

# **Simulation of hydrologic data using fractals to investigate climate change**

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California Department of Water Resources  
Sacramento, April 15, 2016

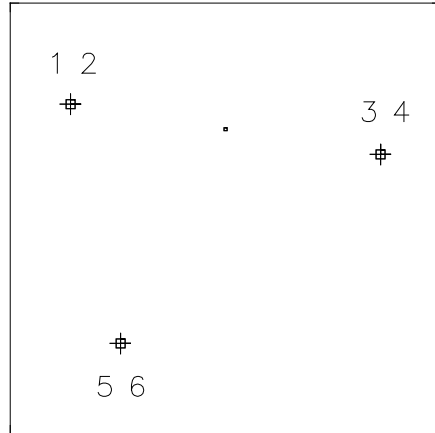
# Outline

- The fractal-multifractal method
- Encoding rainfall
- Simulating and downscaling rainfall
- Encoding streamflow
- Classifying and predicting streamflow
- Downscaling streamflow
- Conclusions and potential collaboration
- Encoding temperature

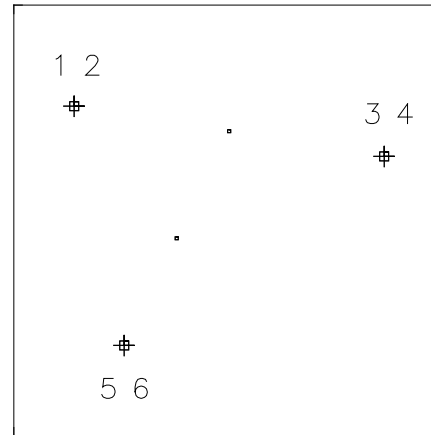
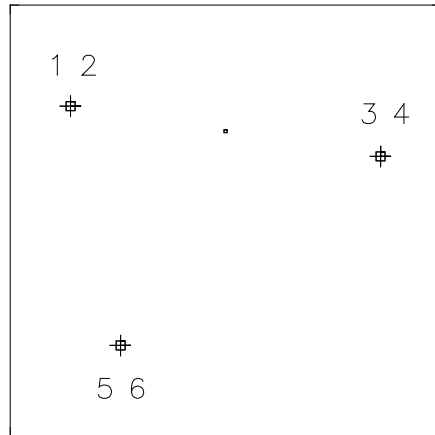
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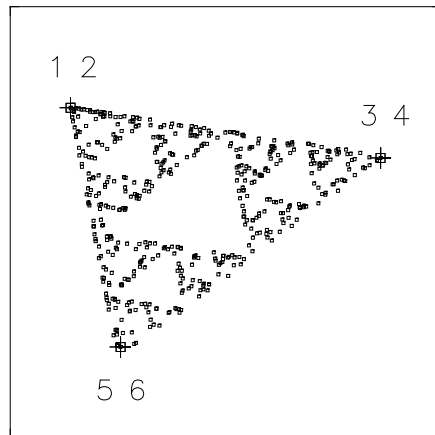
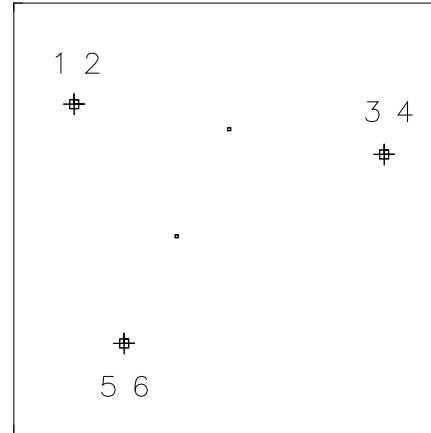
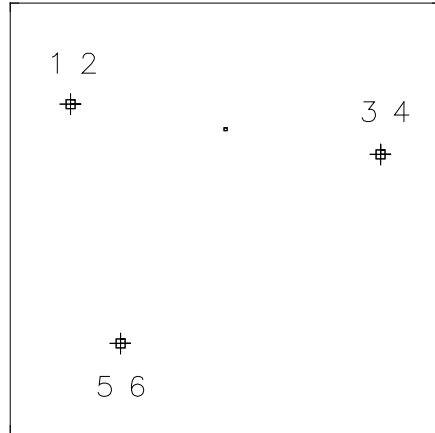
# The chaos game



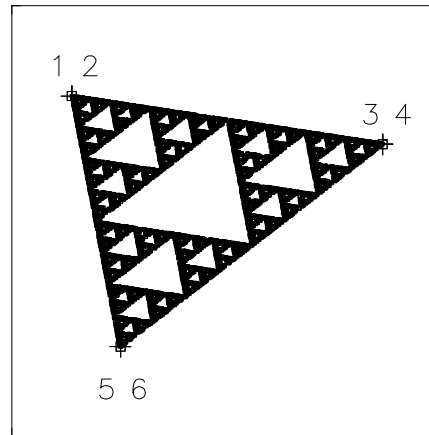
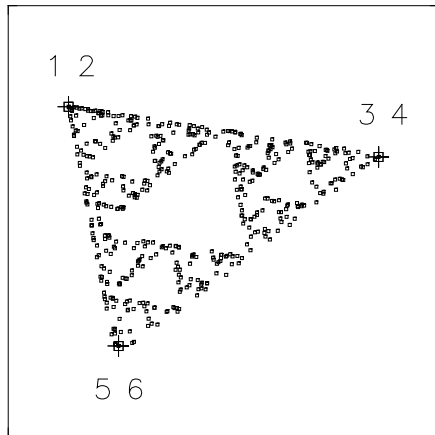
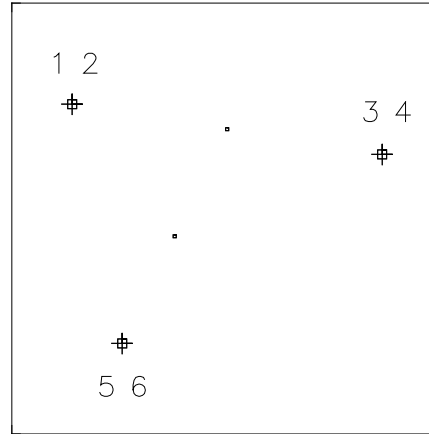
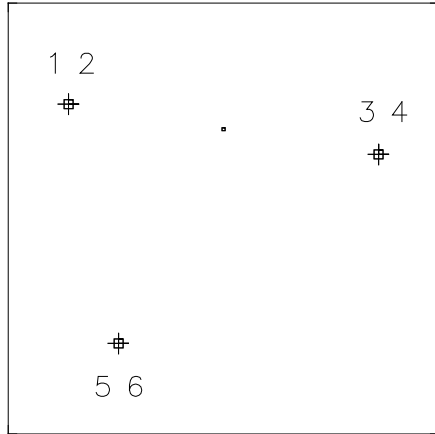
# The chaos game



# The chaos game



# The chaos game



**a fractal**

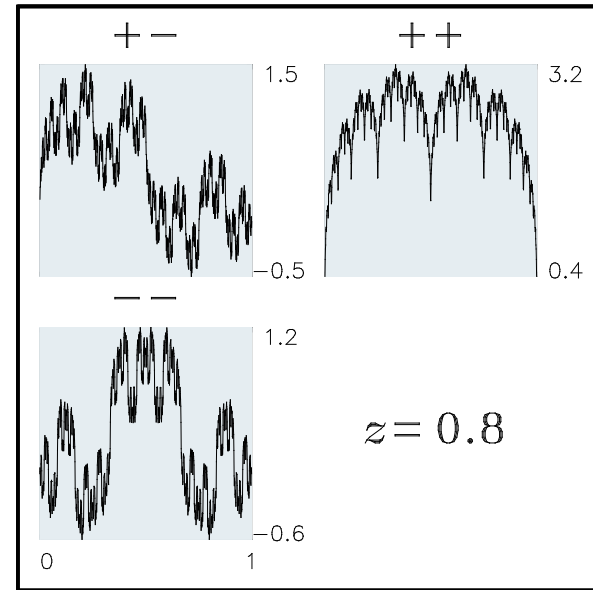
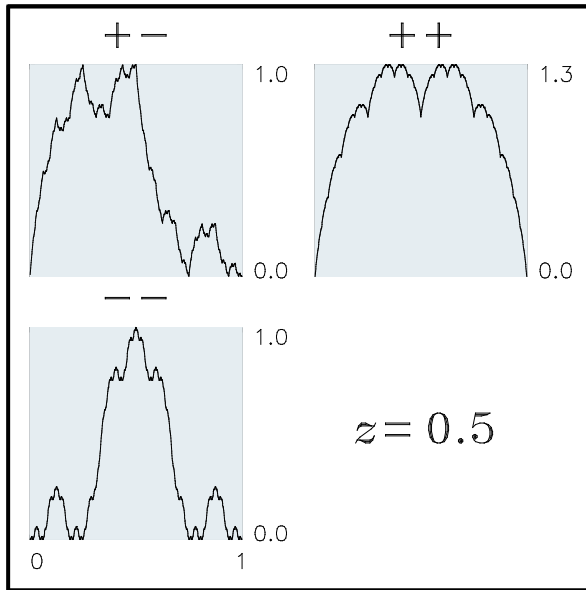
## With other rules and a coin

$$w_1(x, y) = \left( \frac{x}{2}, x + d_1 \cdot y \right), w_2(x, y) = \left( \frac{x}{2} + \frac{1}{2}, 1 - x + d_2 \cdot y \right)$$

# With other rules and a coin

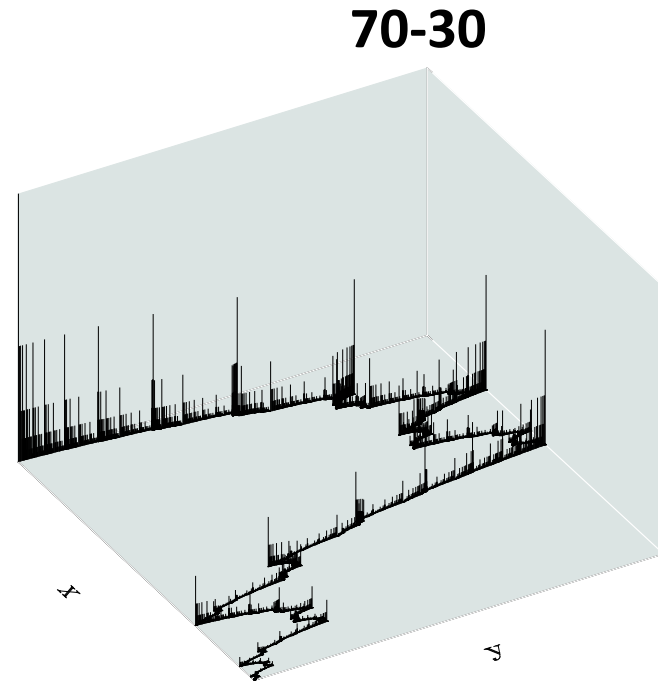
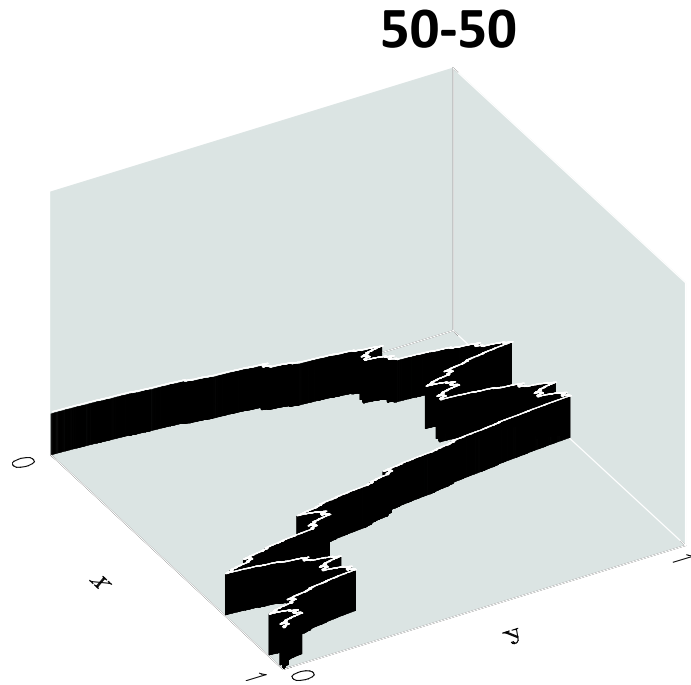
$$w_1(x, y) = \left( \frac{x}{2}, x + d_1 \cdot y \right), w_2(x, y) = \left( \frac{x}{2} + \frac{1}{2}, 1 - x + d_2 \cdot y \right)$$

$$z = |d_1| = |d_2|$$



more fractals...

