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Encoding and Simulation of Daily Rainfall Records via Adaptations of the Fractal-Multifractal Method

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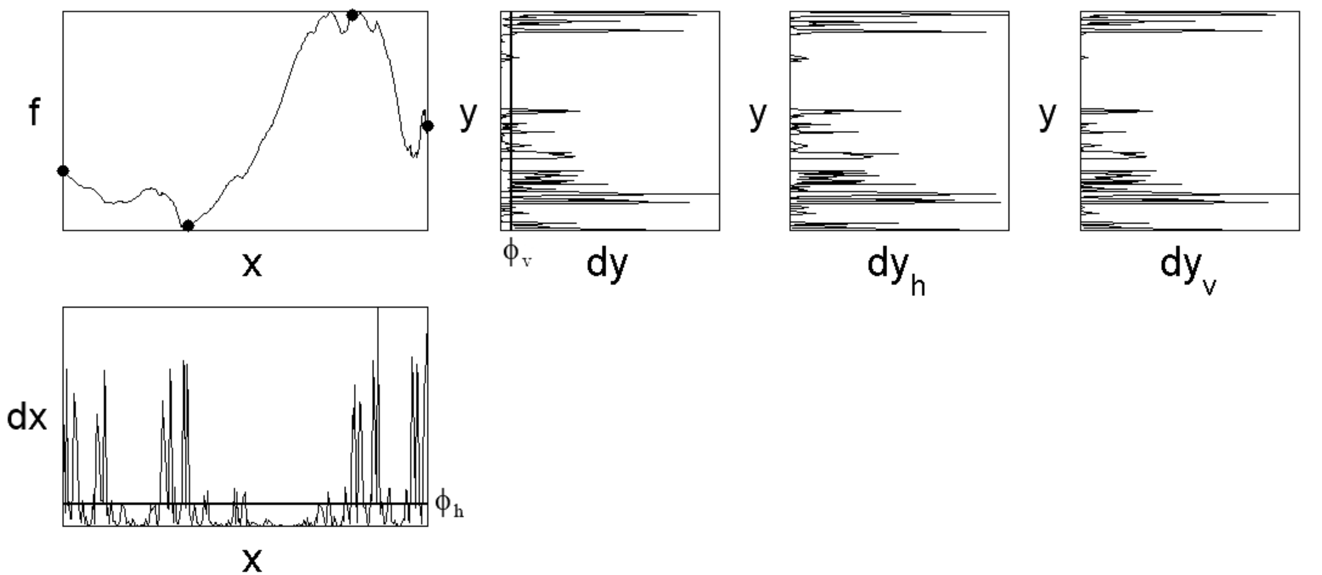
Outline

- The fractal-multifractal method and an extension
- Relevant calculations and the inverse problem
- Some encodings for Tinkham, Washington
- Some encodings for Laikakota, Bolivia, for a year and 20 years
- Some simulations for Laikakota, Bolivia
- Summary and Conclusions

The Fractal-Multifractal Approach and Adaptations

(Puente, 1996; Maskey et al., 2015)



from a **multifractal** dx to a **derived** dy via a **fractal** function

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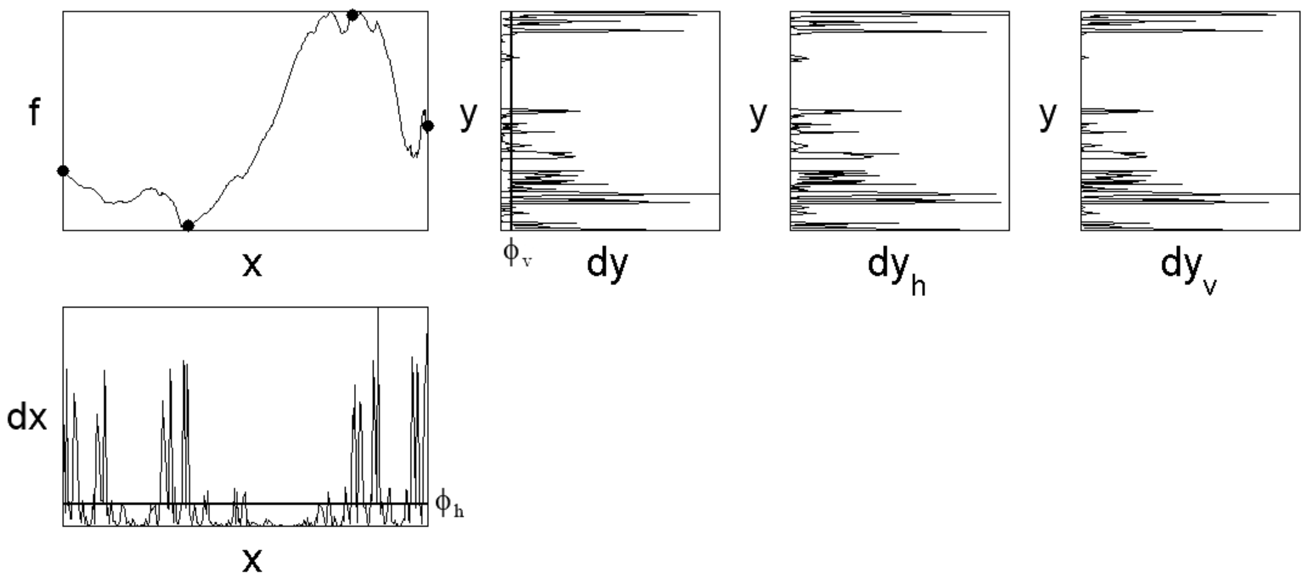

from a **multifractal** dx to a **derived** dy via a **fractal** function

dy_h is found pruning dx below ϕ_h (small eddies)

dy_v is obtained trimming below ϕ_v (rain traces)

