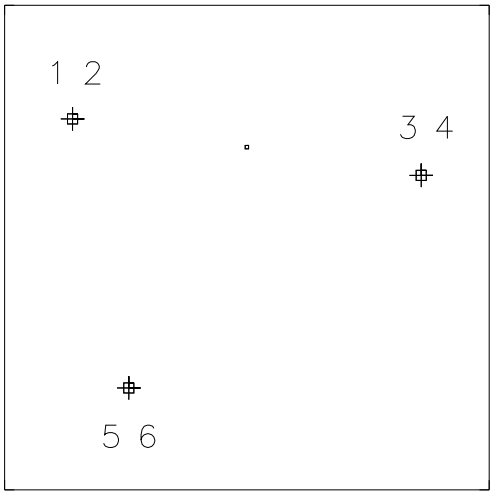

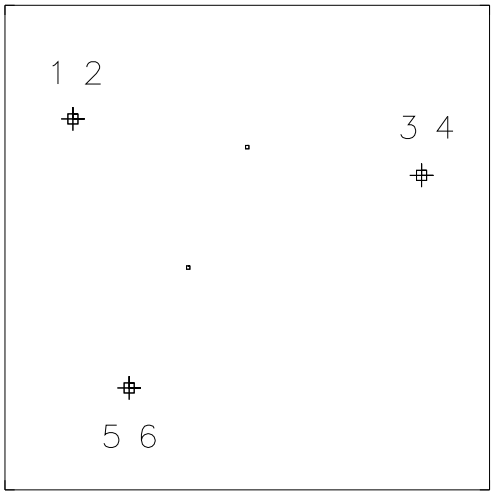

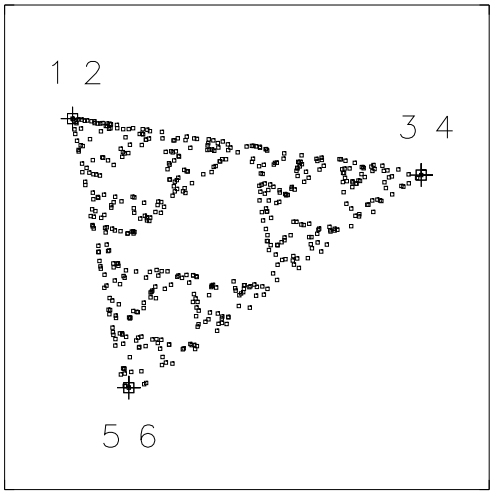

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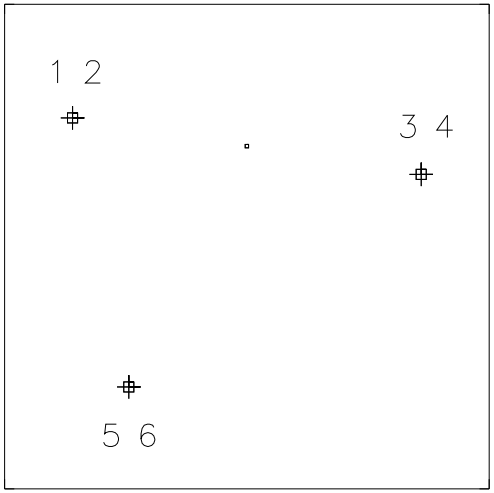

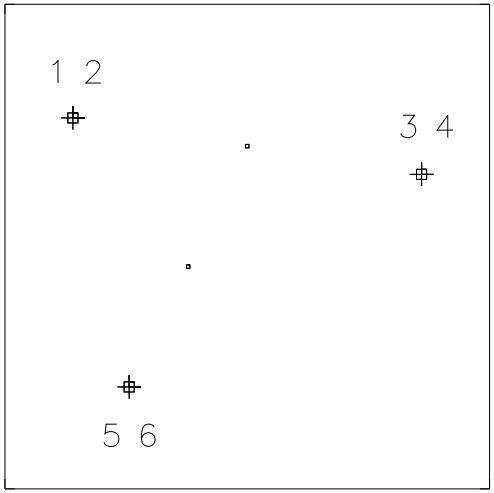

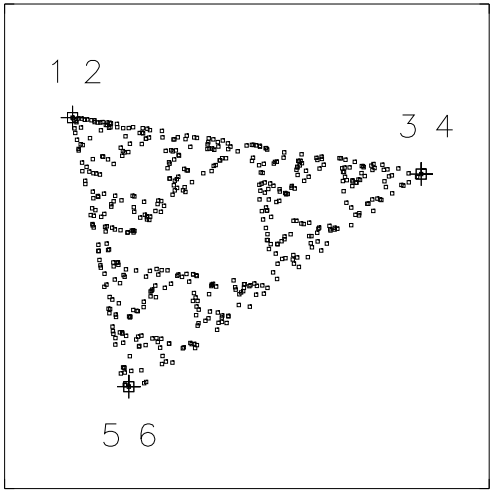

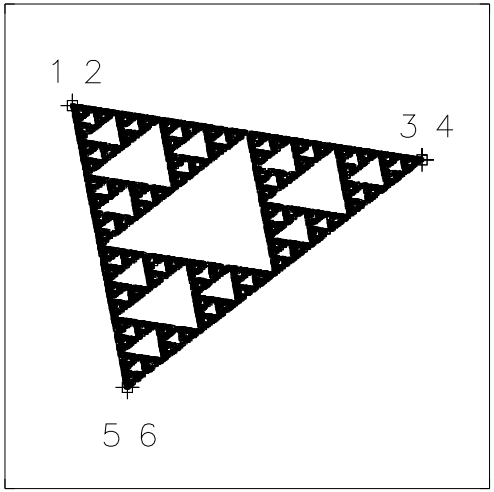

Fractal Interpolating Functions

(Barnsley, 1988)

- Instead of *"move to the mid point"* use the simple maps:

$$w_1(x, y) = (x/2,\ x + d_1 \cdot y)$$
$$w_2(x, y) = (x/2 + 1/2,\ 1 - x + d_2 \cdot y)$$

with the *scalings* $|d_1| < 1$, $|d_2| < 1$.

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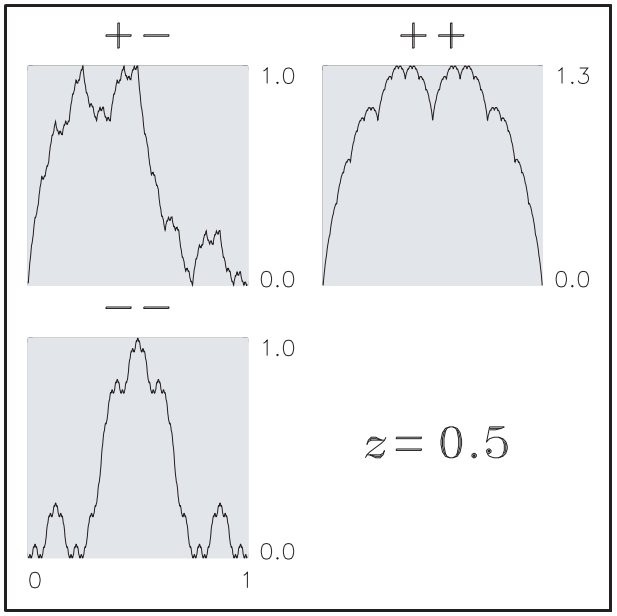
  with the *scalings* $|d_1| < 1$, $|d_2| < 1$.

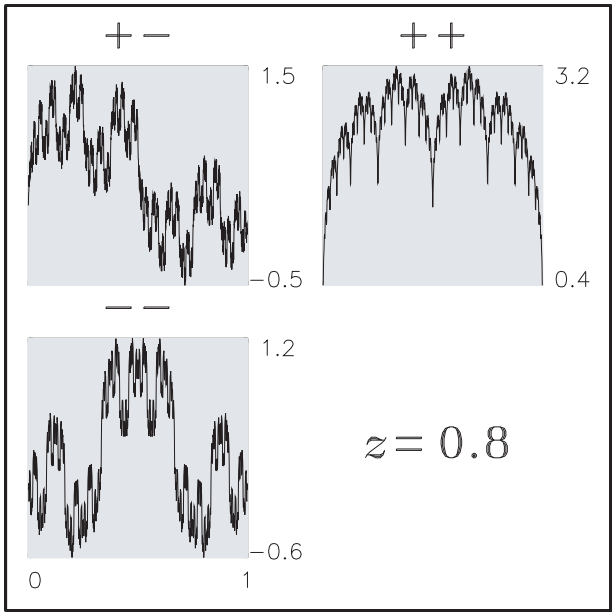
- Iterations yield a unique attractor shaped as a **"wire."**

- Such a set interpolates the points $\{(0, 0), (1/2, 1), (1, 0)\}$.

Fractal Interpolating Functions
(Barnsley, 1988)

- For $z = |d_1| = |d_2|$ and different signs the wires are:





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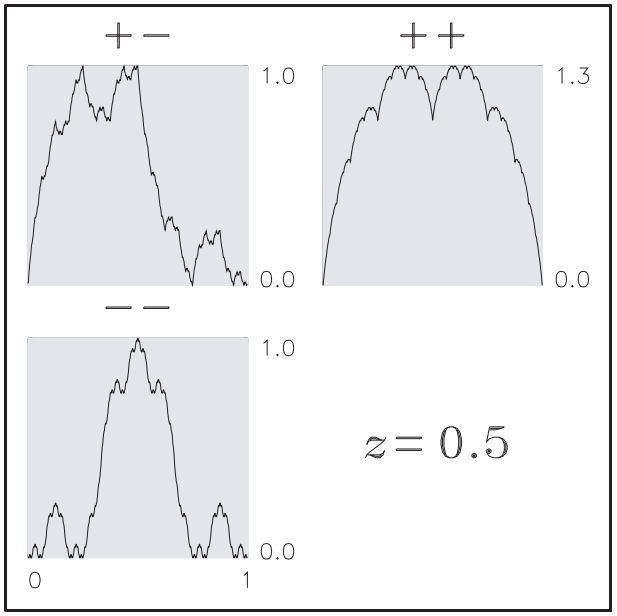
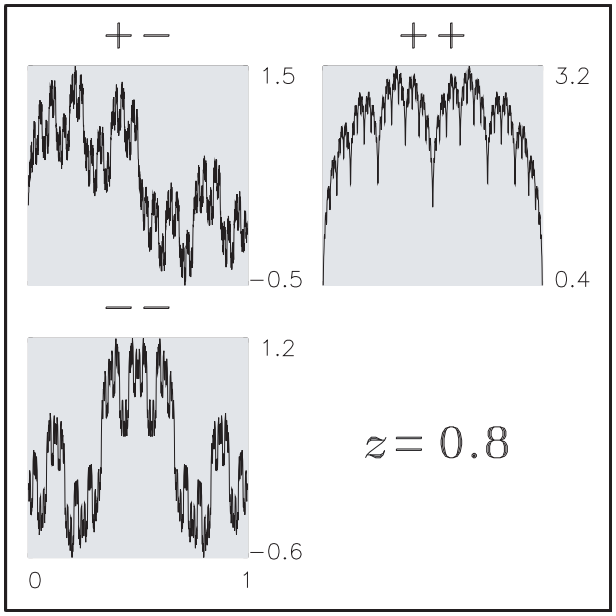


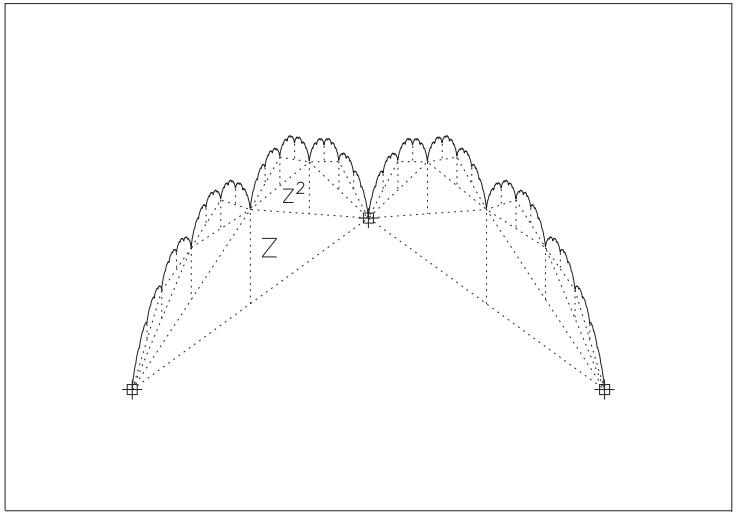


- As $z \to 1$ beyond 1/2, the wires' dimension increases.
- The wires **fill-up** the plane when $z \to 1$.