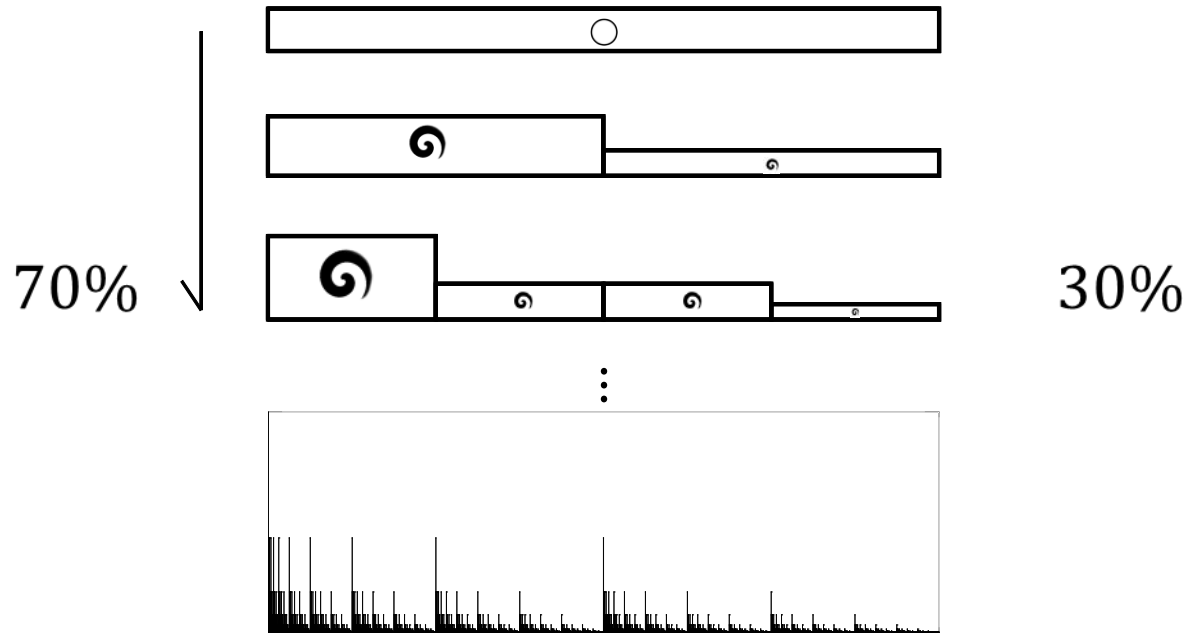
# Using fair or biased coins



different textures over the same “wire”

# From x: a turbulent cascade

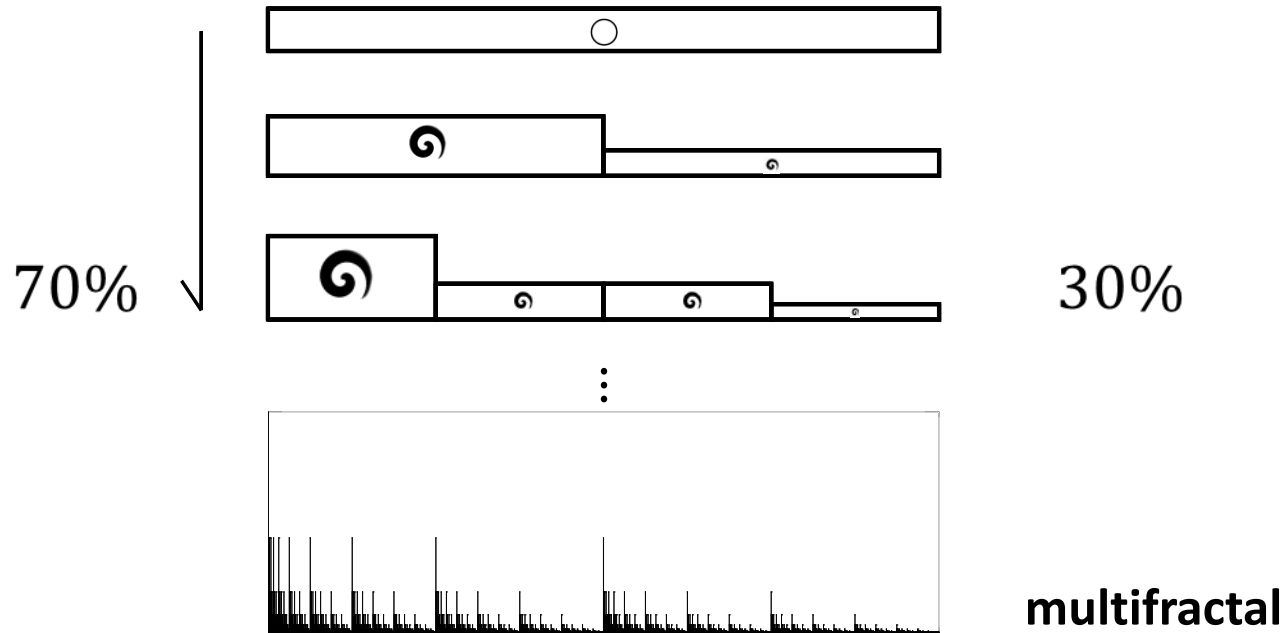
(Meneveau and Sreenivasan, 1987)



a sequence of **eddies** that divide the energy **universally**

# From $x$ : a turbulent cascade

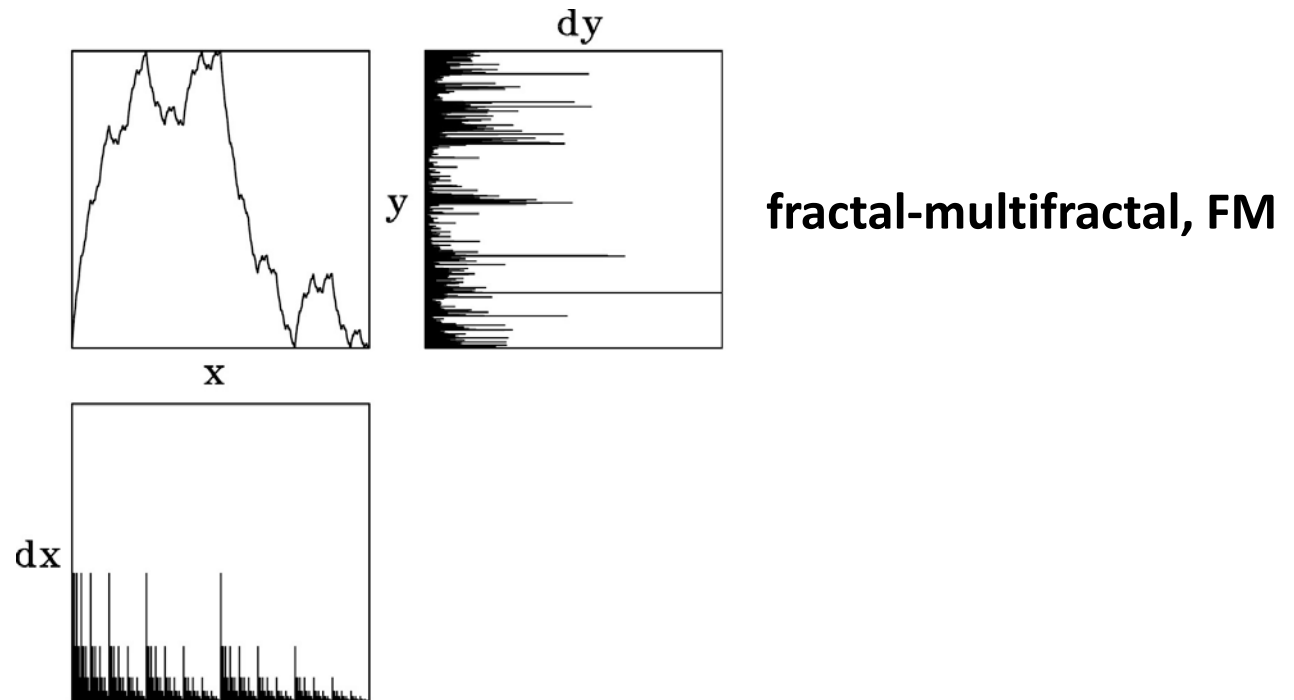
(Meneveau and Sreenivasan, 1987)



a sequence of **eddies** that divide the energy **universally**  
a collection of intertwined **thorns** with **dusty** supports

# From $y$ : a Platonic vision to complexity

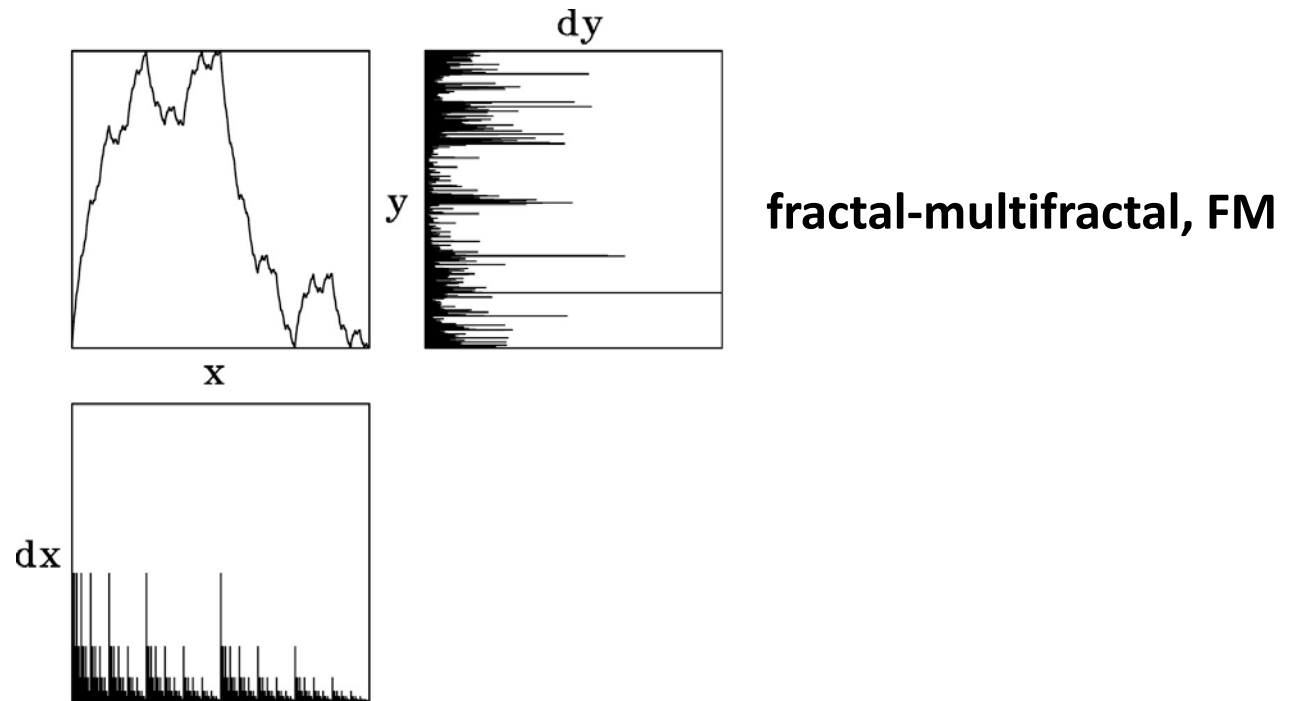
(Puente, 1992, 1994)