The Fractal-Multifractal Approach and Adaptations

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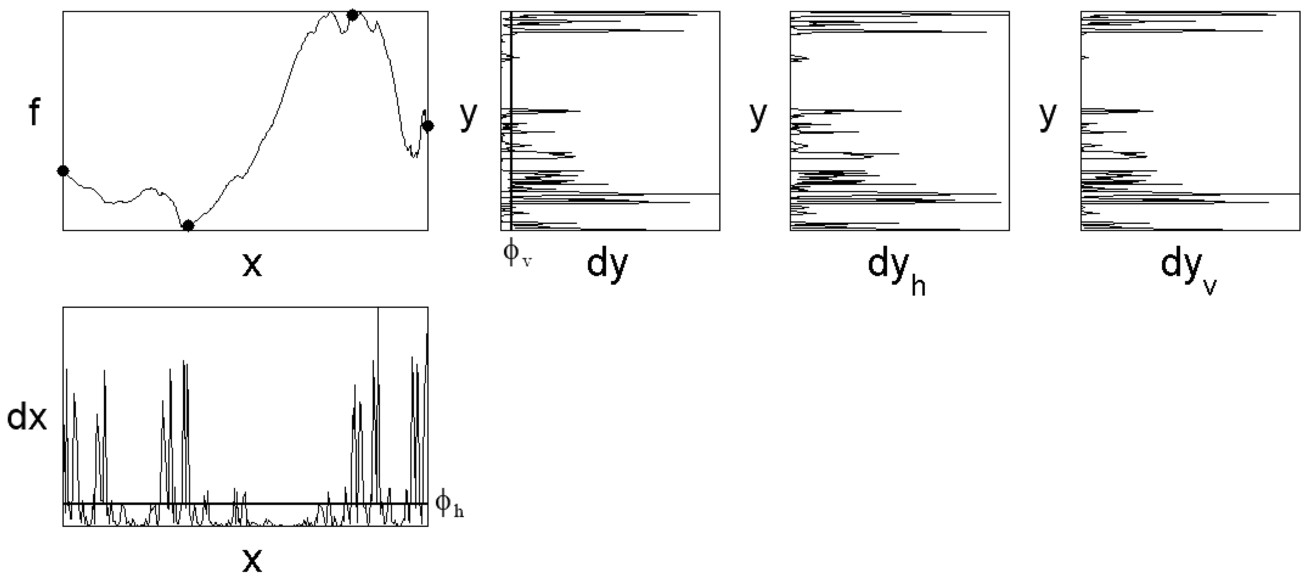

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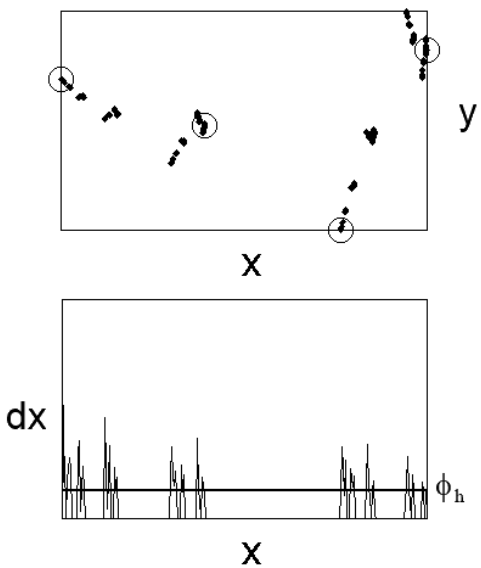
dy_h and dy_v contain many zeroes and appear to be **random**

A Generalization of the Approach and Adaptations

(Cortis et al., 2010; Maskey et al., 2015)



from a Cantorian **multifractal** dx to a **derived** dy via a **fractal** function

dy_h is found pruning dx below ϕ_h (small eddies)

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dy_h and dy_v contain many zeroes and appear to be **random**

Relevant Calculations

(Barnsley, 1988; Fernández-Martínez et al., 2010; Huang et al., 2013)

- construction via iteration of affine maps:

$$w_n \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_n & 0 \\ c_n & d_n \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e_n \\ f_n \end{pmatrix}$$

s.t.

$$w_n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_{n-1} \\ y_{n-1} \end{pmatrix}; \; w_n \begin{pmatrix} x_N \\ y_N \end{pmatrix} = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