Fractal Interpolating Functions

(Barnsley, 1988)

- The attracting wires are **simple** as they are self-affine:



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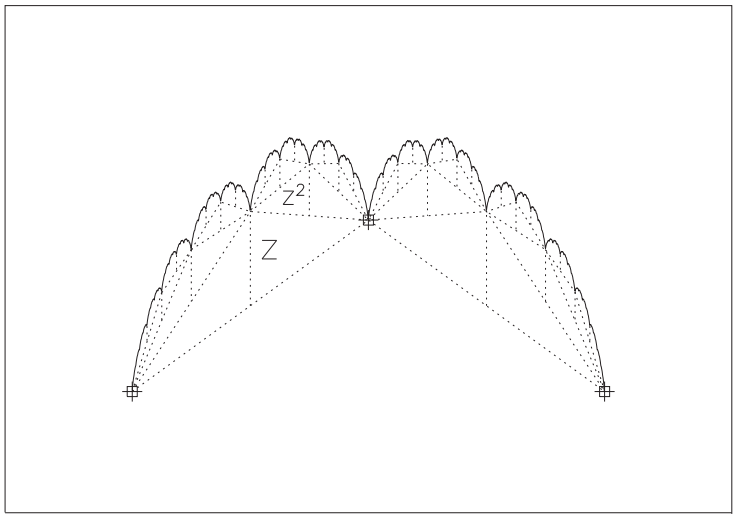

- Other cases have **deterministic** sequences of *ups* and *downs*:

  \+ **-**, *up* and *down* on each level,

  - -, all *down* on first level, then all *up*, and so on, alternating.

Fractal Interpolating Functions
(Barnsley, 1988)

- The same wire is found *point by point* via independent tosses of any "coin" (Elton's theorem).

Fractal Interpolating Functions
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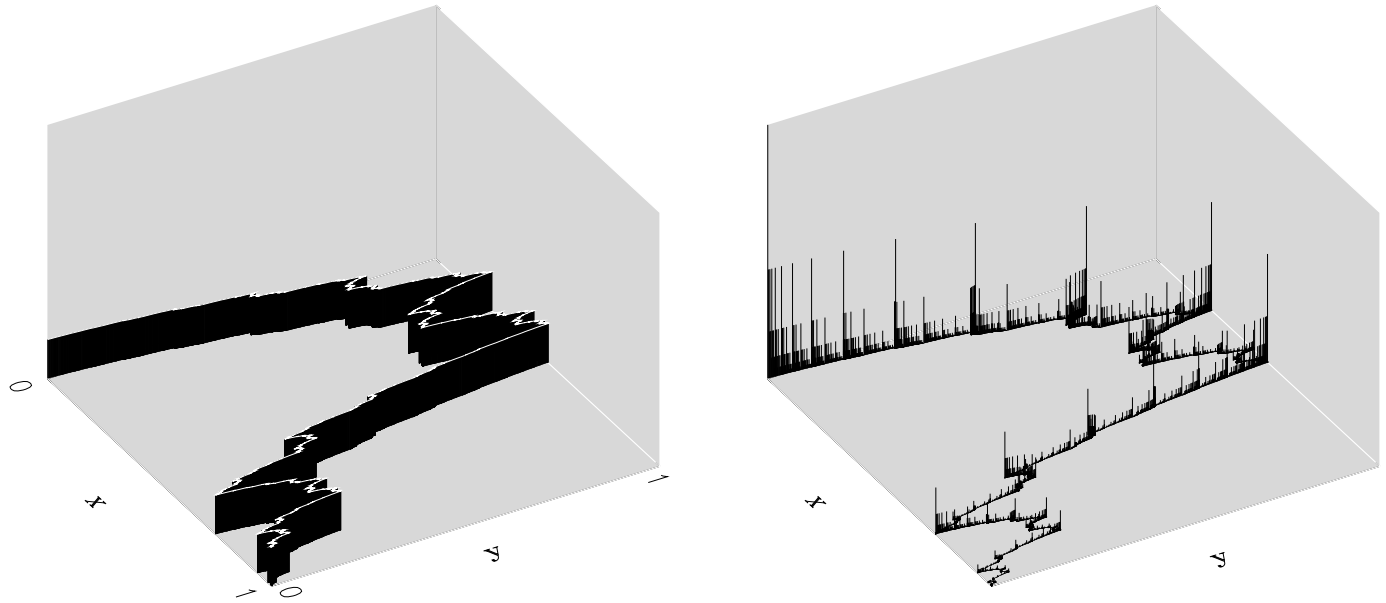
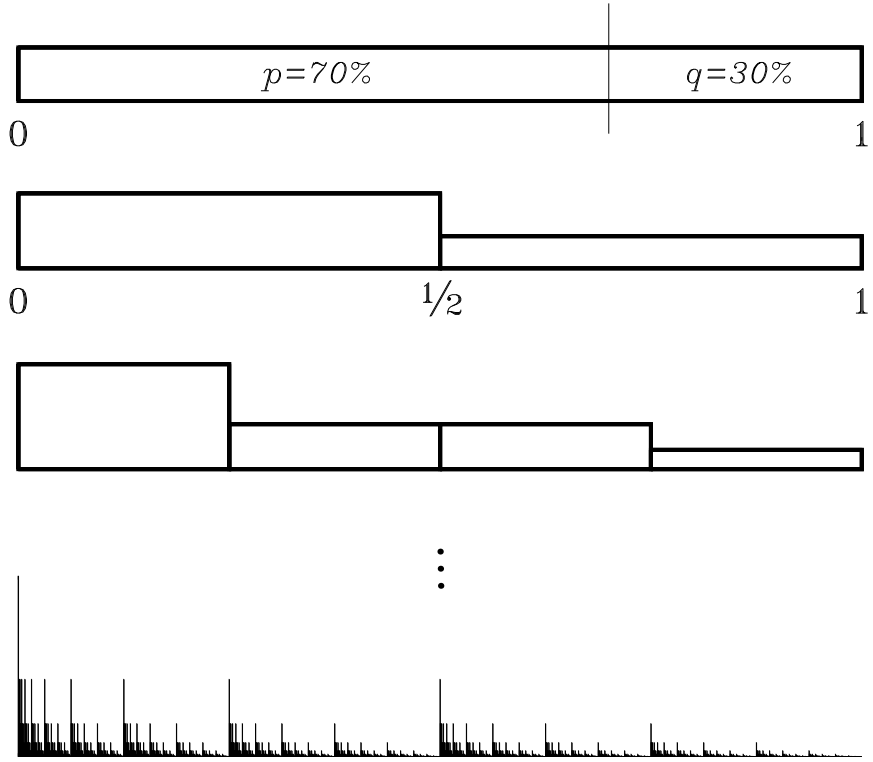
- The same wire is found *point by point* via independent tosses of any "coin" (Elton's theorem).
- However, usage of **fair** or **biased** coins results in rather distinct *textures* over the attractor:



Multifractal Measures
(Mandelbrot, 1989)

- Biased "**shadow**" over x corresponds to a simple **cascade**:



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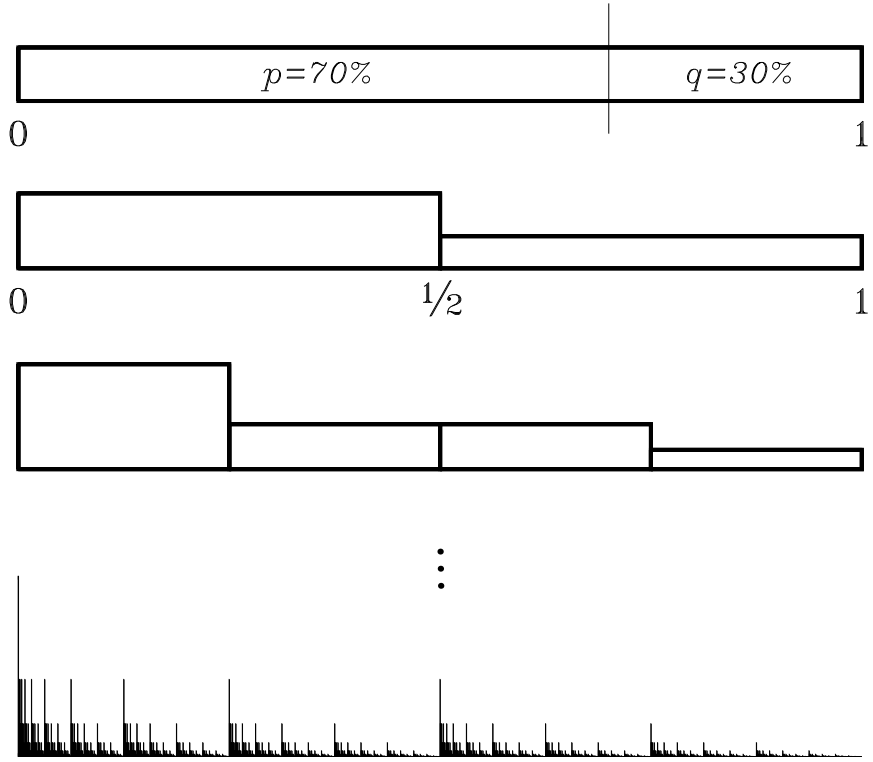
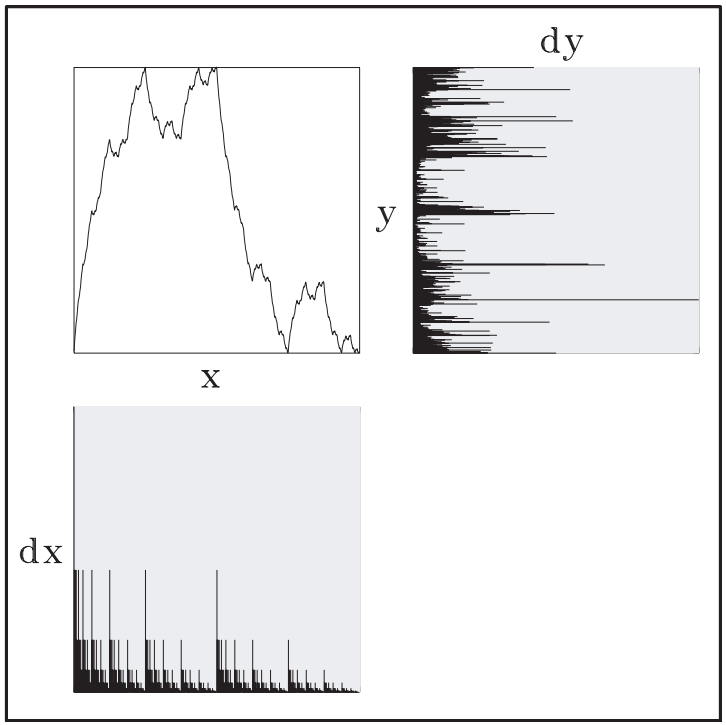


- Such describes dissipation in *fully developed turbulence*.
- Also related to *wealth inequalities* of nations (Puente, 2006a,b).