$dx$  is the “illumination” of the **wire** and  $dy$  is its **derived** projection

# From $y$ : a Platonic vision to complexity

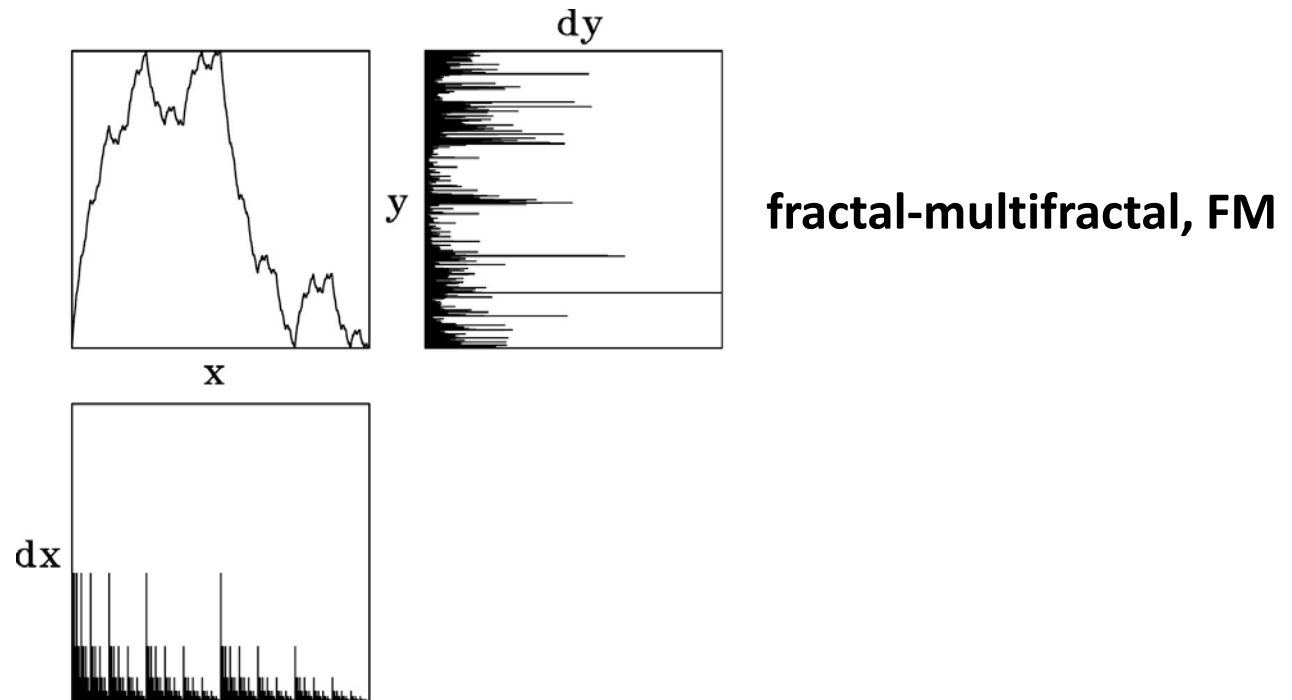
(Puente, 1992, 1994)



$dx$  is the “illumination” of the **wire** and  $dy$  is its **derived** projection  
 $dy$  is a **transformation** of turbulence, a non-trivial **physical** cascade

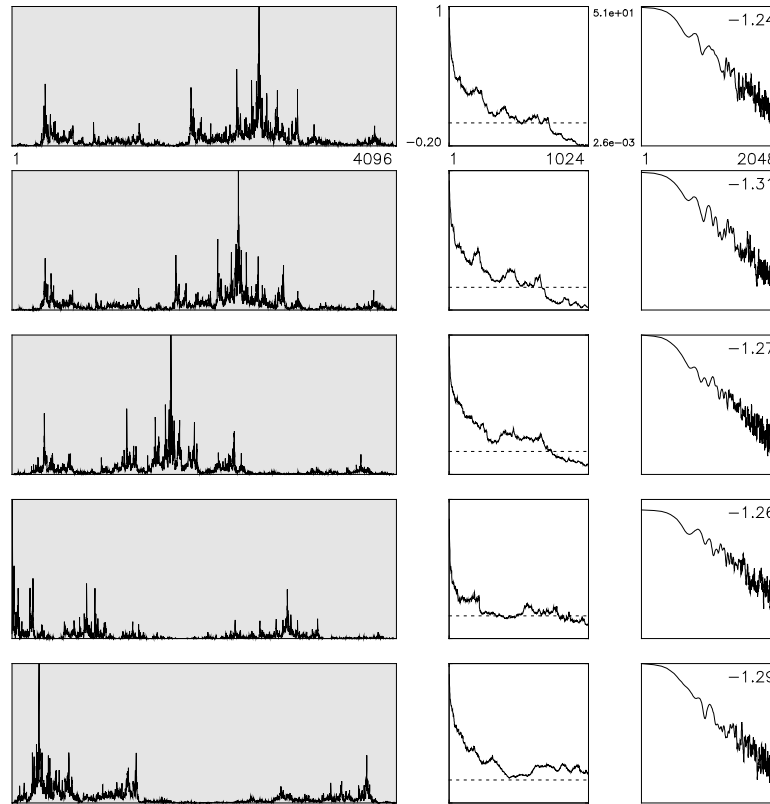
# From $y$ : a Platonic vision to complexity

(Puente, 1992, 1994)



$dx$  is the “illumination” of the **wire** and  $dy$  is its **derived** projection  
 $dy$  is a **transformation** of turbulence, a non-trivial **physical** cascade  
the “shadow”  $dy$  appears to be **random**, but it is not

# Projections galore



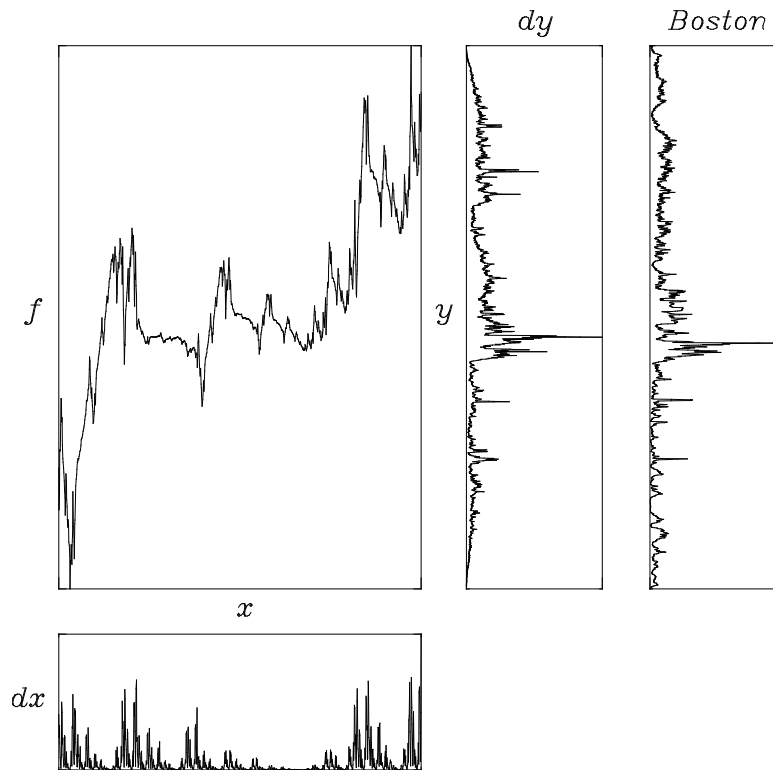
complex data sets are thought of as distributions  
varying FM parameters yields similar statistics as **natural** sets

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# Encoding a rainfall storm in Boston

(Puente and Obregón, 1996)

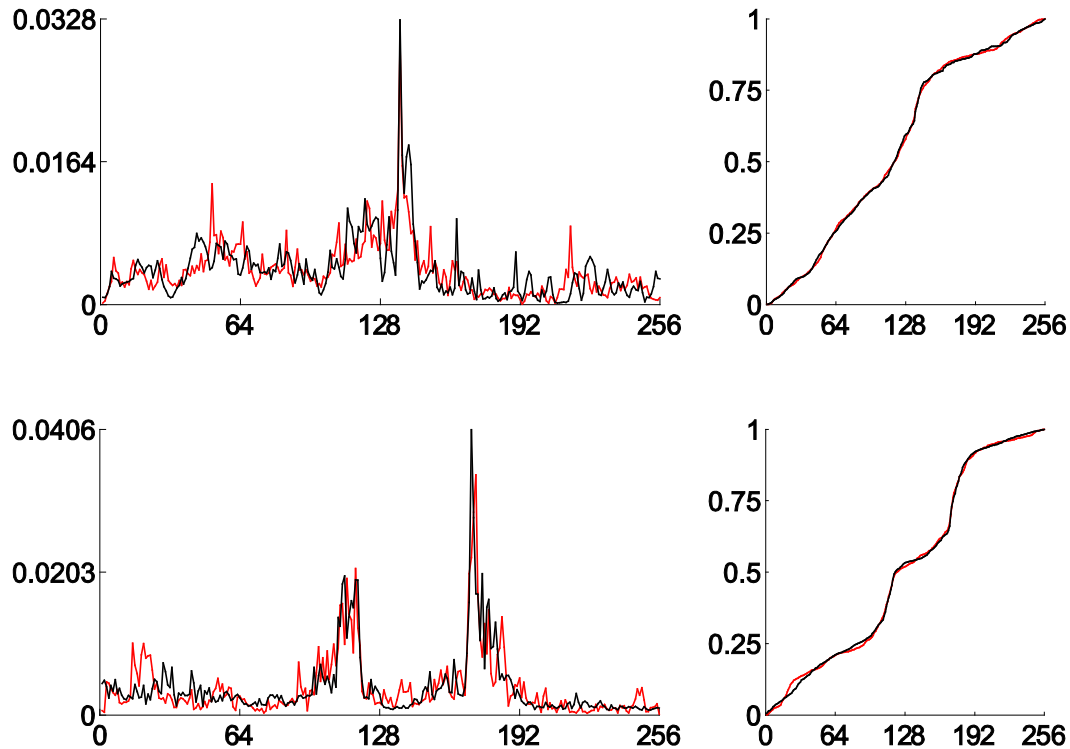


a model (center) of real data (right)

the model preserves **statistical** and **chaotic** characteristics

# Encoding storms in Boston and Iowa City

(Puentes et al., 2010)