- FM parameters: $(x_0, y_0), \ldots, (x_N, y_N); \; d_n; \; p_n; \; \phi_v$

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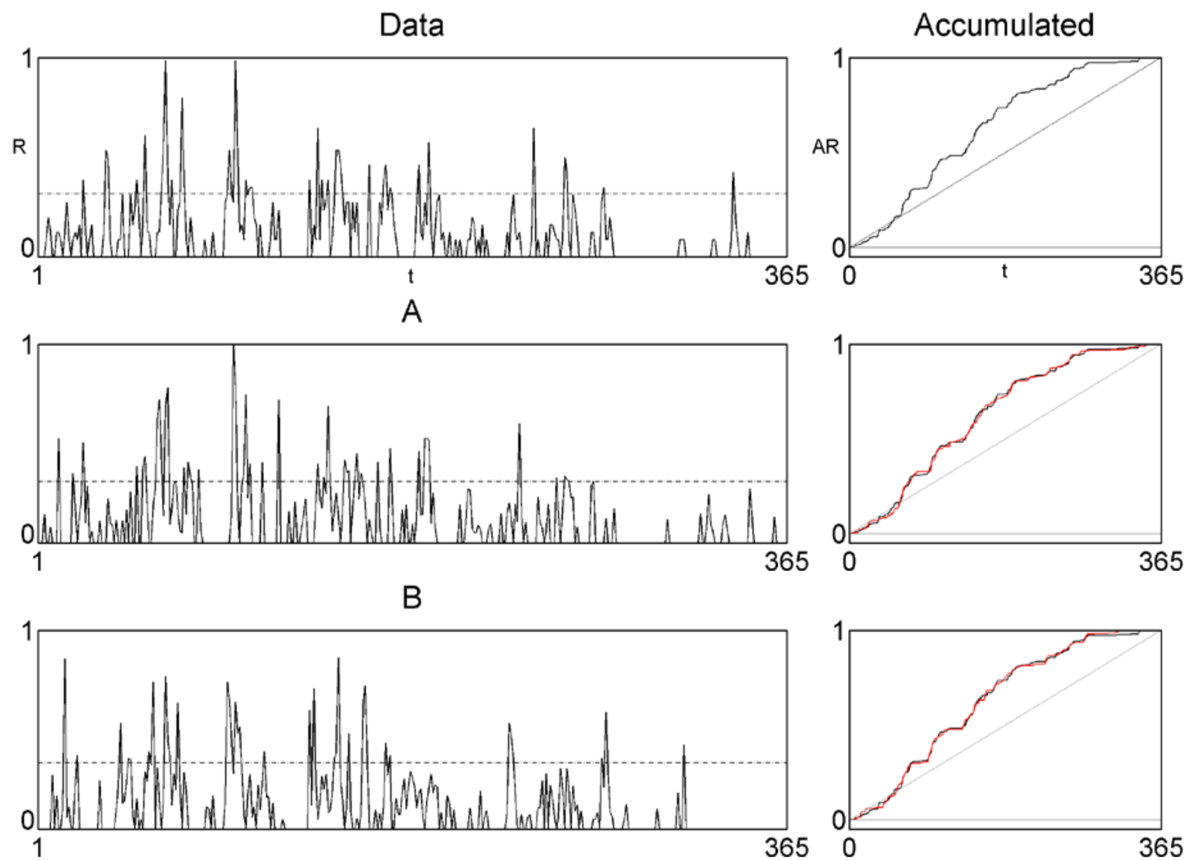
- FM parameters: $(x_0, y_0), \ldots, (x_N, y_N); d_n; p_n; \phi_v$
- inverse problem using generalized particle swarm
- objective, root mean square error:

  encodings, accumulated rain

  simulations, histogram, entropy, zeroes

Some Encodings in Tinkham, Washington, 93-94

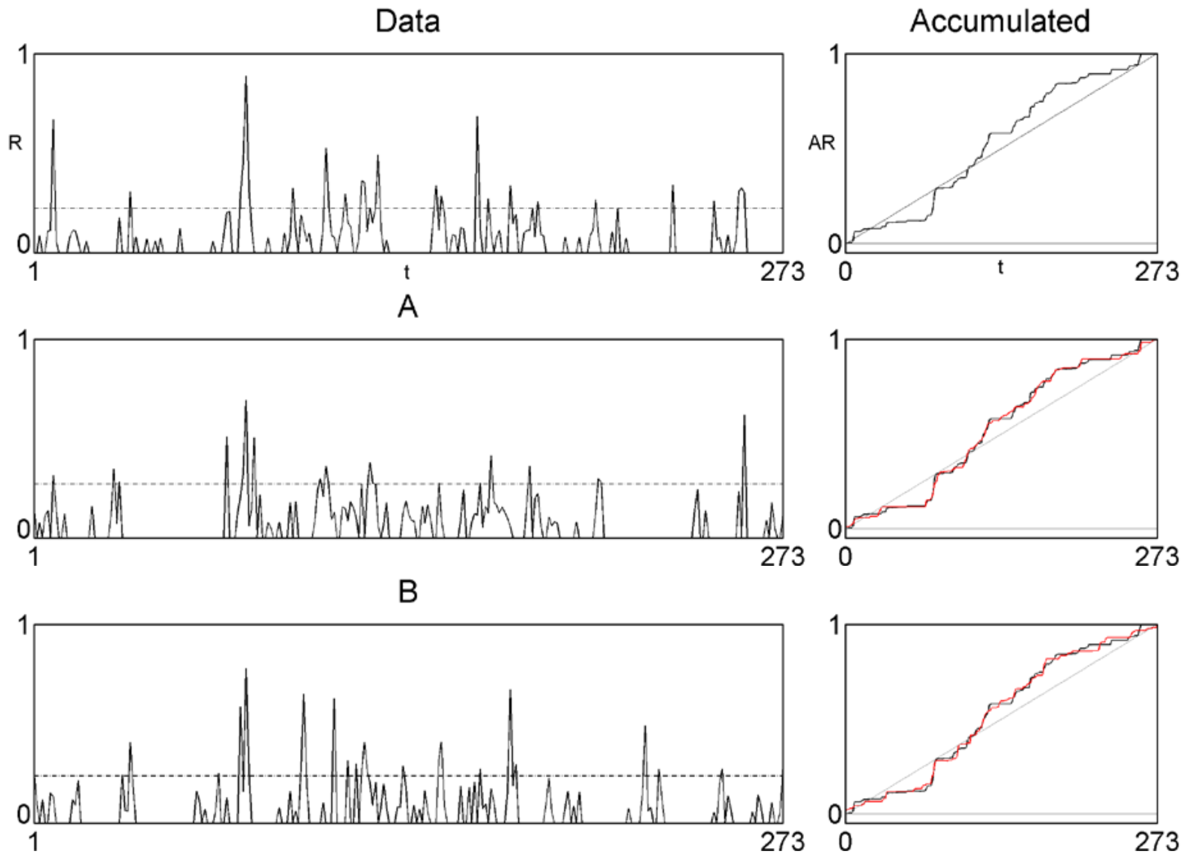


Cantorian input
vertical threshold

Cantorian input
vertical threshold

RMSEAR, MAXEAR: 1.1, 3.1% for A; 1.2, 2.7% for B

Some Encodings in Laikakota, Bolivia, 65-66

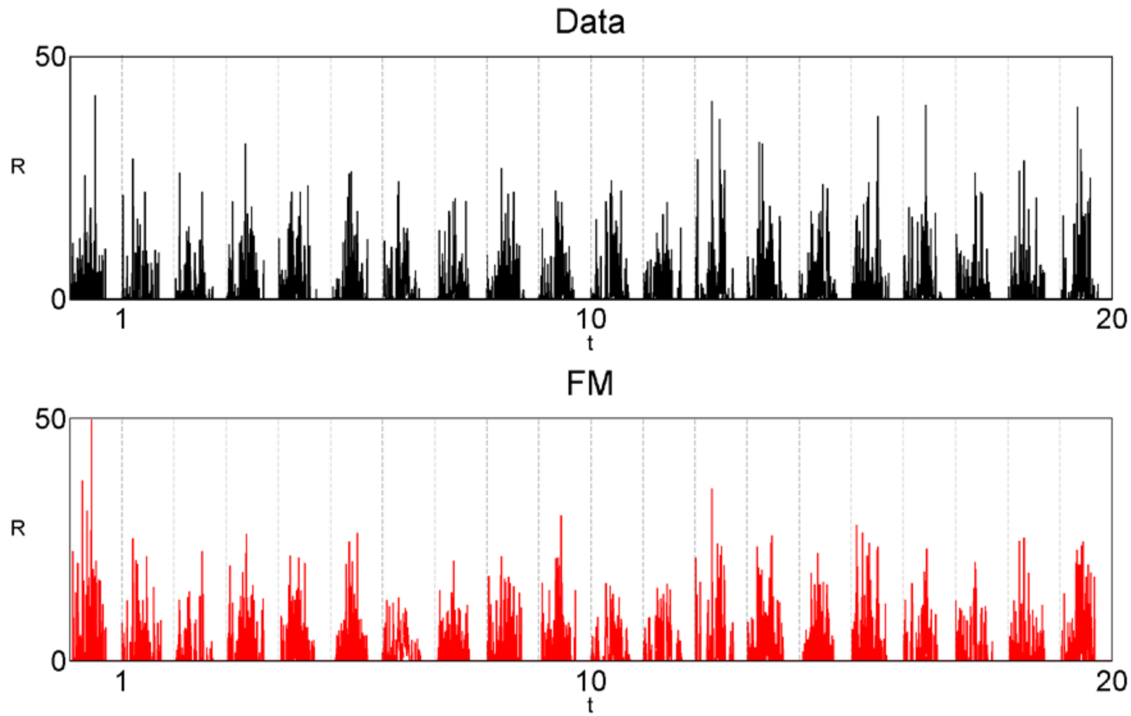


Cantorian input vertical threshold

Cantorian input vertical threshold

RMSEAR, MAXEAR: 1.4, 4.4% for A; 1.8, 3.5% for B