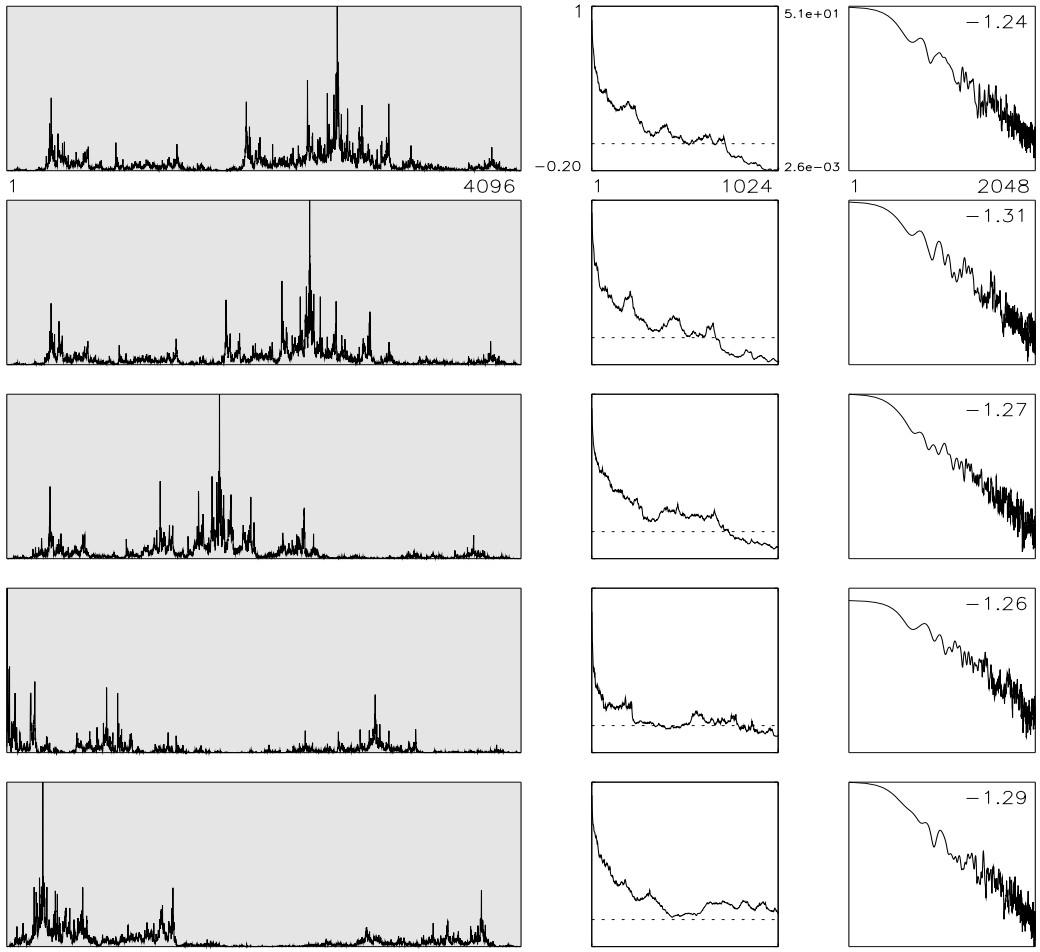
A Platonic Approach to Complexity

(Puente, 1992, 1994a, 1996)



- dx is multifractal "*illumination*" on a wire function f.
- dy is defined as a derived distribution ($dy = dx \circ f^{-1}$).
- dy is interpreted as a "*transformation*" of turbulence.

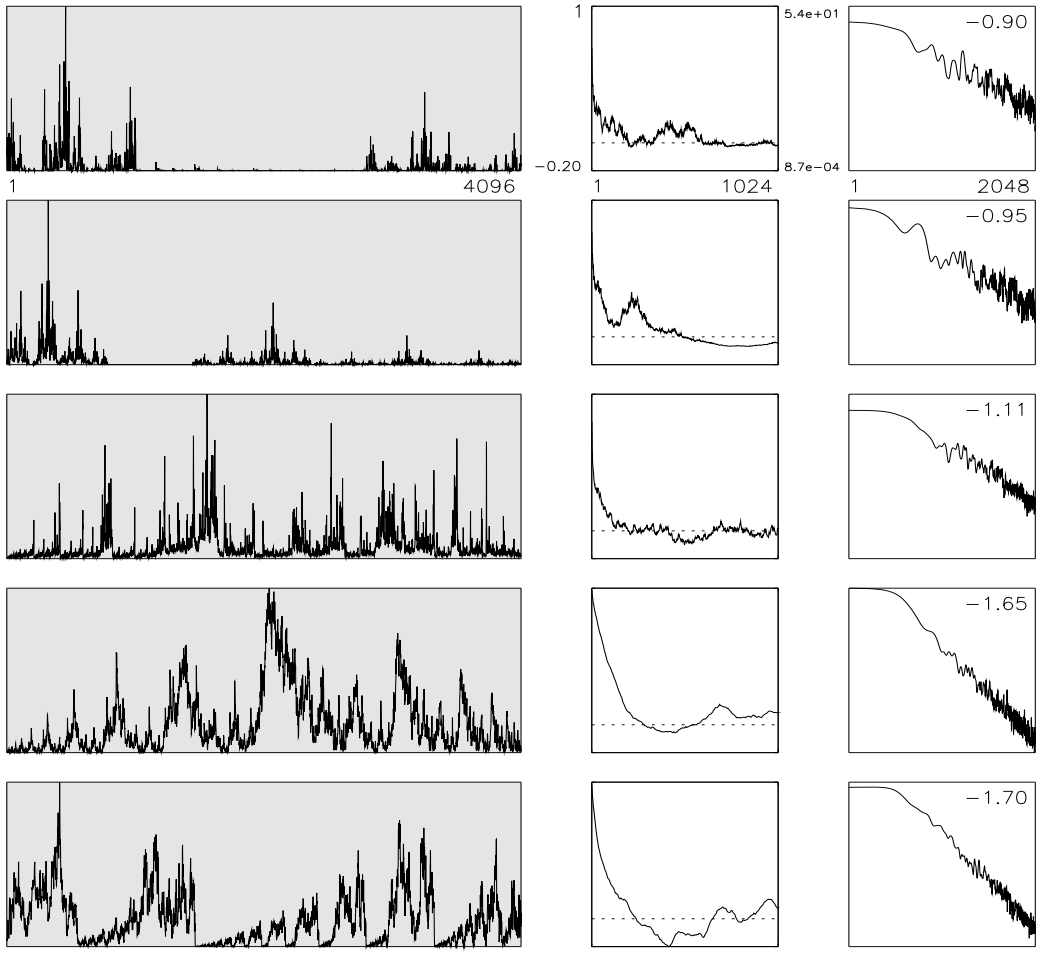
Projections Galore
(Puente, 2004)



- Varying interpolating points...

Other Platonic Designs

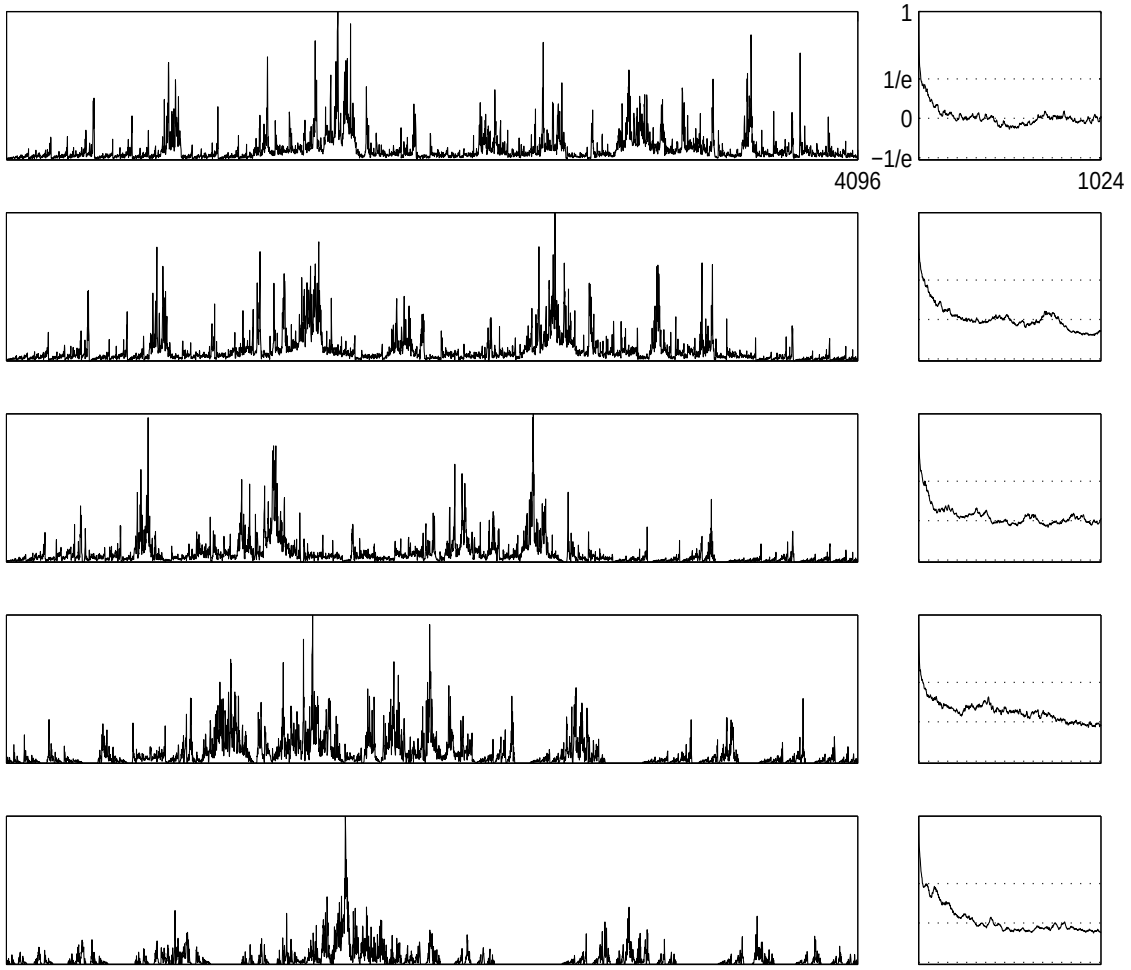
(Puente, 2004)



- Distinct shapes and statistics...