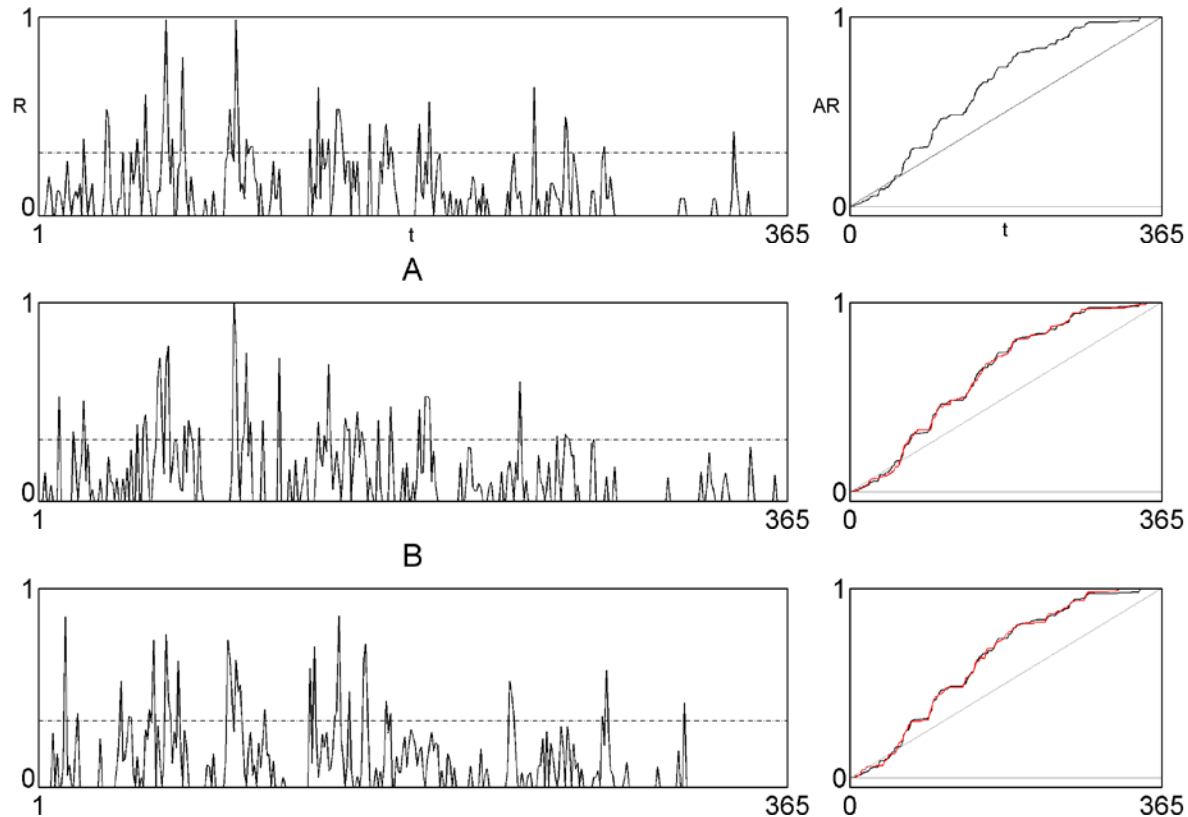


RMSEAR 0.7, 1.0%; MAXEAR 1.8, 2.1%

compressions greater than 350:1

# Encoding rainfall in Tinkham, WA

(Maskey et al., 2015)



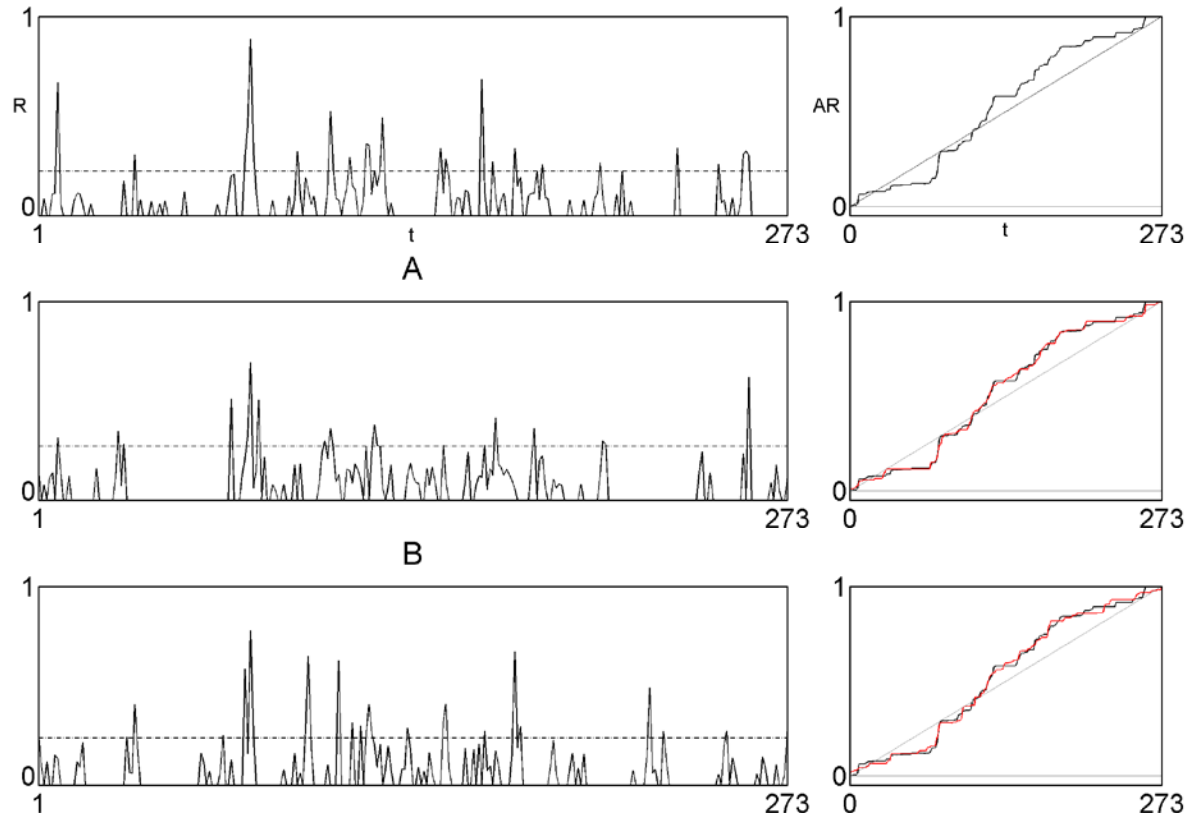
**Cantorian input  
plus threshold**

RMSEAR 1.1, 1.2%; MAXEAR 3.1, 2.7%

compressions 33:1

# Encoding rainfall in Laikakota, Bolivia

(Maskey et al., 2015)



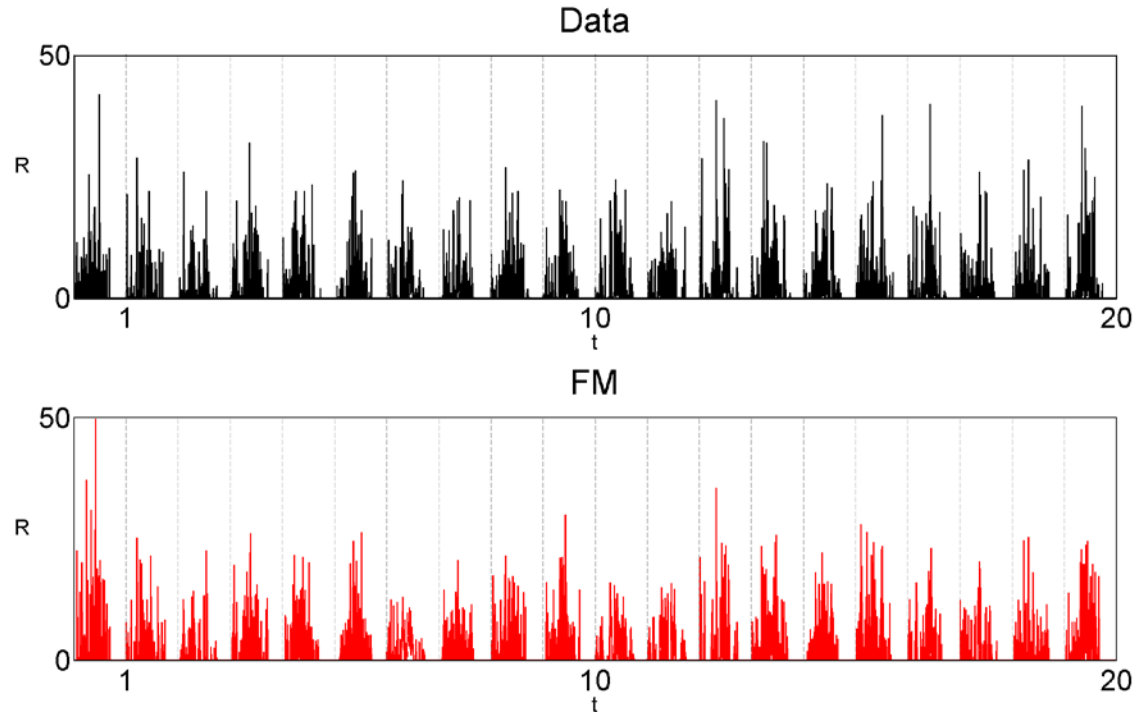
**Cantorian input  
plus threshold**

RMSEAR 1.4, 1.8%; MAXEAR 4.4, 3.5%

compressions 25:1

# Encoding rainfall in Laikakota, Bolivia

(Maskey et al., 2015)



**Cantorian** input  
plus threshold

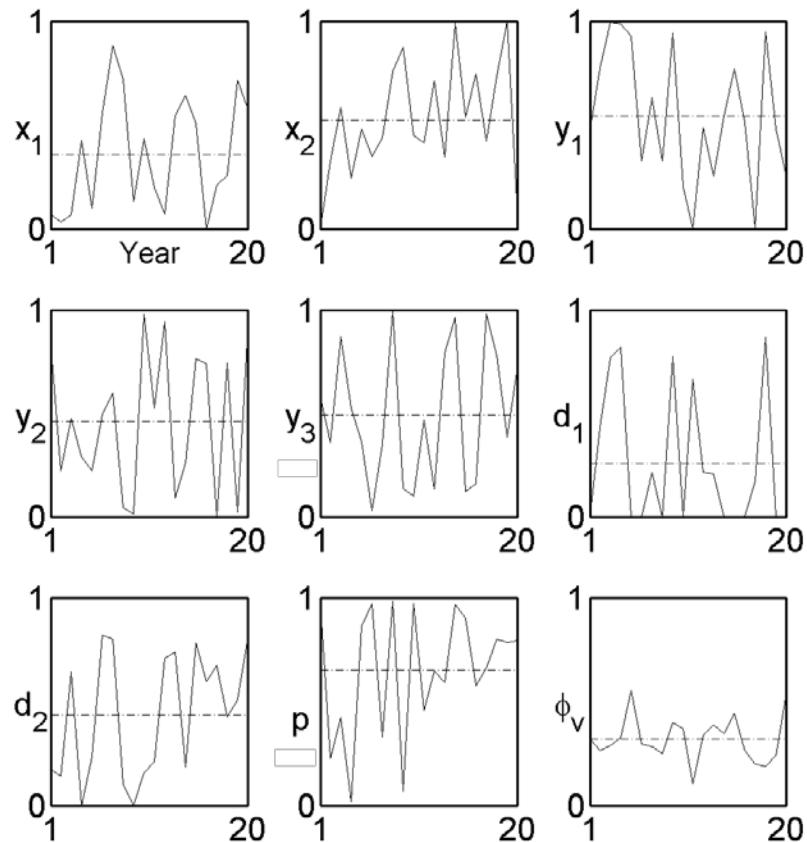
scale in mm/day

RMSEAR  $1.8 \pm 0.3\%$ ; MAXEAR  $5.6 \pm 1.3\%$

compressions 25:1

# Dynamics of rainfall in Laikakota, Bolivia

(Maskey et al., 2015)