Twenty Years in Laikakota, Bolivia, 64-83



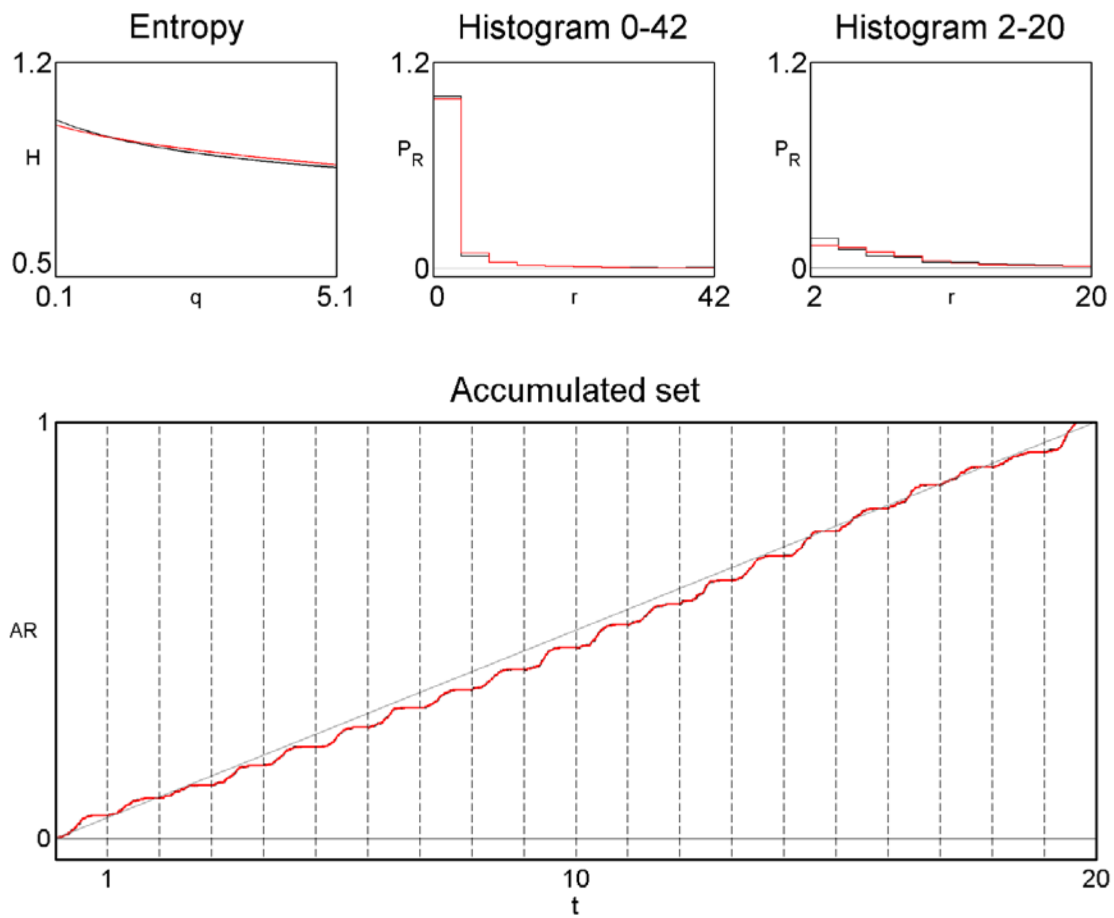
Cantorian input
vertical threshold

scale in mm/day

two maps, nine FM parameters

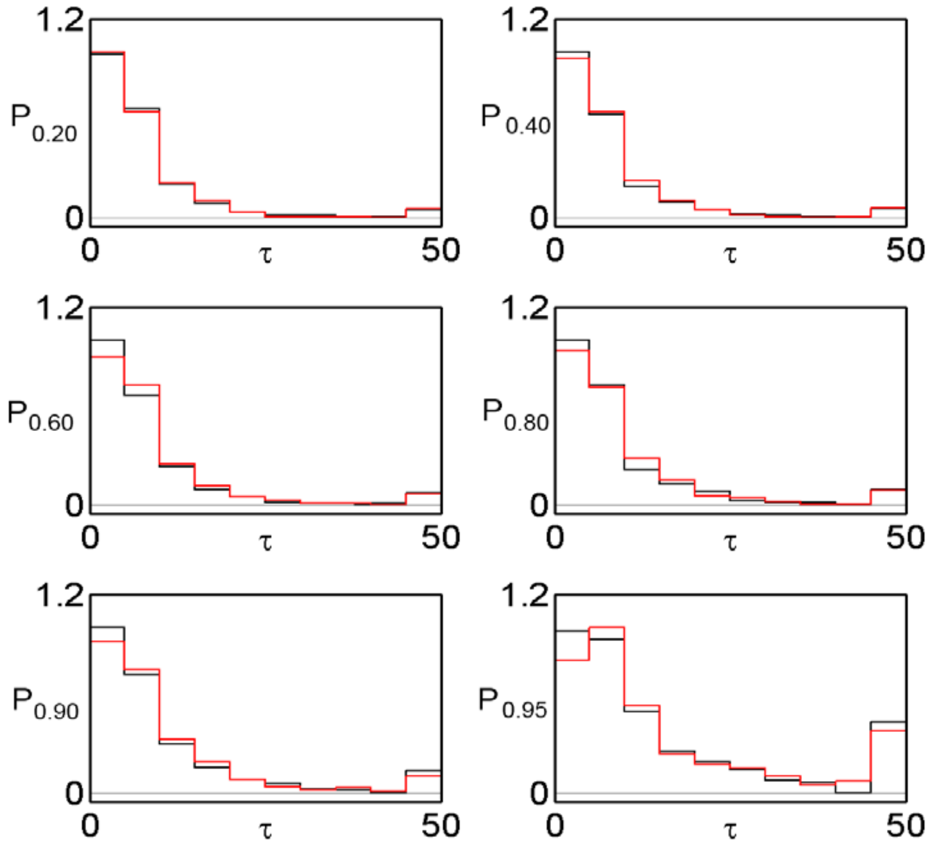
RMSEAR, MAXEAR: 1.8 ± 0.3%, 5.6 ± 1.3%

Statistics in Laikakota, Bolivia, 64-83



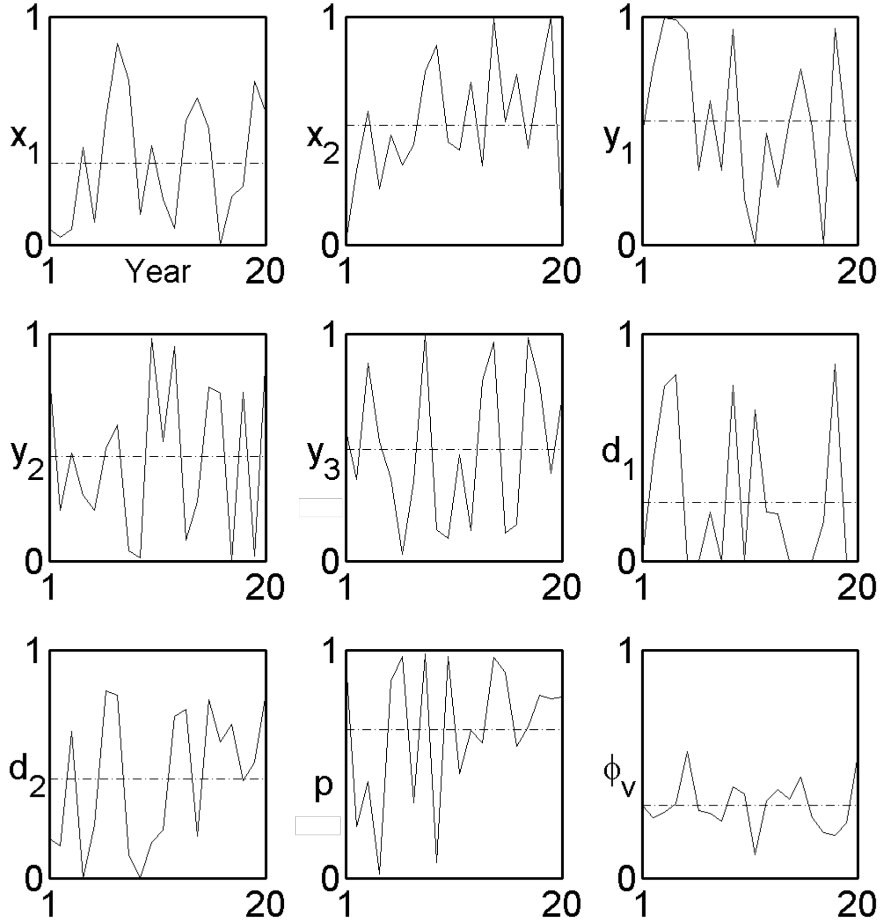
RMSEAR, MAXEAR: 0.08, 0.65%

Inter-arrival Times in Laikakota, Bolivia, 64-83



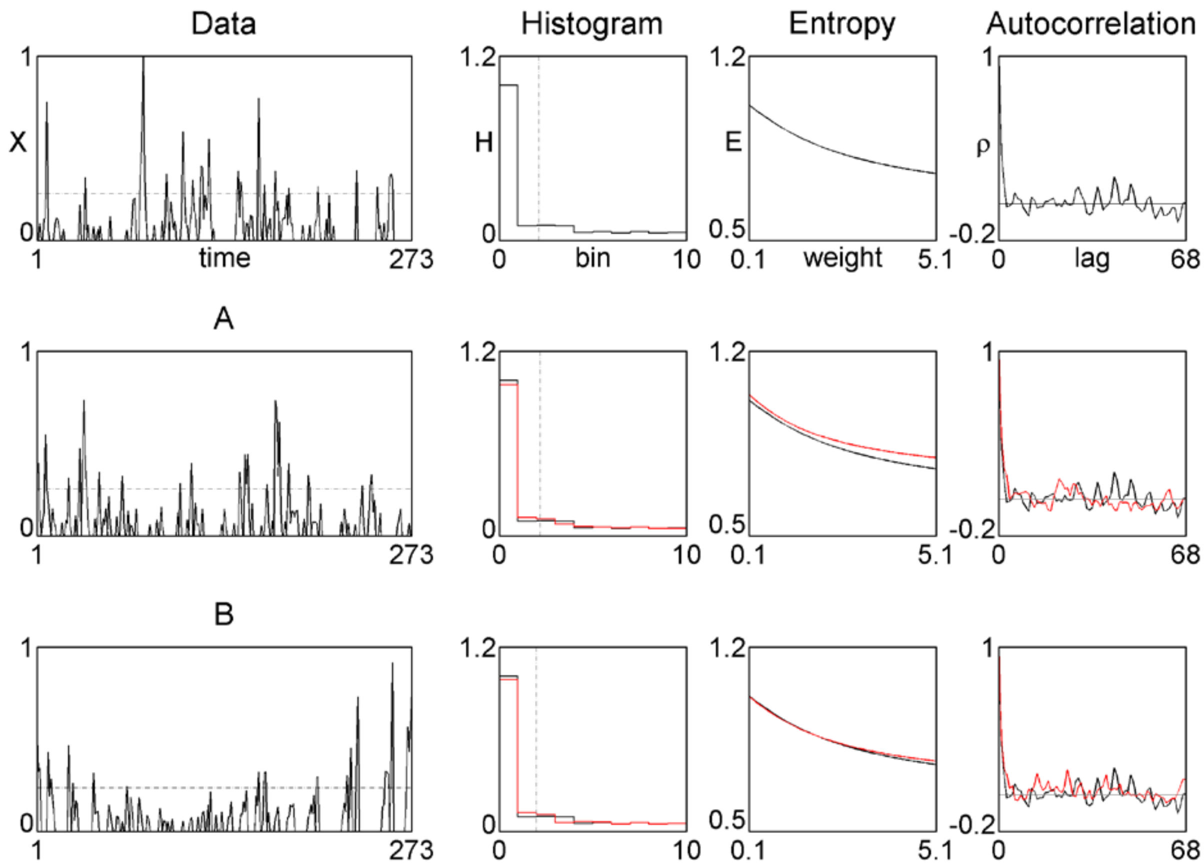