Yet More Platonic Designs

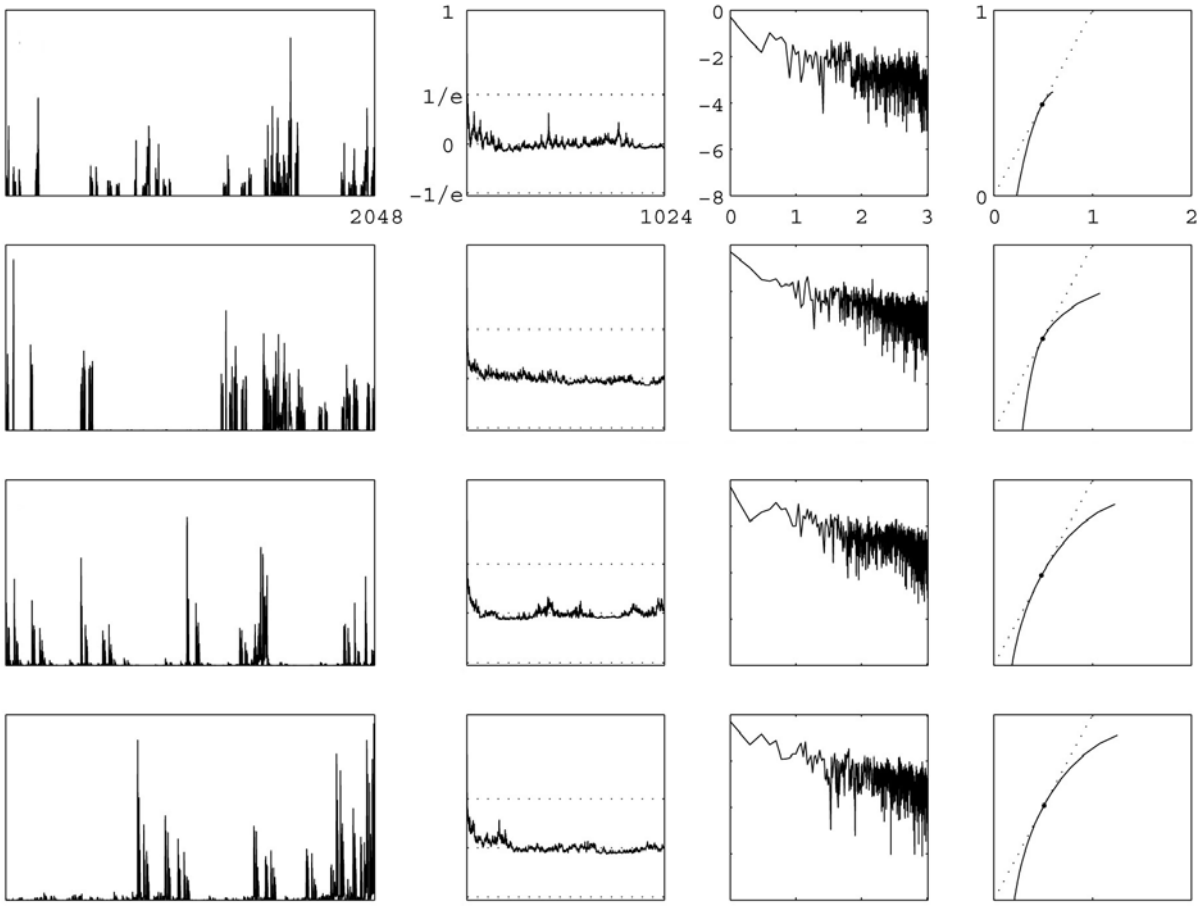
(Cortis et al., 2008)



- Adding a nonlinear *cosine* perturbation on y component...

Real and Simulated Rainfall

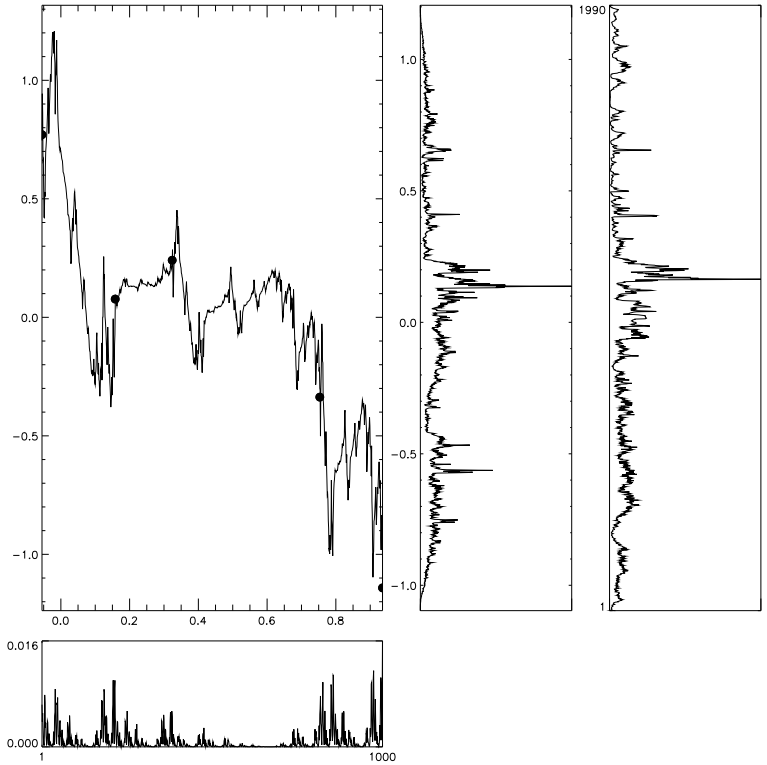
(Puente and Cortis, 2009)



- Above: *real* at La Honda, California.
- Below: simulations via *Cantorian* dx's.

A Detailed Storm in Boston

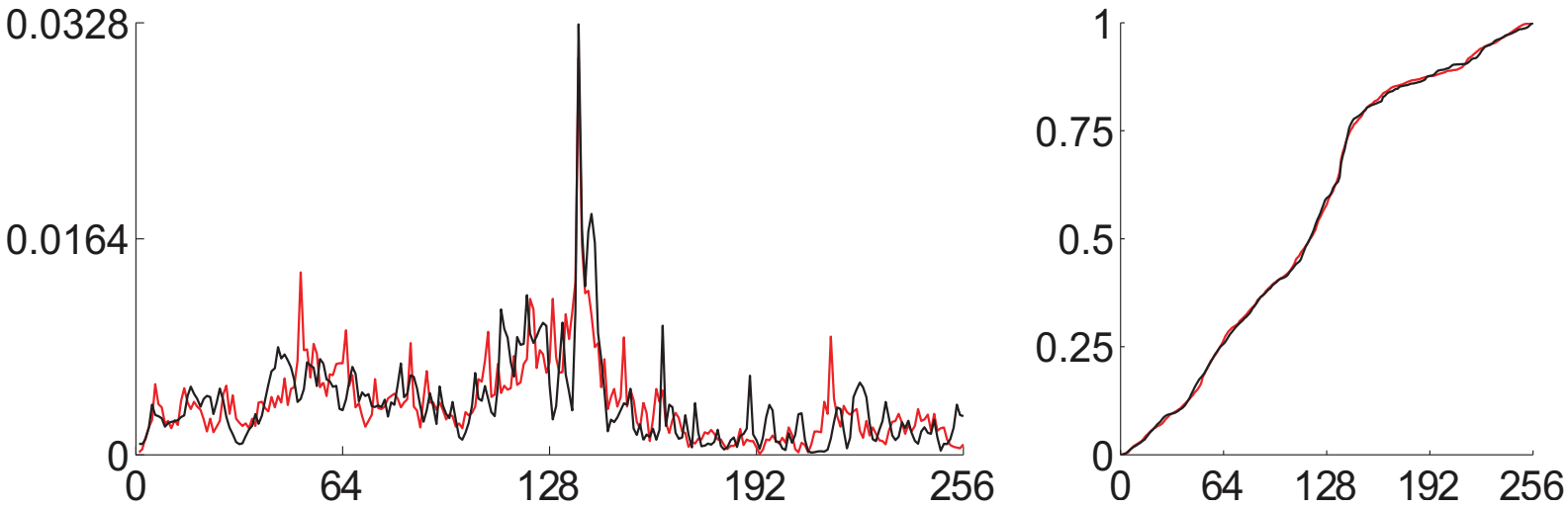
(Puente and Obregón, 1996)



- Required five interpolating points and 14 parameters.
- Obtained via preservation of moments and multifractality.
- Fits correlation, spectrum and chaotic nature of real data.

A Detailed Storm in Boston Revisited

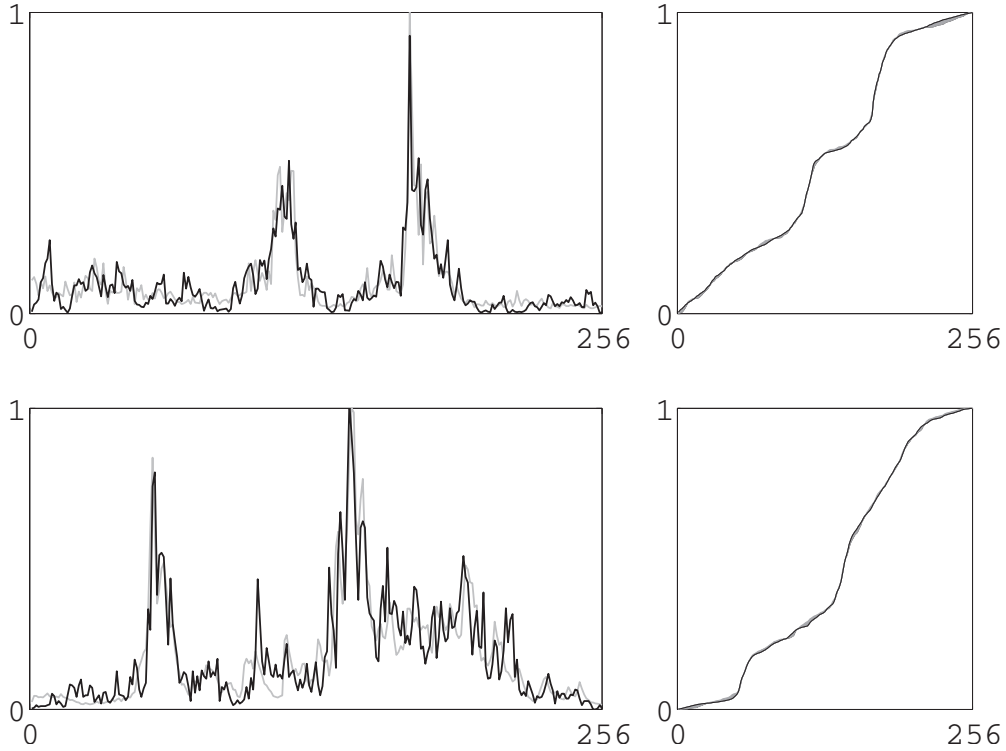
(Puente et al., 2010)



- Via modified particle swarm procedure.
- Minimizing cumulative distribution.
- LS-DATA = 0.23%, LS-CUML = 0.68%, MX-CUML = 1.75%.
- Compression ratio = 140:1.