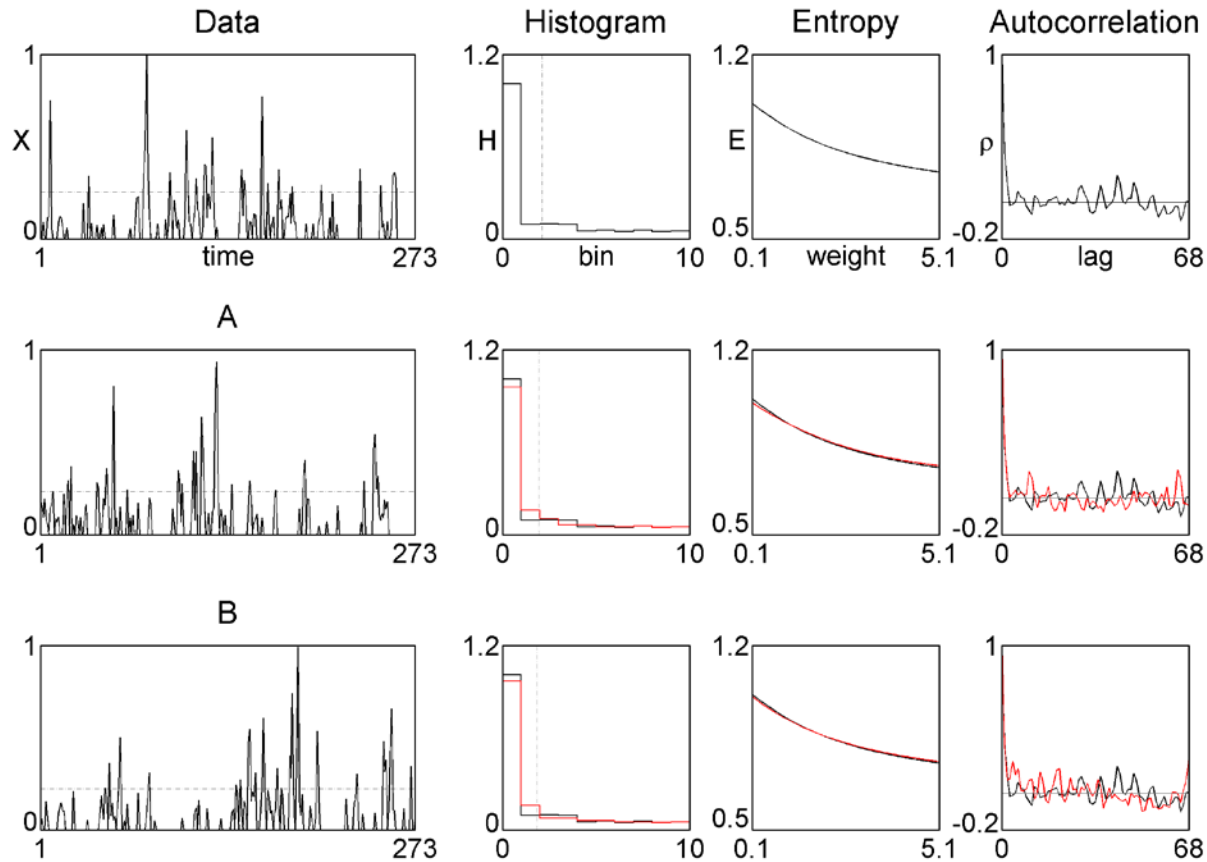
to assess rainfall complexity and climate change, also temperature...

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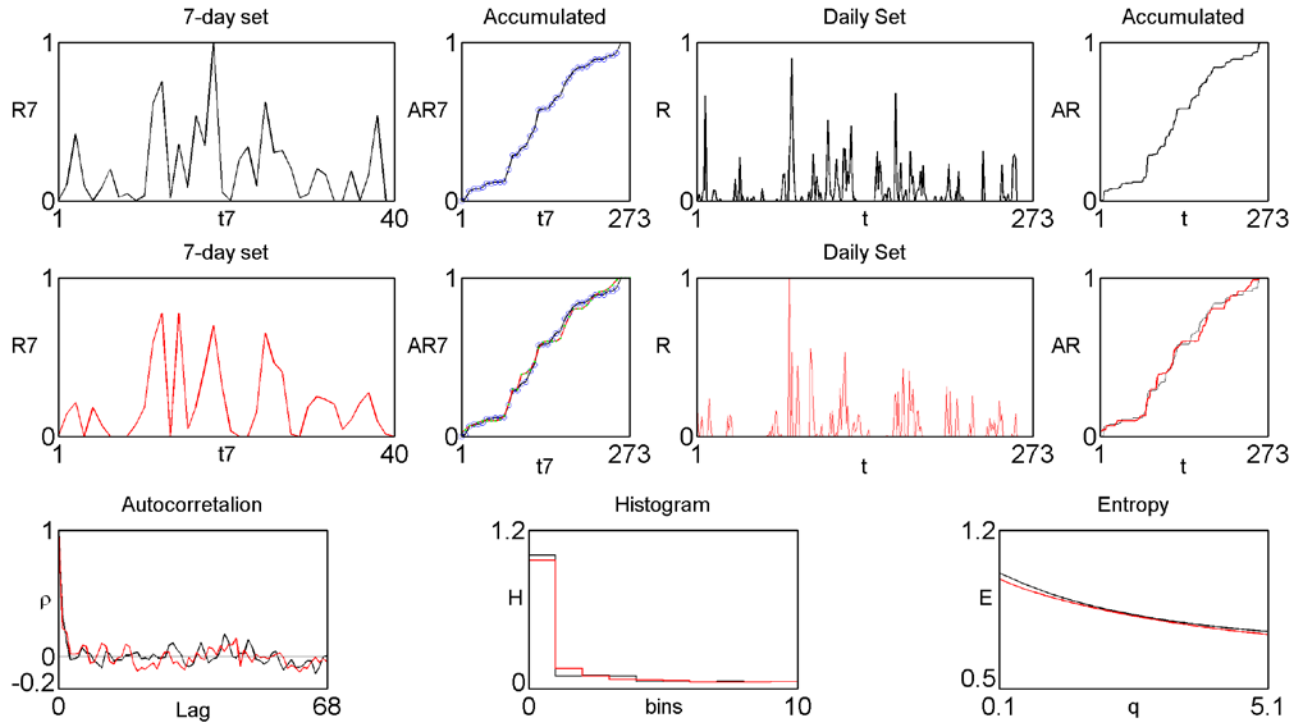
# Simulating rainfall in Laikakota, Bolivia

Based on histogram, entropy and consecutive zeroes



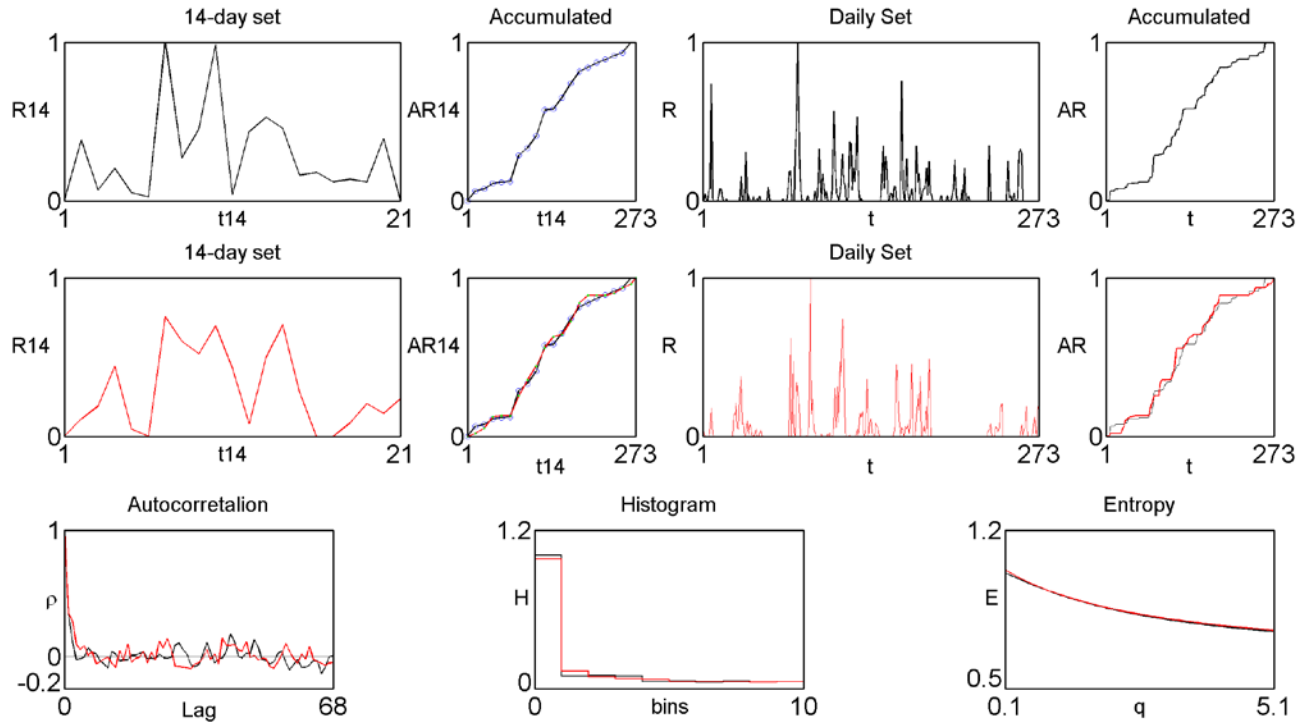
# Downscaling rainfall in Laikakota, Bolivia

Based on cumulative set every 7 days



# Downscaling rainfall in Laikakota, Bolivia

Based on cumulative set every 14 days

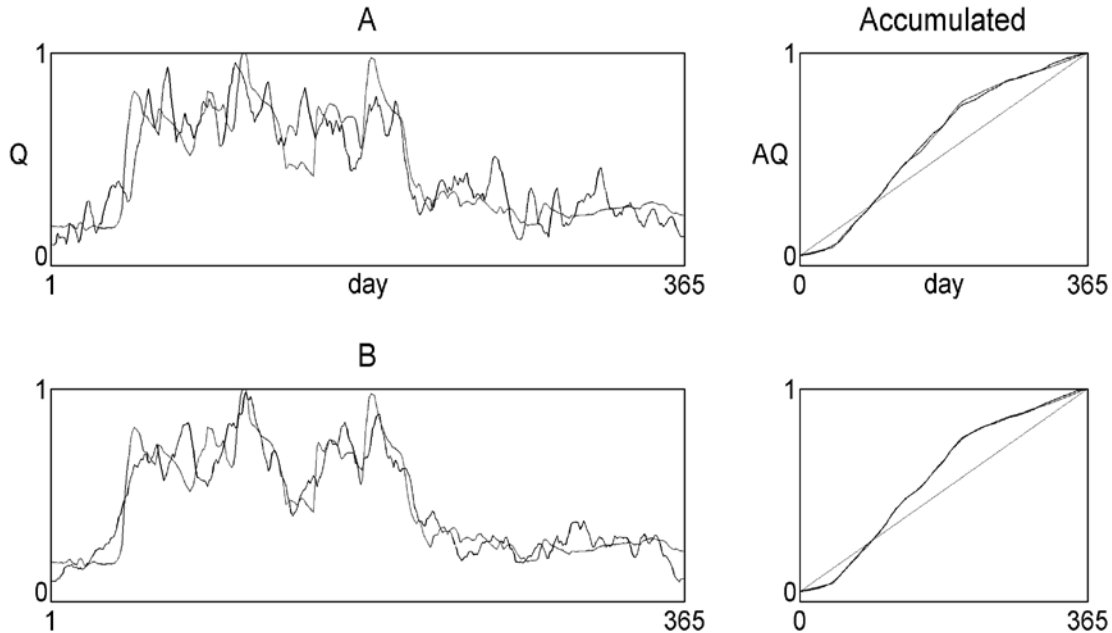


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# Encoding streamflow in Sacramento

