Title Page

Today we are going to talk about recent advancements towards the applicability of the Fractal-Multifractal approach for the encoding and simulation of daily rainfall records, which are highly intermittent in nature.

Slide 2

This talk starts with a brief overview of the fractal-multifractal method and one of its extensions, which we adapted to approximate the daily rainfall records. Then we will go over the strategy of our approach through relevant calculations, including an inverse problem. This will be followed by a demonstration of the Fractal-Multifractal method to encode rainfall records, first over a year in Washington and, then, over a year and 20 years in Bolivia. In addition, we will present, by the end, some plausible rainfall simulations for the Bolivia site. Then, summary with conclusions are advanced.

Slide 3

While available physical and stochastic models require a host of geophysical parameters and still do not capture finer details inherent in the natural records, following Mandelbrot's seminal work on fractal geometry, Puente thought that "random looking" natural patterns could be modeled as shadows of attractors obtained as transformations of multifractal measures.

As shown here, an input multinomial multifractal measure dx when transformed by a fractal function f, passing by four points, produces a random looking pattern dy that contains a non-trivial structure.

Slide 4

In order to approximate the intermittency present in rainfall records, one can adapt the fractal-multifractal method in two ways. First, we may prune the input measure dx below a horizontal threshold ϕ_h to account only for large enough eddies, or second, we may trim the derived measure dy below a vertical threshold ϕ_v in order to exclude rain traces.

Slide 5

As such, the derived measure dy in the second column is now modified, either horizontally or vertically, into the last two patterns dy_h and dy_v, which contain many zeroes and the overall feel of rainfall sets.

Slide 6

A variant of the original form of FM method may also be used so that the input is a Cantorian measure and the transforming attractor is a cloud of points rather than a continuous function. Once again, derived measures dy, following adaptations, turn out to approximate the intermittency of rainfall sets.

Slide 7

The mathematics behind the constructions depend on the successive iterations of affine maps as shown here, subject to contractile initial conditions for each map that guarantee the convergence to a stable attractor. As a result, the parameters that determine a given FM set are: the end points by which the maps pass, the vertical scalings dn, the frequencies used to carry the iterations, pn, and the value of a threshold, horizontal or vertical.

Slide 8

Although the procedure is simple and deterministic, the FM parameter space is highly complex. This leads to a numerical solution of the inverse problem for a given set, which we performed via a generalized particle swarm optimization scheme that minimized either one of the following objective functions: root mean square error in accumulated rain for encodings and root mean square error in histogram or entropy for simulation purposes. In addition, we accounted for the number of zeros in the records while doing simulations.

Slide 9

In order to test the capability of the FM approach, daily rainfall records gathered in Tinkham, Washington and Laikakota, Bolivia, were encoded. This slide shows two encodings in Tinkham for the water year 1993-94. Notice how the FM approach based on a Cantorian input, and relying on 9 parameters, captures the intermittency present in the records. Such is reflected in the close fitting of the accumulated rain with rather small root mean square error and maximum error in accumulated rain.

Slide 10

This slide presents two examples of encodings at the Bolivia site. As seen, results are comparable to the ones shown for Tinkham and the fitting errors are rather small, certainly within the limits of accuracy in rainfall measurements.

Slide 11

After such remarkable encodings of rainfall records were found, this slide shows twenty successive years of rainfall in Bolivia from 1964-83, once again encoded via the Cantorian approach and a vertical threshold. As seen, the FM approach is indeed capable of representing such twenty years one by one, and the overall fits, relying on 9 parameters, have a root mean square error and maximum error in accumulated records, which have averages of 1.8 and 5.6%, with rather small standard deviations.

Slide 12

On such 20 years of encodings, various statistical attributes were evaluated in order to assess the goodness of FM method. As seen here, the FM approach (in red) fits very well both Renýi's entropy function and the histogram of the records. As shown below, the overall accumulated records (accounting for the total amount of rain on individual years) become indistinguishable from the observations.

Slide 13

The goodness of FM approach may also be seen by the rather close fittings on inter-arrival times of rainfall sets for different thresholds. Overall, the geometric approach performs rather well in all fronts.

Slide 14

As you may be wondering, here is the evolution of the FM parameters over the 20 years. Yes, the parameters exhibit, as expected, noticeable variability, which perhaps may be employed to assess rainfall complexity in nature.

Slide 15

Till now, we have shown that acceptable encodings of daily rainfall records may be found. Now we will show that FM method may also be used to find suitable rainfall simulations, by preserving the record's histogram, entropy and number of zeros. This slide shows two examples found by minimizing histogram errors for a data set in Bolivia, one via a fractal function and continuous input and the other via a Cantorian representation. As seen, the two patterns shown are indeed plausible, as they have similar histograms and entropy functions (with Nash-Sutcliffe efficiencies close to 100%). They also have similar autocorrelation functions that decay rather quickly.

Slide 16

This slide shows two simulations of the same Bolivia records based on preserving Renýi's entropy function. These examples turn out to be also plausible, as Nash-Sutcliffe efficiencies are once again rather close to a 100%.

Slide 17

Finally, this page shows yet other plausible simulations found minimizing the discrepancies in the number of zero values, that is, non rainy days present in the records. For this case, we tried to preserve the maximum of consecutive non rainy days as well as the number of zeros in the set. As seen, the results are good in all fronts.

Slide 18

So, in summary:

The Fractal-multifractal approach may be used to encode and simulate complex rainfall sets gathered daily and for the duration of a water year.

Such encodings do not only capture the accumulated rain patterns, but also suitable statistical information such as histogram, entropy and inter-arrival distributions for various thresholds.

Simulations may be done based on histograms, entropy, and distribution of zero values. They represent alternatives that complement stochastic methods.

The evolution of FM parameters may perhaps be used to quantify the complexity of rainfall sets.