Other Storms in Iowa City

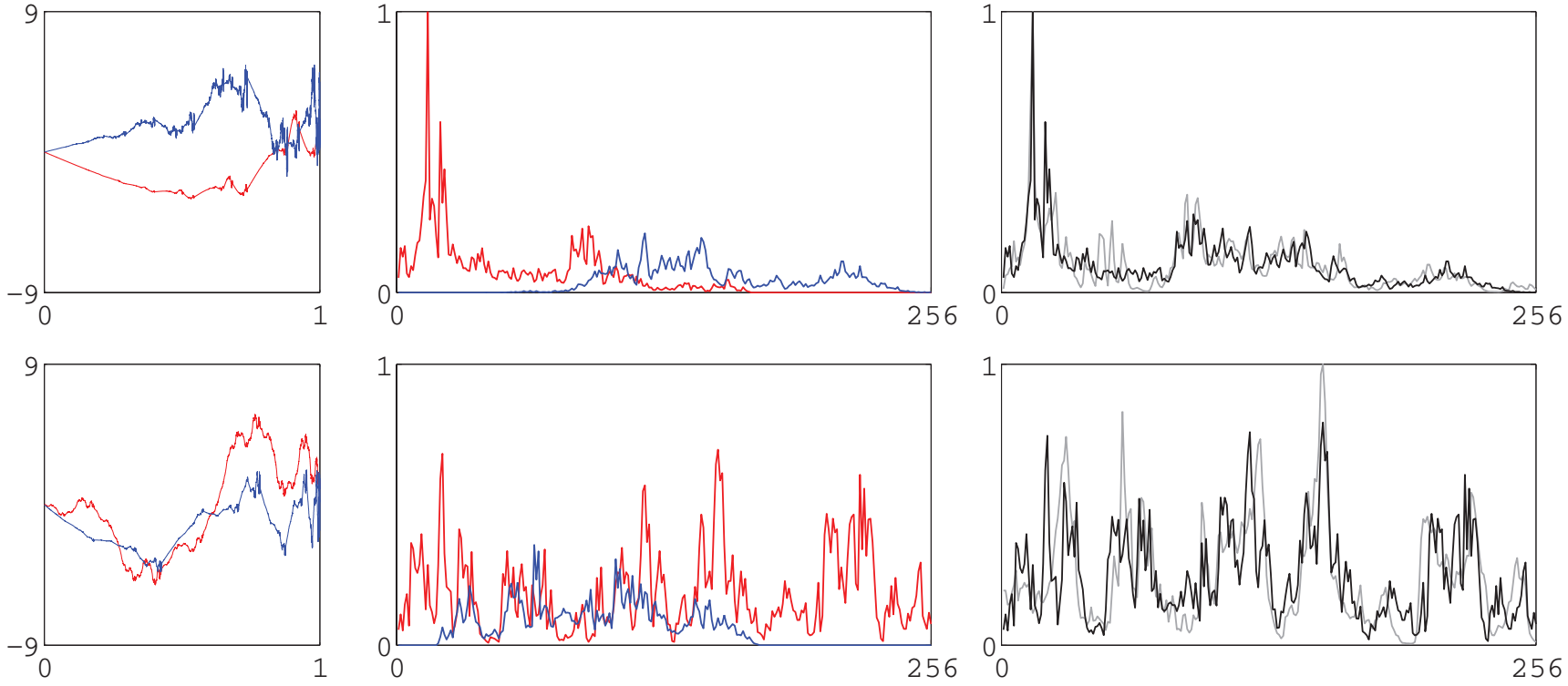
(Huang et al., 2012a)



- Via a generalization using maps with overlapping domains
- MX-CUML = 1.50%, 1.46%
- Compression ratios = 431:1, 531:1

Yet Another Generalization

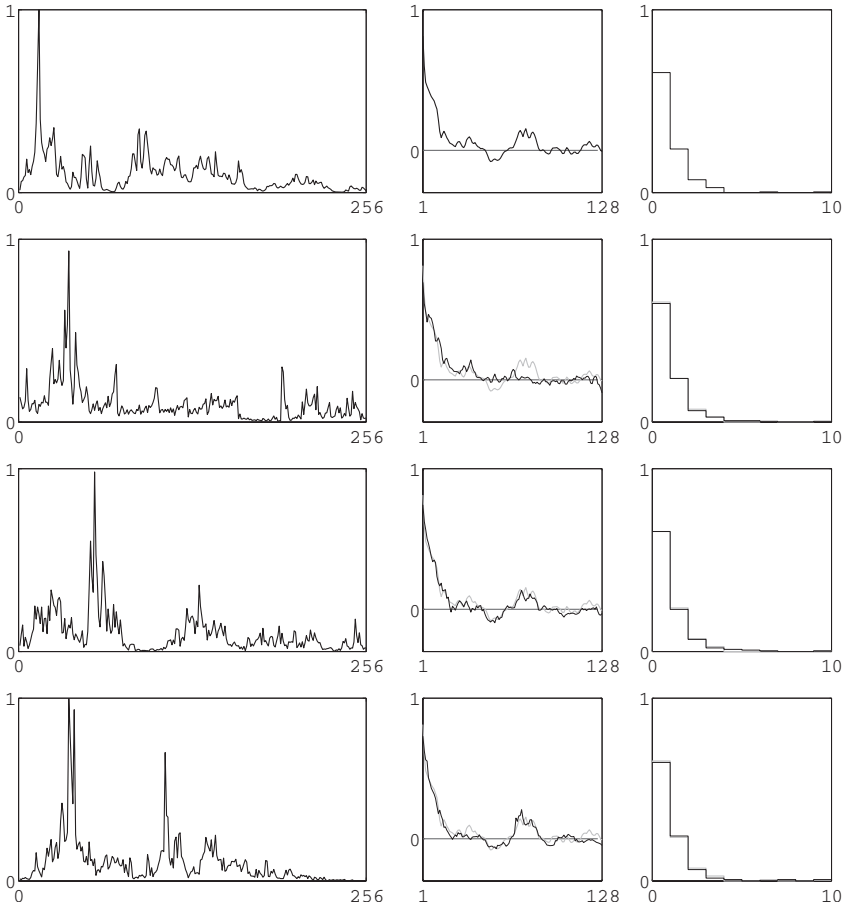
(Huang et al., 2012b)



- Thinking about "gravitons" from particle physics
- More parameters but still sizable compression . . .

Other Rainfall Simulations

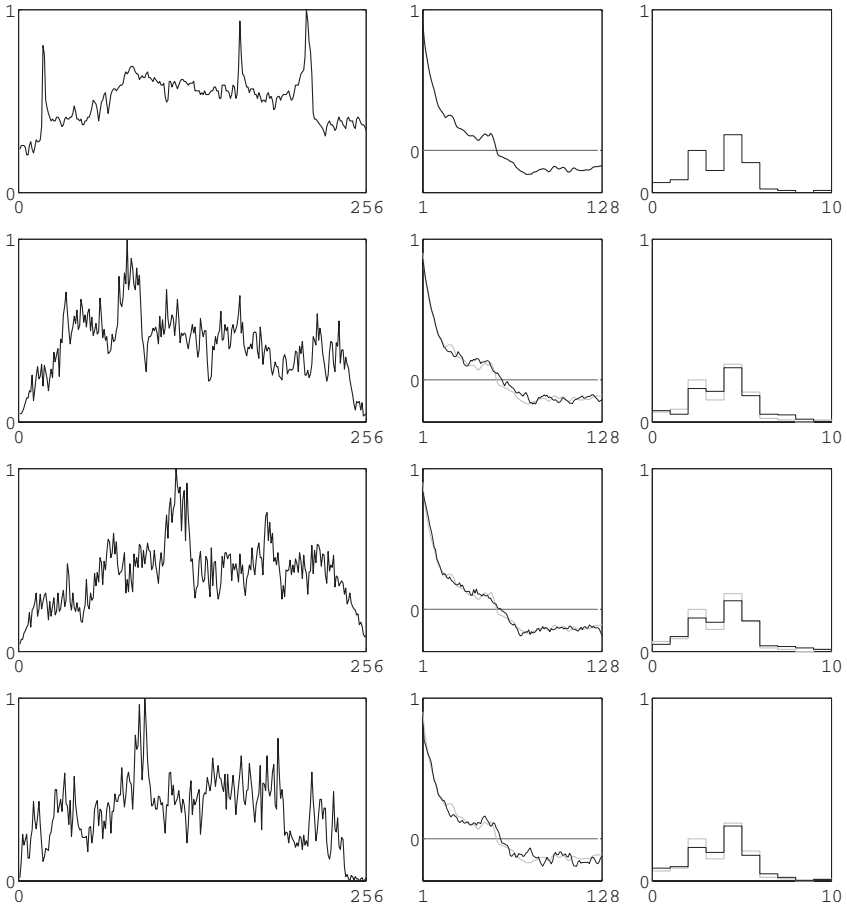
(Huang et al., 2012c)



- As a storm event ($\approx$ 9 hrs) from Iowa City
- Preserving autocorrelation and rainfall distribution

Some Runoff Simulations

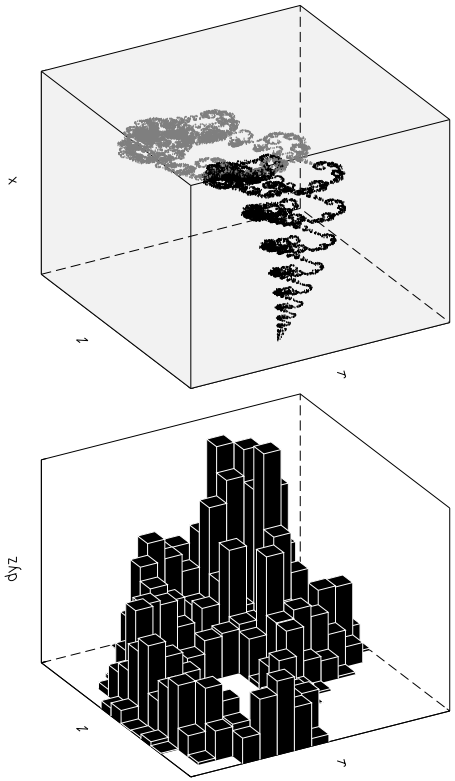
(Huang et al., 2012c)



- A 256 day period at Salt River near Mecca, California
- Preserving autocorrelation and runoff distribution

Extensions to Higher Dimensions

(Puente, 1994b)

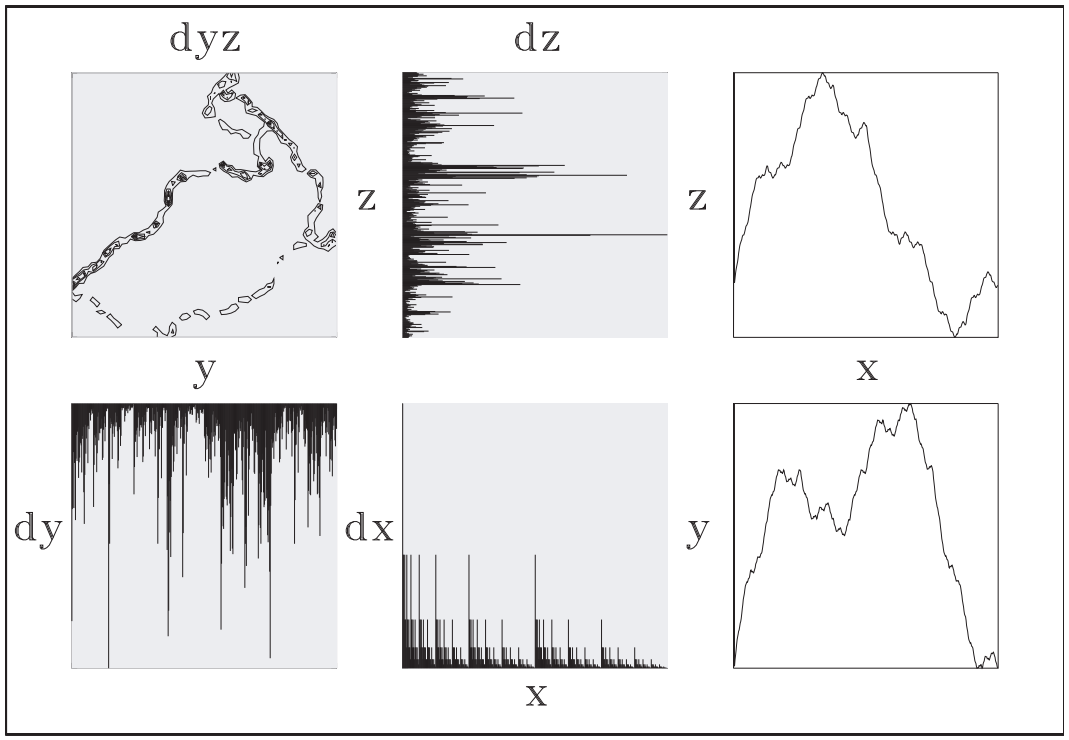


- Mappings:

$$w_n \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_n & 0 & 0 \\ c_n & r_n^{(1)} cos\theta_n^{(1)} & -r_n^{(2)} sin\theta_n^{(2)} \\ k_n & r_n^{(1)} sin\theta_n^{(1)} & r_n^{(2)} cos\theta_n^{(2)} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} e_n \\ f_n \\ g_n \end{pmatrix}$$