Water year 1973-74



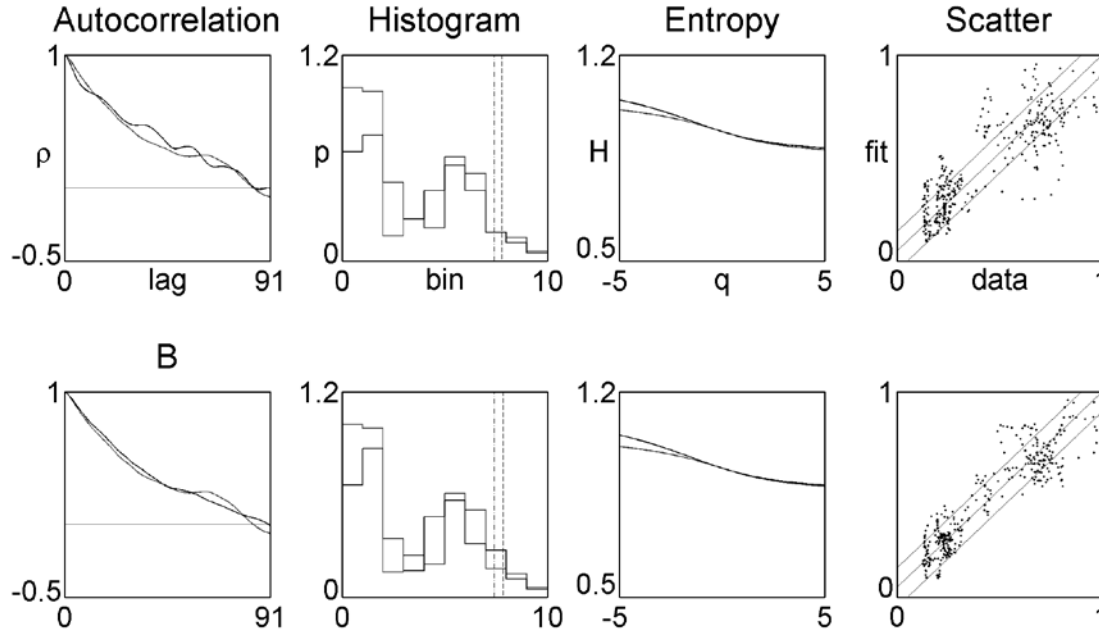
A: wire 2 maps, B: wire 3 maps

RMSEAR 1.0%, 0.5%; MAXEAR 2.3%, 1.1%

compressions greater than 40:1

# Encoding streamflow in Sacramento

Water year 1973-74

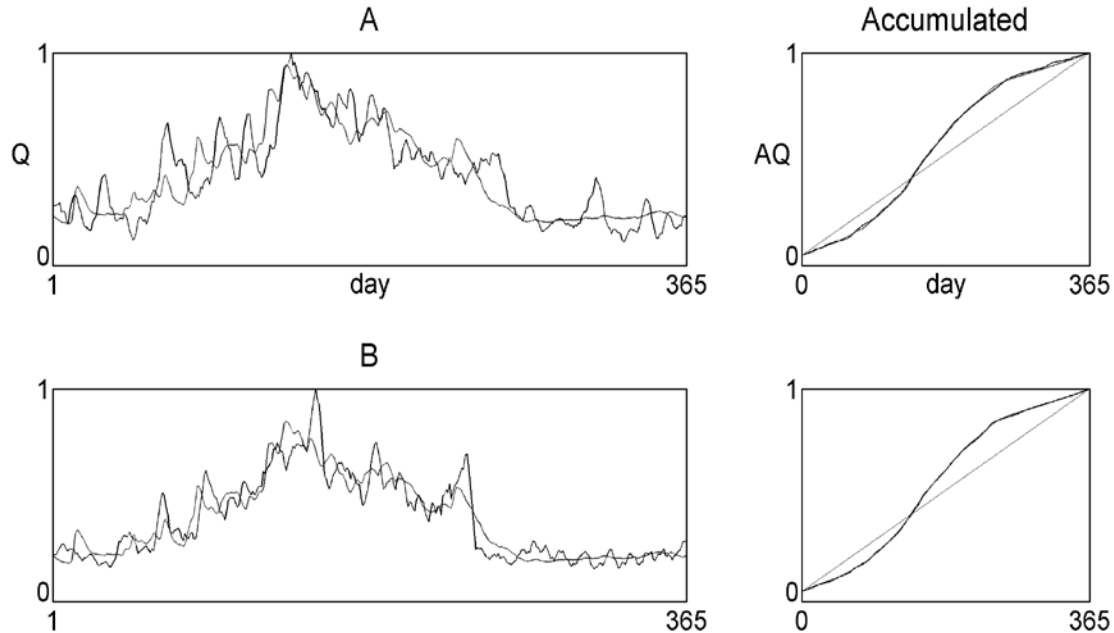


A: wire 2 maps, B: wire 3 maps

NSD 73.3%, 85.0%; NSA 95.7%, 98.1%; NSE 93.9%, 92.8%

# Encoding streamflow in Sacramento

Water decade 1956-65



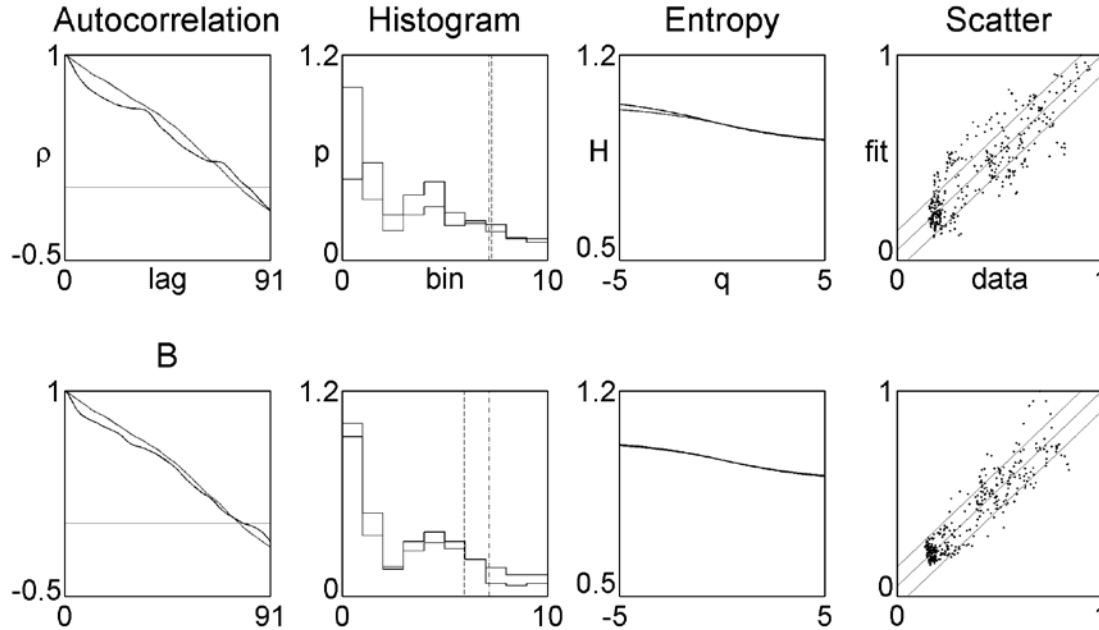
A: wire 2 maps, B: wire 3 maps

RMSEAR 0.7%, 0.3%; MAXEAR 1.5%, 0.7%

compressions greater than 40:1

# Encoding streamflow in Sacramento

Water decade 1956-65

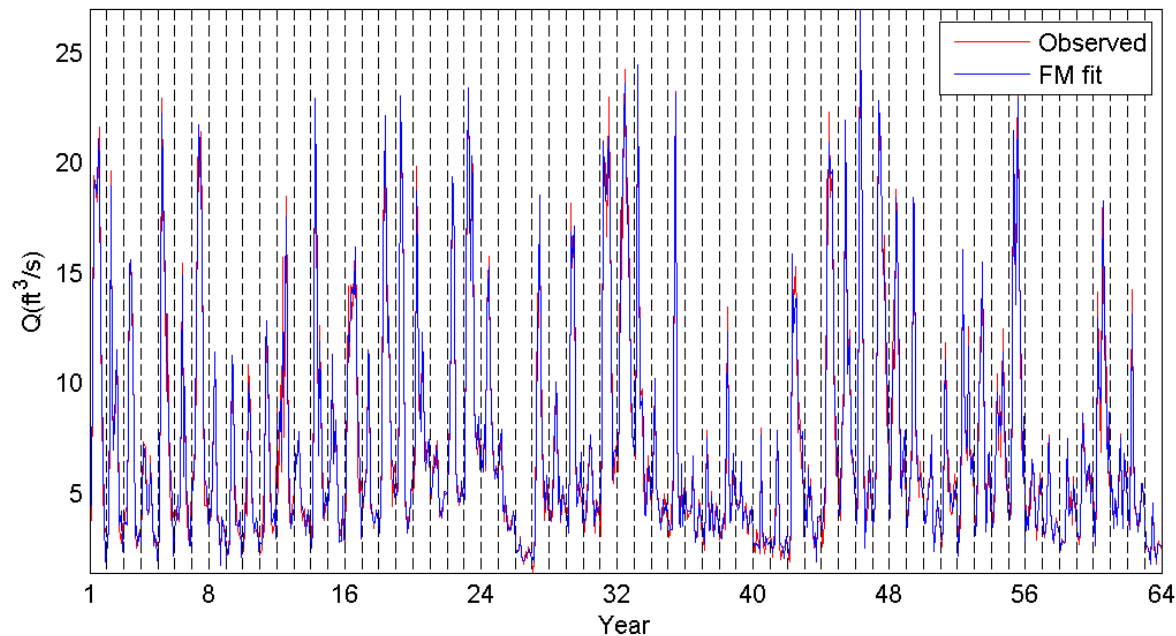


A: wire 2 maps, B: wire 3 maps

NSD 72.9%, 84.0%; NSA 92.4%, 96.9%; NSE 96.1%, 99.6%

# Encoding streamflow in Sacramento

By year 1950-2014