more excellent fits…

Parameters in Laikakota, Bolivia, 64-83



perhaps useful to assess rainfall complexity…

Some Simulations for Laikakota, Bolivia, 65-66

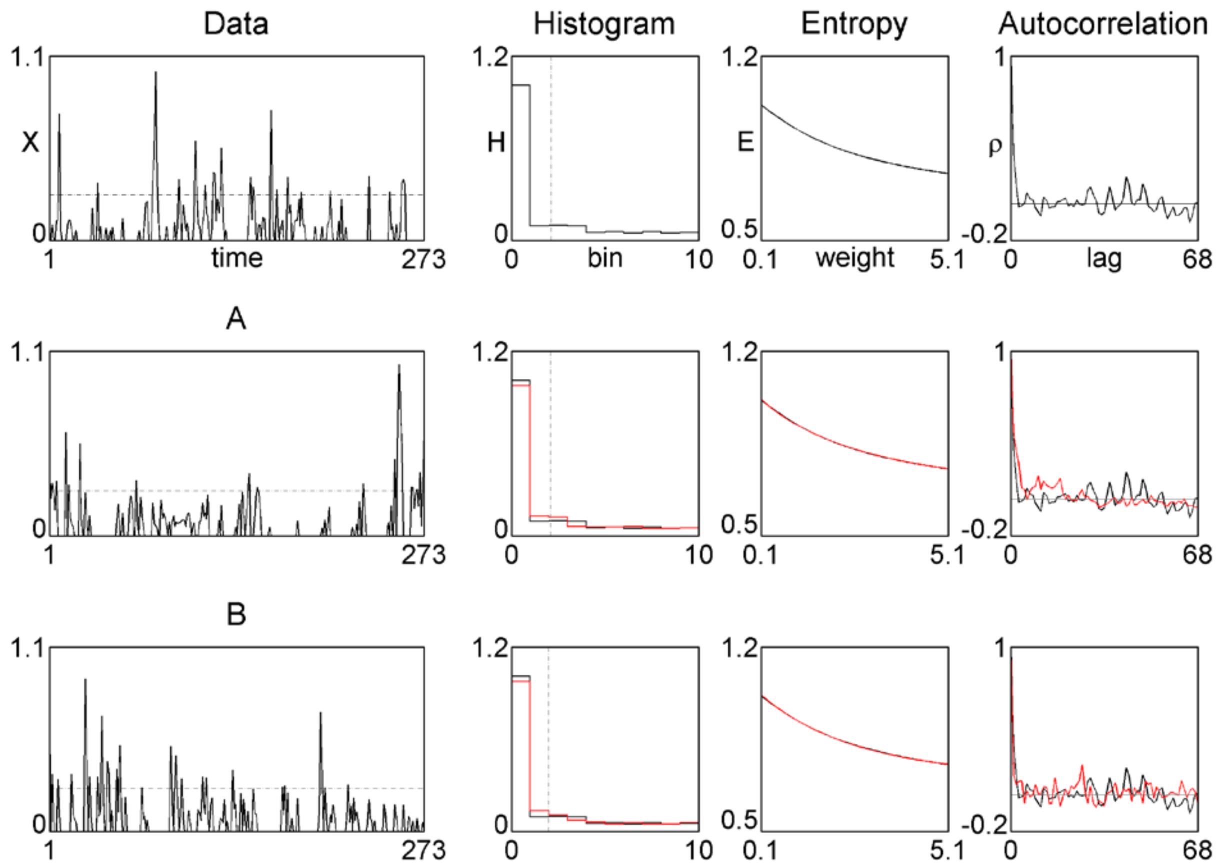


continuous input
vertical threshold

Cantorian input
vertical threshold

histogram based: NSH, NSE: 99.8, 81.2% for A; 99.6, 99.1% for B

Some Simulations for Laikakota, Bolivia, 65-66

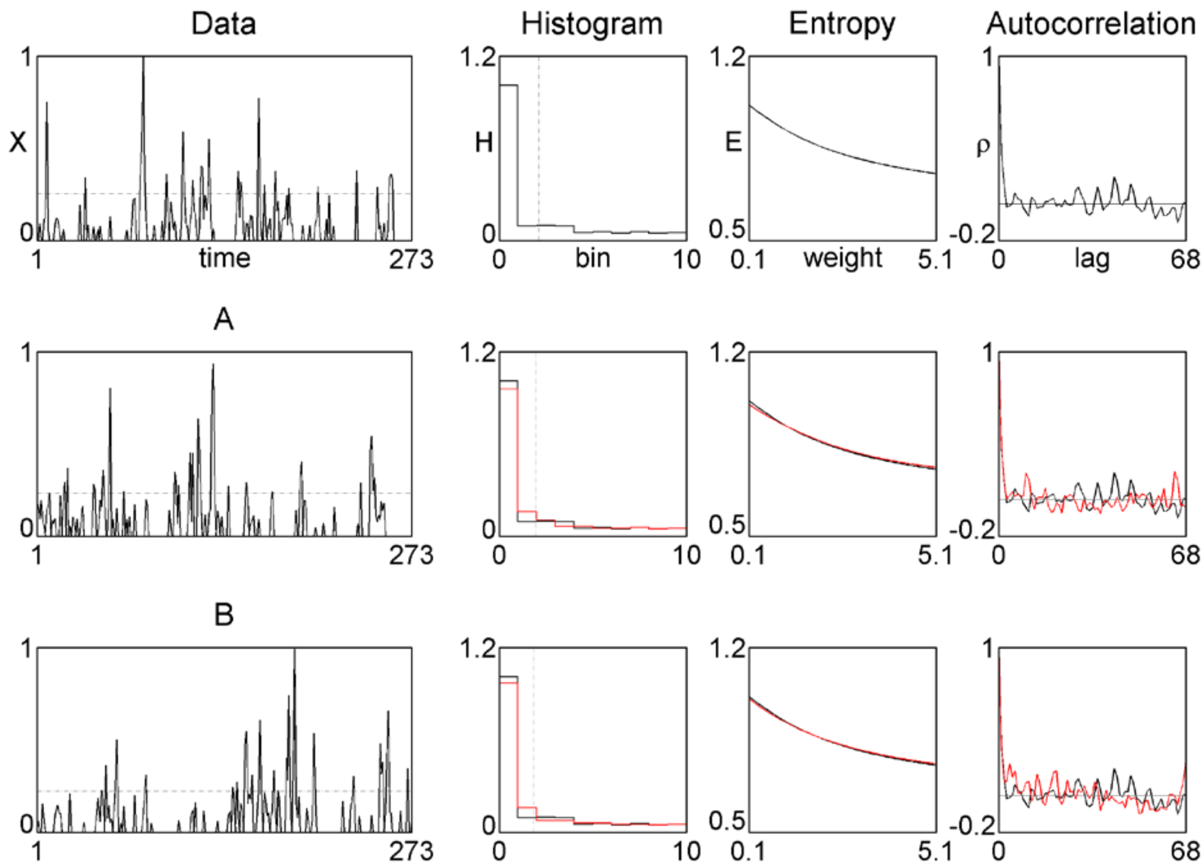


continuous input
vertical threshold

Cantorian input
vertical threshold

entropy based: NSH, NSE: 99.5, 100% for A; 99.6, 100% for B

Some Simulations for Laikakota, Bolivia, 65-66



continuous input
vertical threshold

Cantorian input
vertical threshold

zeroes based: % wrong zeros -4.0% for A; -1.2% for B

Summary and Conclusions

- The Fractal-multifractal approach may be used to **encode** and **simulate** complex rainfall sets gathered daily and for the duration of a water year.

- Encodings capture not only the accumulated rain patterns, but also suitable statistical information such as histogram and inter-arrival distributions for various thresholds.

- Simulations may be done based on histograms, entropy, and distribution of zero values. They represent alternatives that complement stochastic methods.

- The evolution of FM parameters may perhaps be used to quantify the **complexity** of rainfall sets.

Selected Bibliography

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