Projections Along Several Axes

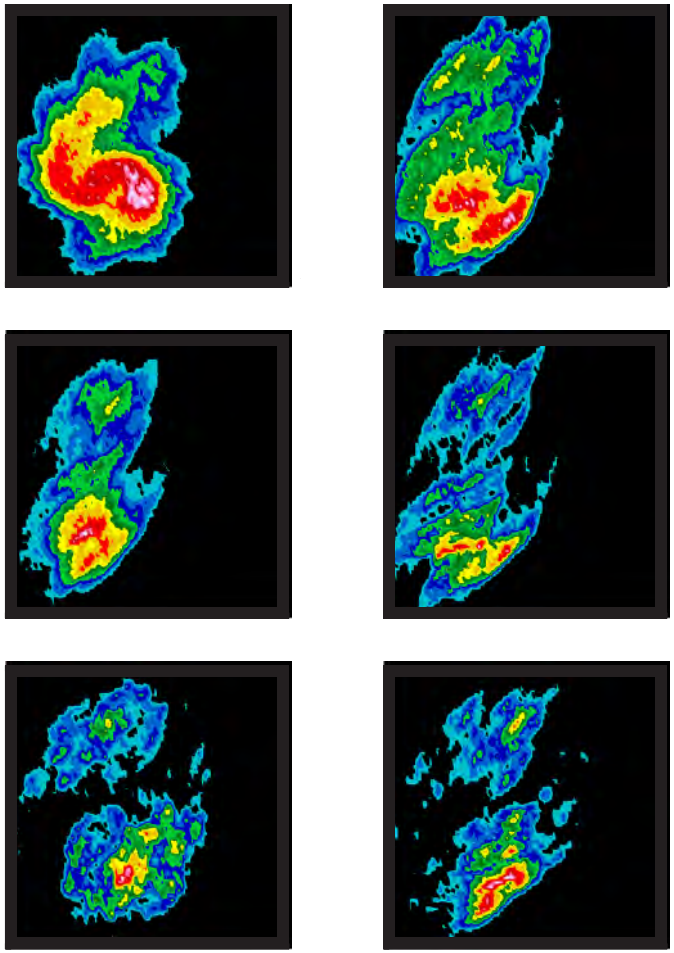
(Puente et al., 1999)



- Could a single wire represent rainfall dy and runoff dz jointly?
- Could the ideas apply to multiple sites?

Deterministic Rainfall Patterns in Space

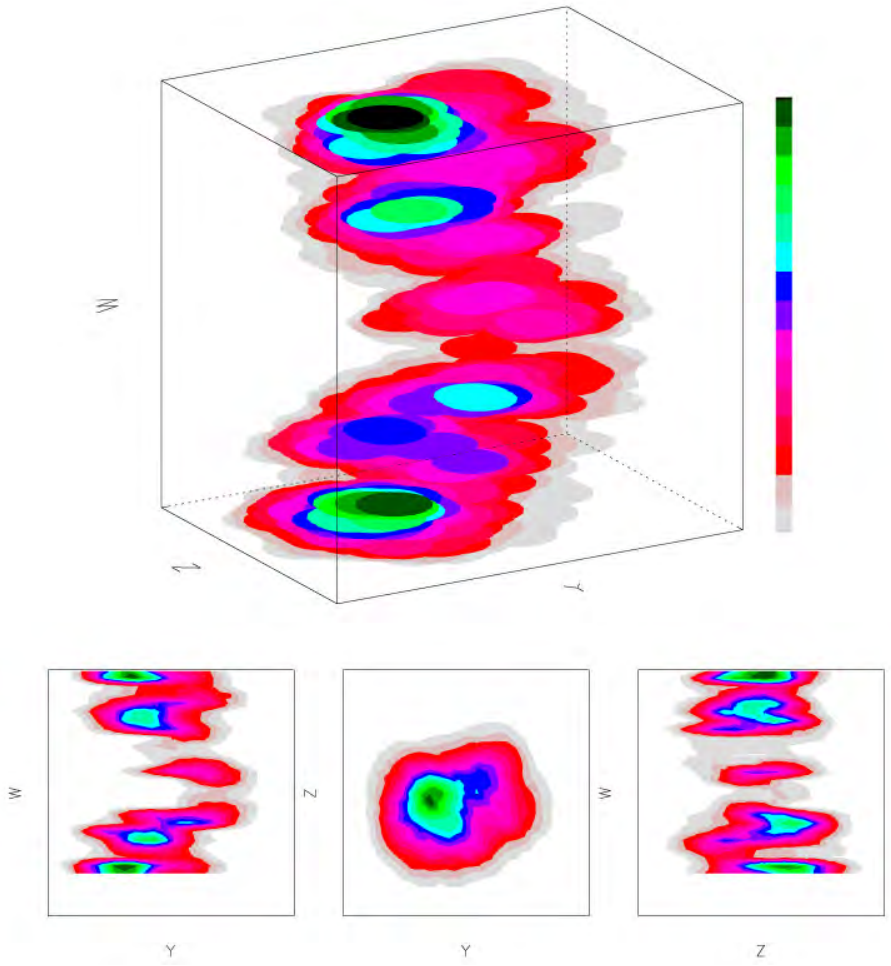
(Puente, 2004)



- From single wires obtained iterating two linear maps.

Complex Pollution Patterns

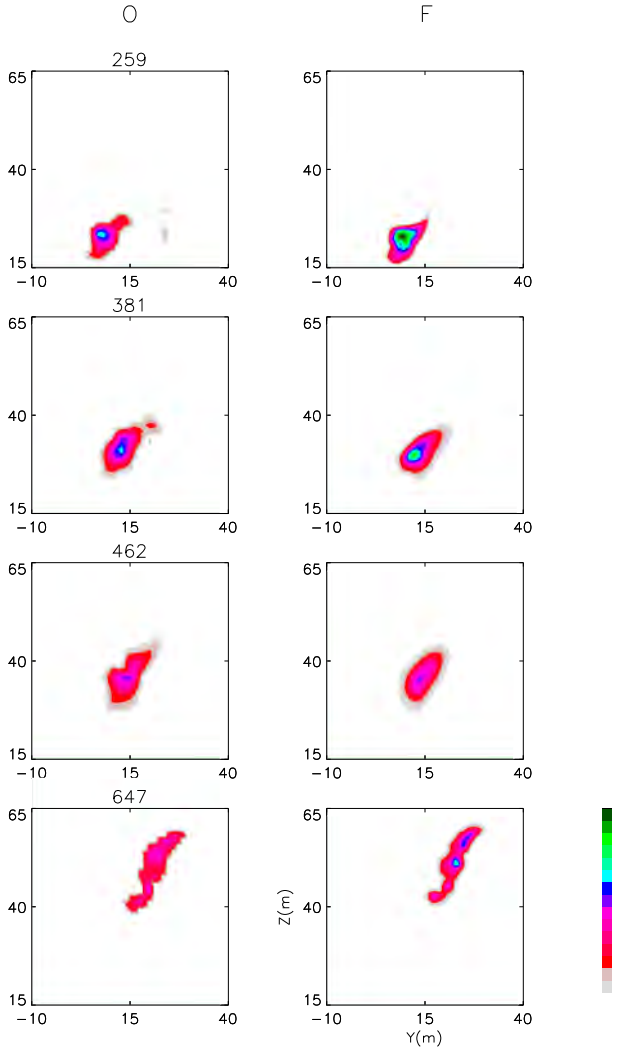
(Puente, 2004)



- From a wire in four dimensions.

Pollution Sets at the Borden Site

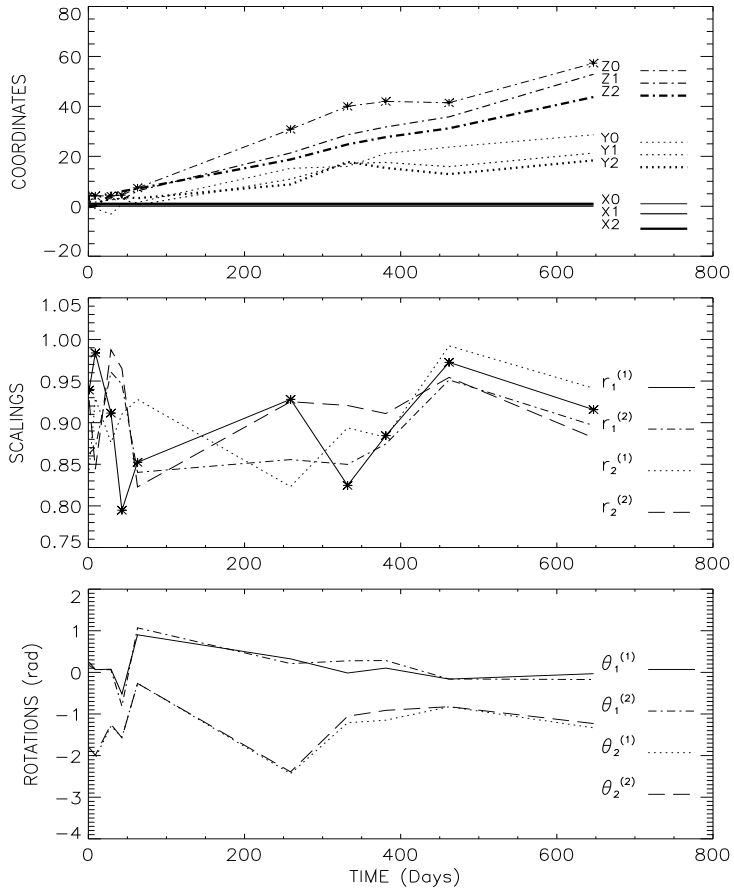
(Puente et al., 2001a)



- Sets: observed, **O**, and fitted via shadows of simple wires, **F**.