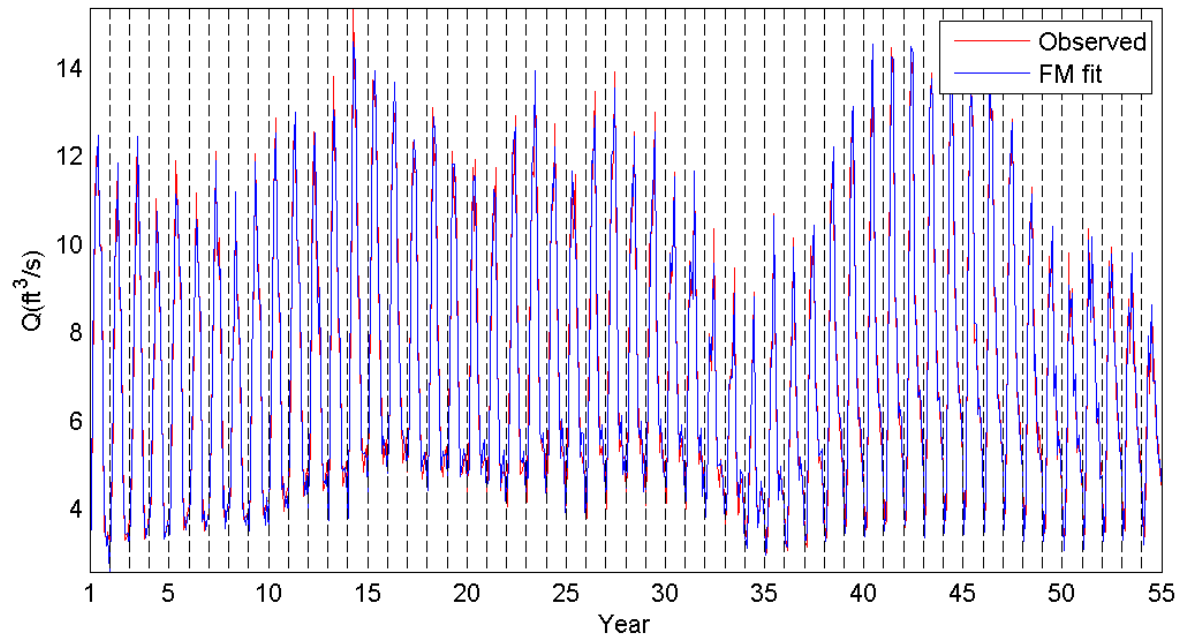


wire 3 maps, scale in  $10^4$  x cfs

RMSEAR  $0.8 \pm 0.3\%$ ; MAXEAR  $1.8 \pm 0.5\%$ ; NSD  $63.9 \pm 19.0\%$

# Encoding streamflow in Sacramento

By decade 1959-2014

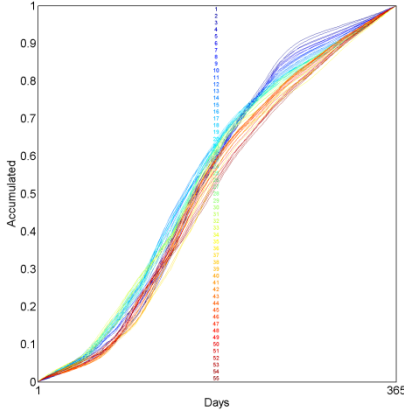
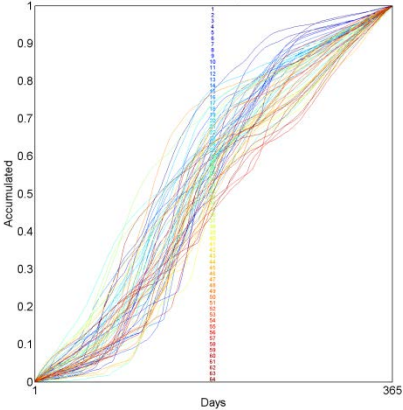


wire 3 maps, scale in  $10^4 \times \text{cfs}$

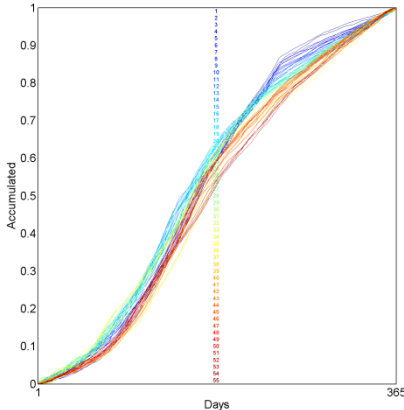
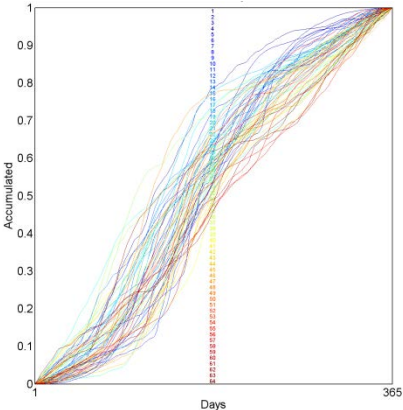
RMSEAR  $1.8 \pm 0.4\%$ ; MAXEAR  $3.6 \pm 0.7\%$ ; NSD  $92.2 \pm 12.5\%$

# Encoding streamflow in Sacramento

Accumulated by year and by decade



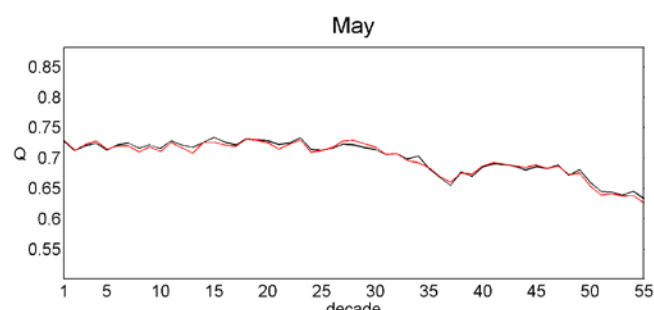
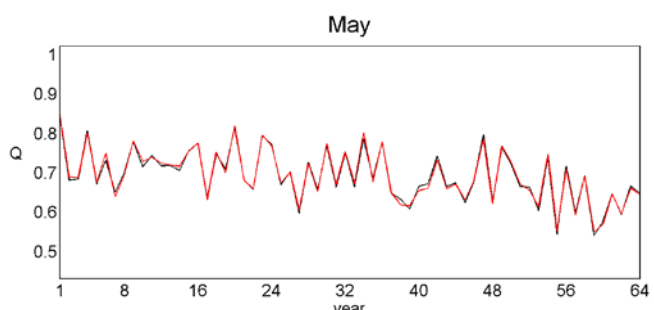
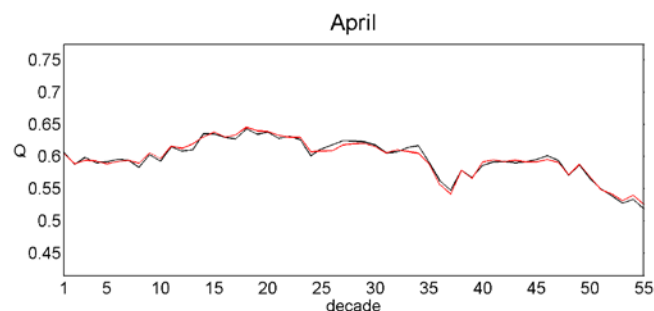
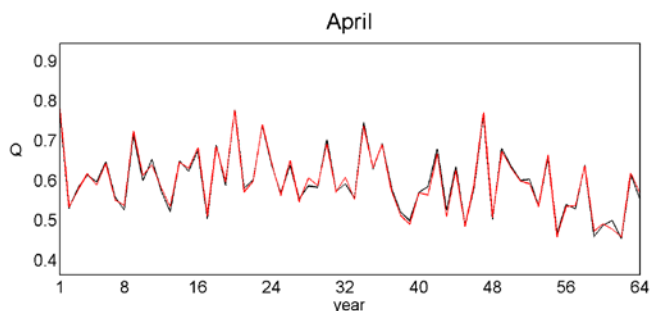
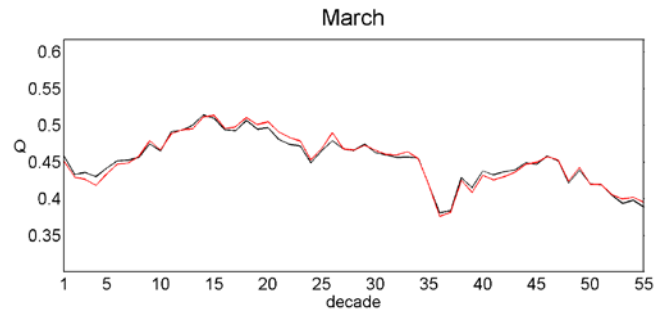
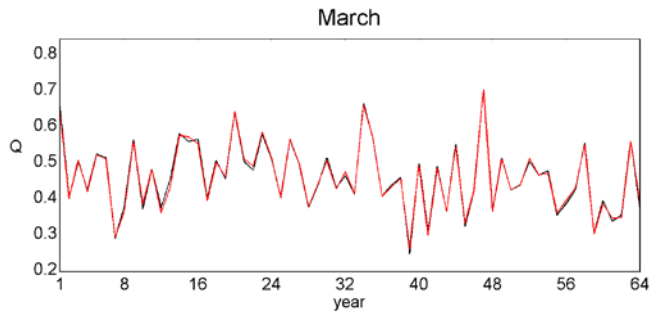
data