Pollution Dynamics at the Borden Site

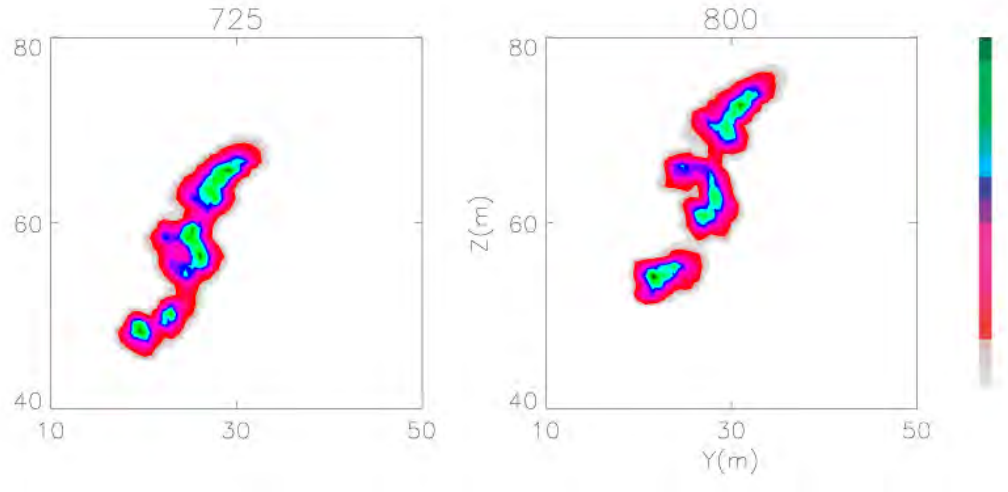
(Puente et al., 2001a)



- Trends in space of wire parameters allow visualizing the plume.

Pollution Dynamics at the Borden Site
(Puente et al., 2001b)

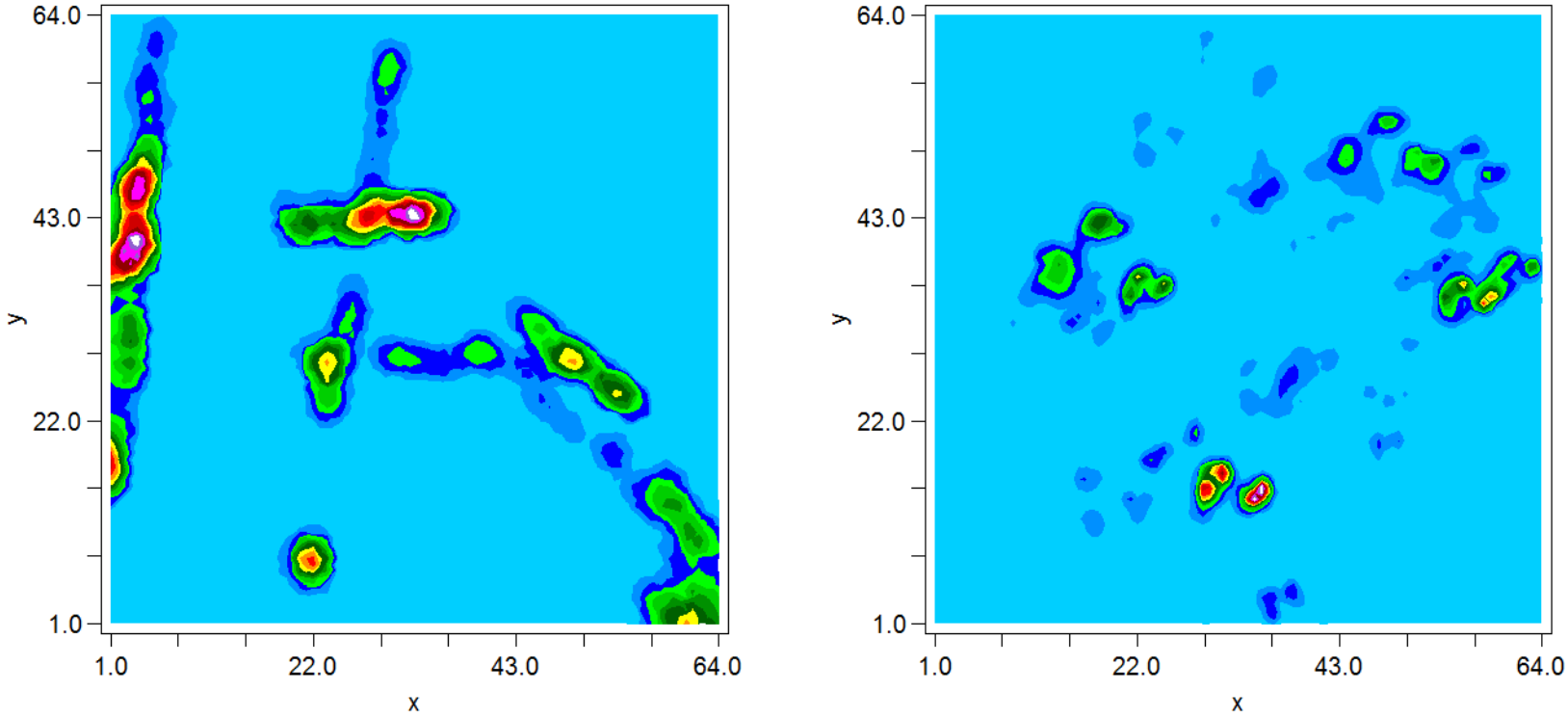
- Same trends in wire parameters allow predicting the plume:



- These predictions turn out to be consistent with observations.
- Such sets are also consistent with predictions via *stochastic* transport models that track the center of mass and dispersion of the plume.

Other Plausible Pollution Patterns

(Huang and Puente, 2011)



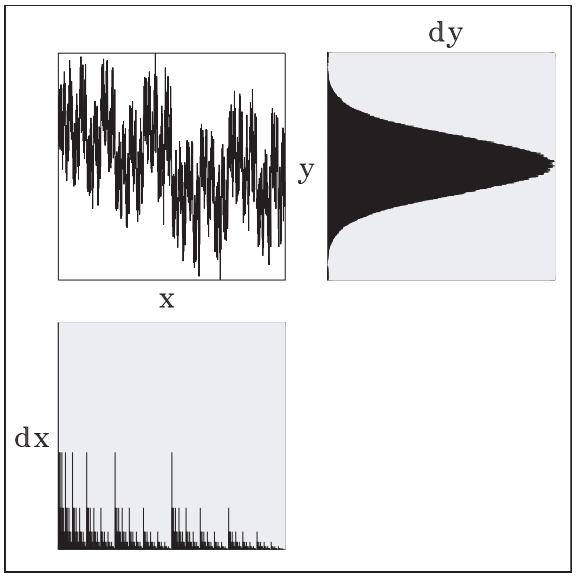
- From attractors in three dimensions.

From Outside the Cave

- The Platonic ideas find **order** in **disorder** and may be used to **simulate** a host of geophysical patterns, in time and in space.
- The "simple" geometric notions, and extensions, may ultimately provide a **compact** deterministic "language" of complexity.
- Finding the parameters for a given set is time-consuming, but if quickly achieved, the ideas may yield new vistas to study the **dynamics** of natural patterns.
- Understanding of the new framework within classical **physics** is yet to be achieved, but the sets surely **parameterize** some complex cascades as employed in stochastic representations.

Exploring the Outer Limit

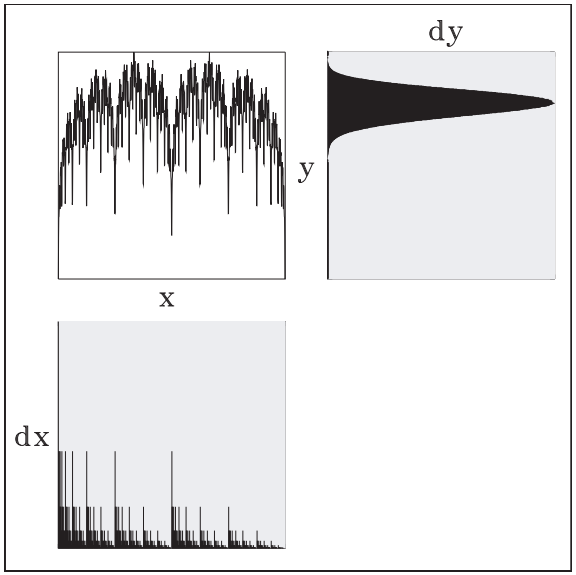
(Puente, 1992, Puente et al., 1996)



- A *plane-filling* function, $d_1 = -d_2 = z \rightarrow 1$, $D \rightarrow 2$, yields a **Gaussian** curve, $dy(y) = \frac{1}{\sqrt{2\pi}}\, e^{-y^2/2}$, **for any** non-discrete dx.
- Such is a surprising bridge from *disorder* to *harmony* yielding an unexpected transformation of *dissipation* into *conduction*.
- The - - case yields, in the limit, oscillations between *two* bells.

A Rather Special Bell

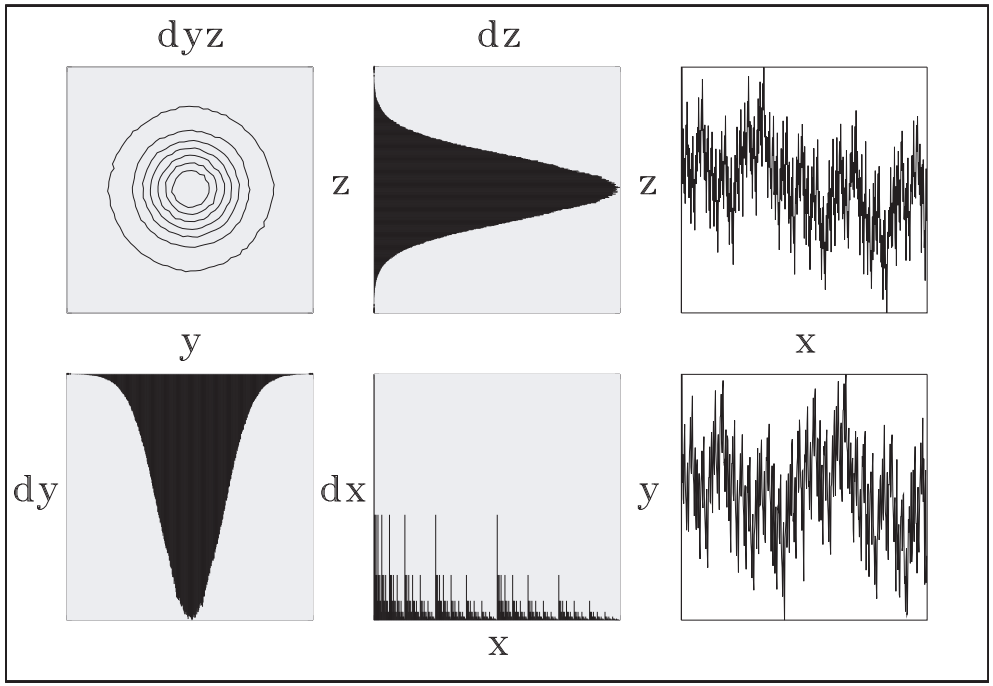
(Puente, 2006c)



- The $+\ +$ case, $d_1 = d_2 = z \to 1,\ D \to 2$, yields a **bell** concentrated at *infinity*, **for any** non-discrete dx: $\mu \to \infty,\ \sigma/\mu \to 0$.
- This lovely case provides yet a more meaningful *universal bridge* to *harmony* for the limit contains no **entropy!**
- The results imply non-trivial *central limit theorems*.

Gaussians Everywhere!

(Puente and Klebanoff, 1994)

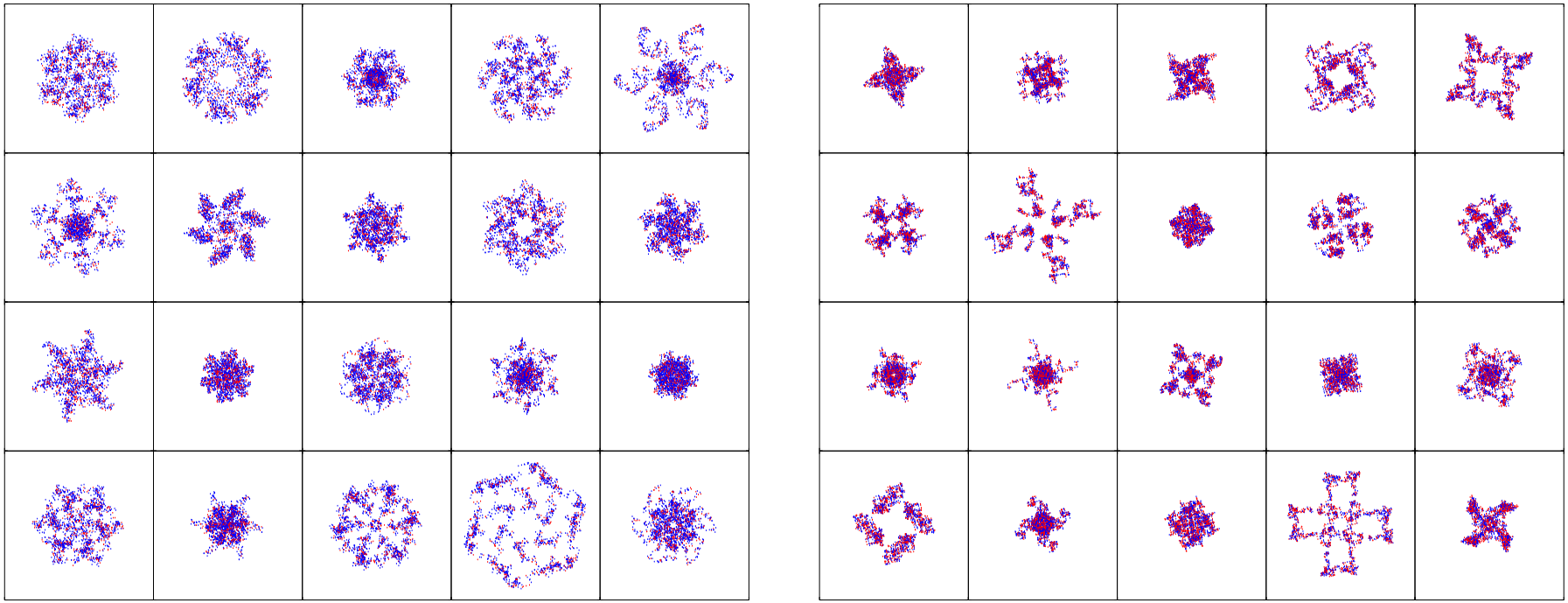


- Always limiting **bells**: $|r_i^{(j)}| \to 1$, $\theta_i^{(1)} = \theta_i^{(2)} + k_i\pi$, for k_i integer.
- *Circular* bells are most common, but also *elliptical* happen.
- There are oscillations among bells depending on *signs* of $r_i^{(j)}$.

Treasures Inside the Bell

(Puente, 1997, 2003a,b)

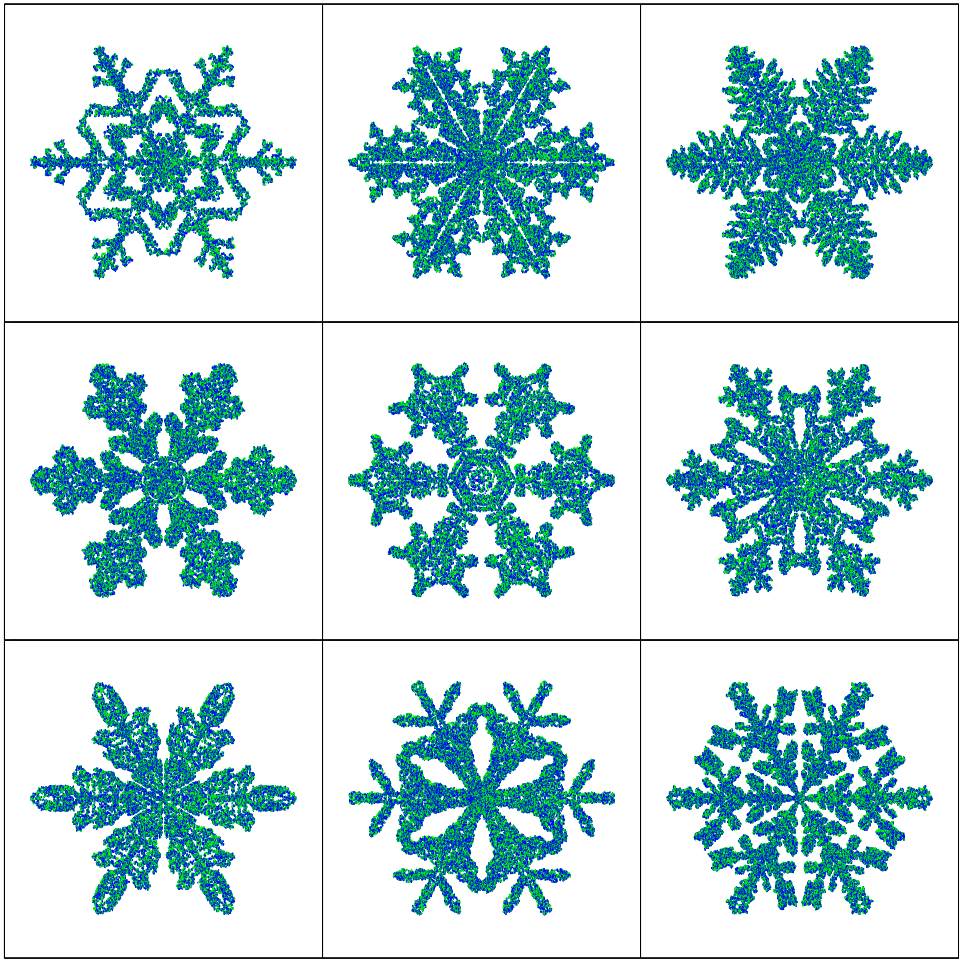
- If angles are *synchronized*, $\theta_1 = \theta_1^{(1)} = \theta_1^{(2)}$, $\theta_2 = \theta_2^{(1)} = \theta_2^{(2)}$ and θ_1 and θ_2 both divide 2π and are multiples of each other, the iterations giving a **bell** produce unforeseen *kaleidoscopes*:



$\theta_1 = \theta_2 = \pi/3$ 2000 dots $\theta_1 = \theta_2 = \pi/2$

Ice Crystals on a Borges' Aleph

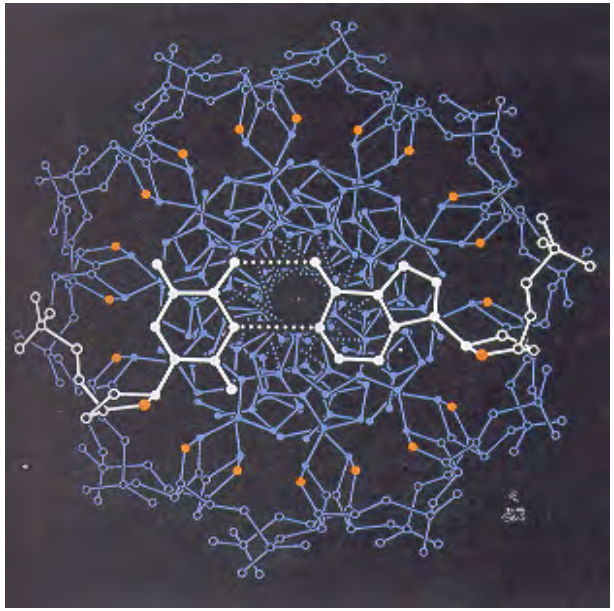
(Puente and Puente, 2004)

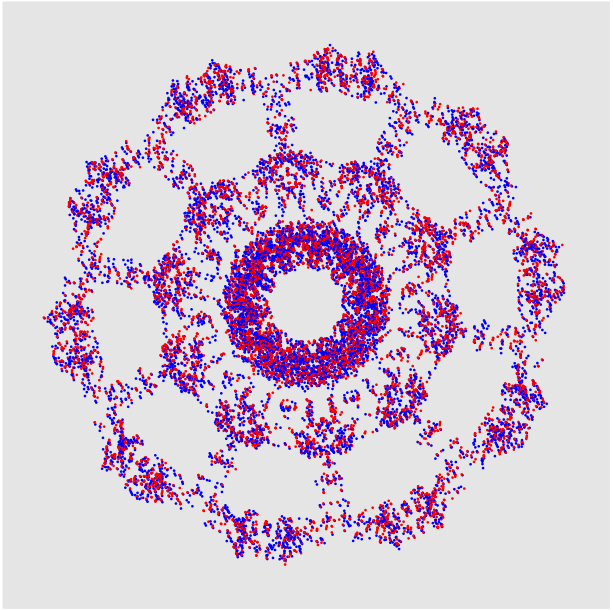


- Filling templates via *fair* "coins" and $\theta_1 = \theta_2 = \pi/3$.

DNA, π, and the Bell

(Puente, 2000, 2003b)





- Not filling a template but just via the binary expansion of π while keeping $\theta_1 = \theta_2 = \pi/5$.
- A rather improbable coincidence requiring 40,000 digits!

The Silent Bell

The bell peals intensely
revealing mighty deed,
in shadows, as by magic,
designs of life crossbreed.

Its central theme exalts
independence as a feast,
reparation of the broken
in enduring code within.

Oh expression of pure beauty,
giving harmony to chance,
Oh quintessential simplicity
ascribing a meaning to π.

In fullness of dimension,
while defeating all strife,
dwells by ardent iteration
a reflection of God's art.

The Amazing Bell

By the mystery of science
graciously shines a state,
an all-embracing alliance
adding liberty a shape.

One day, as if by chance,
boldly there was such gem,
as the shadow off a wire
that fills completely space.

As the ideas hint above
enduring a lasting zest,
here is probable code
in the ever precious bell.

The bell peals silent, oh oh
reflecting its peace,
and inside it gathers
lovely masterpiece.

Symmetric pure beauty, oh oh
oh mighty delight,
this limit in fullness
stores life's designs.

Such vessel contains, oh oh
alephs of all tastes,
diatoms and crystals
including DNA.

But there is a case, oh oh
reason to this song:
the forward selection
that raises it all.

There is clear choice
that rotates the 8.

By loving sincerely
we surely converge.

Notice, this is cogent:
the bell's central theme.

By living in freedom
one fulfills the dream.

There is transformation
that kindles the heart.

By loving in plenitude
we become smart.

For love mends the spiky
and takes to the clouds.

By living the present
one joins blessed crowd.

Oh see, this is truthful:
the plus all the way.

By loving the enemy
we learn how to play.

Dimensional growth,
oh essence of life.

By living in harmony
one nails normal plan.

Oh notice the symbols,
oh irrational might.

By loving simplicity
we experience the light.

Oh listen, you colleague,
let's go out the cave.

By living in unity
we shall all prevail.

Oh notice, my friend,
the plea from a bell.

By loving and loving
joy will have no end...