FM fit

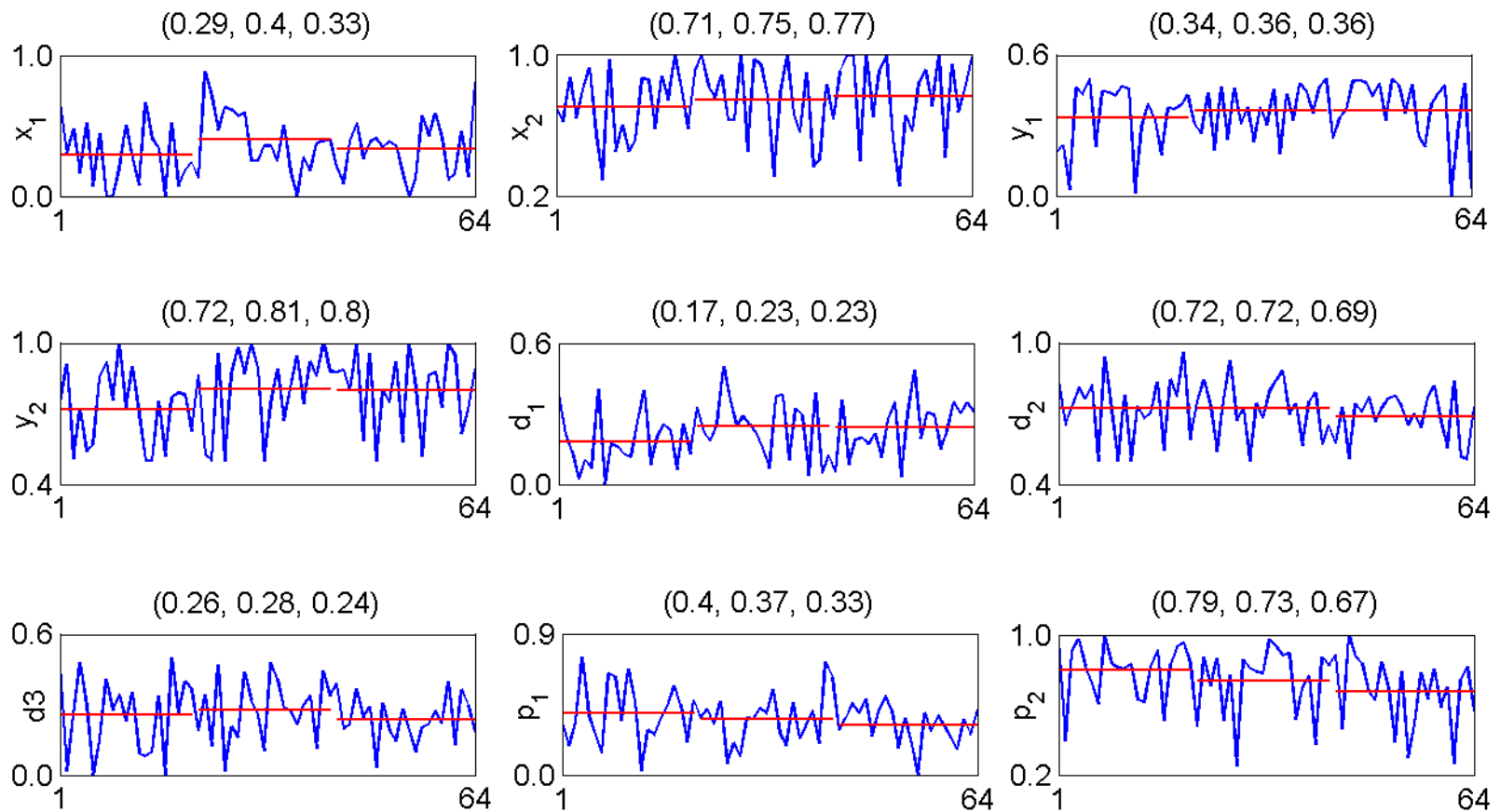
# Encoding streamflow in Sacramento

Spring flows by year and by decade



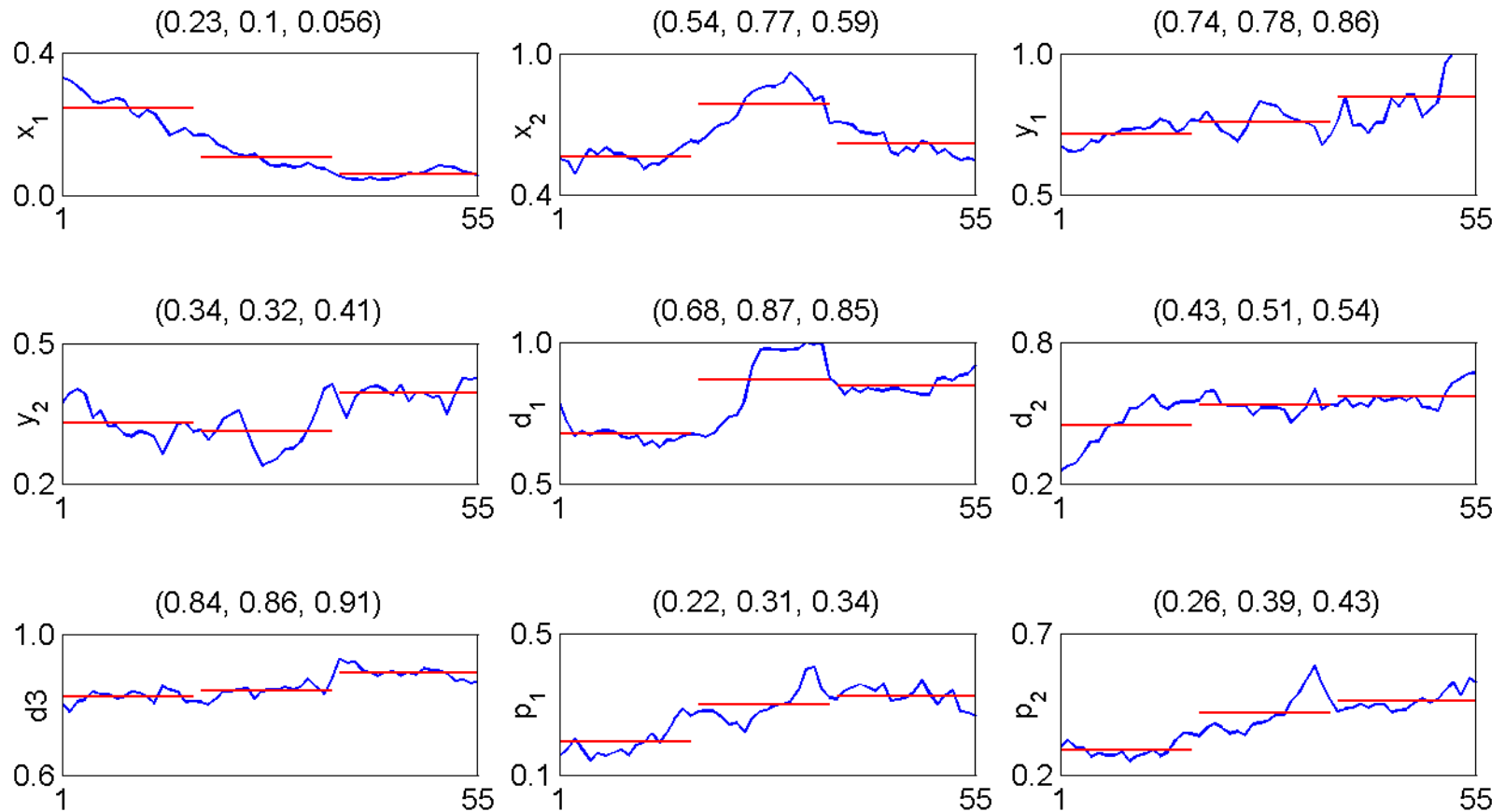
# Dynamics of streamflow in Sacramento

By year 1950 - 2014



# Dynamics of streamflow in Sacramento

By decade 1959 - 2014

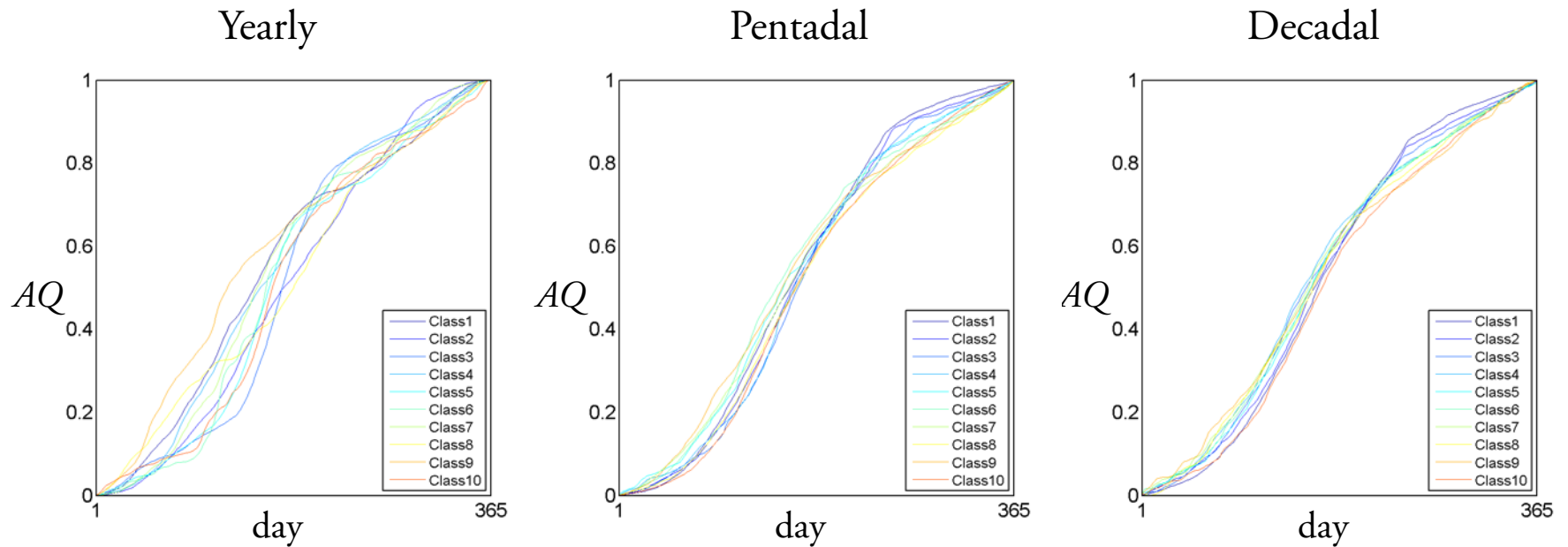


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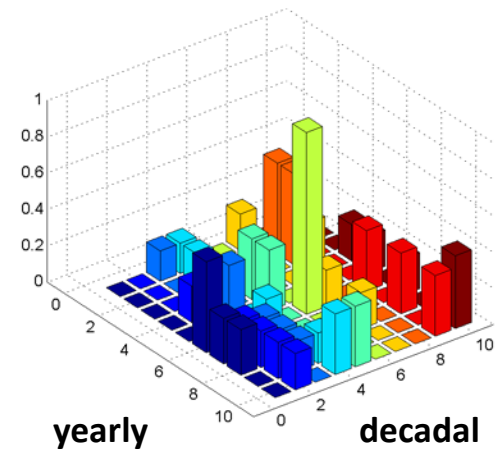
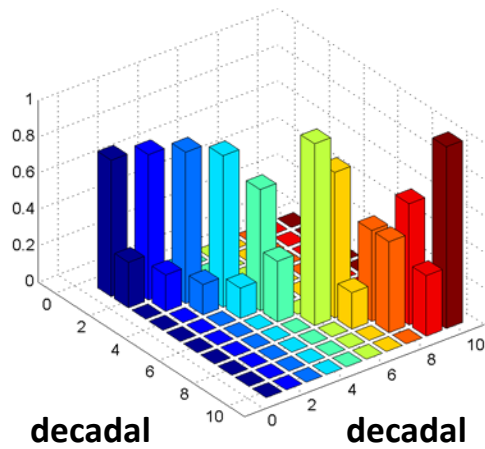
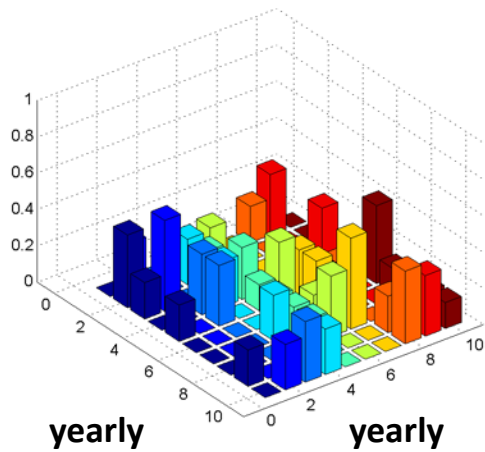
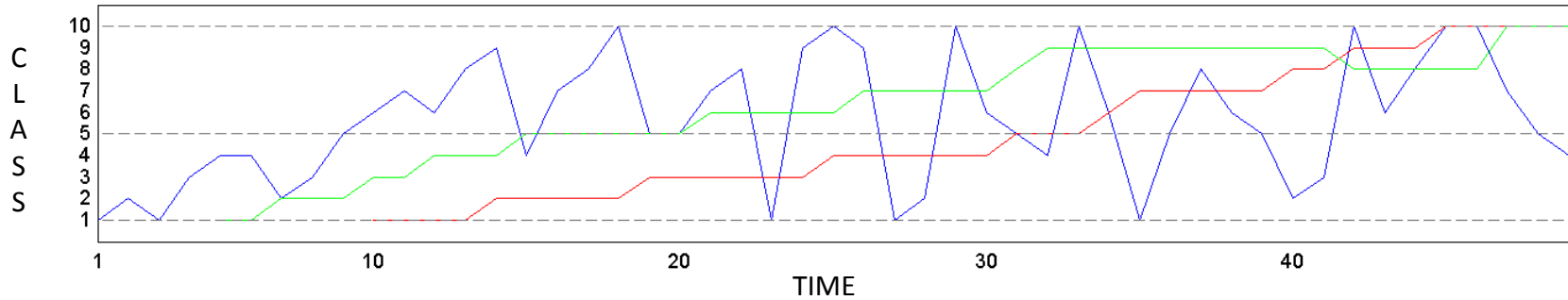
# Classifying streamflow in Sacramento

Based on FM parameters and k-means clustering



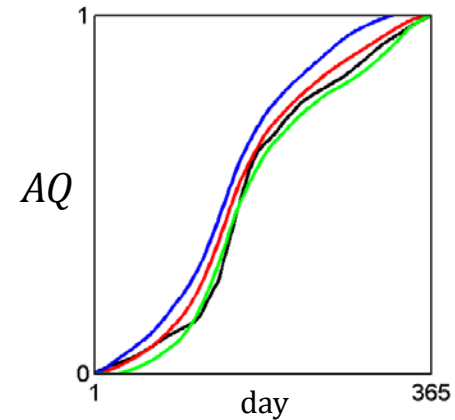
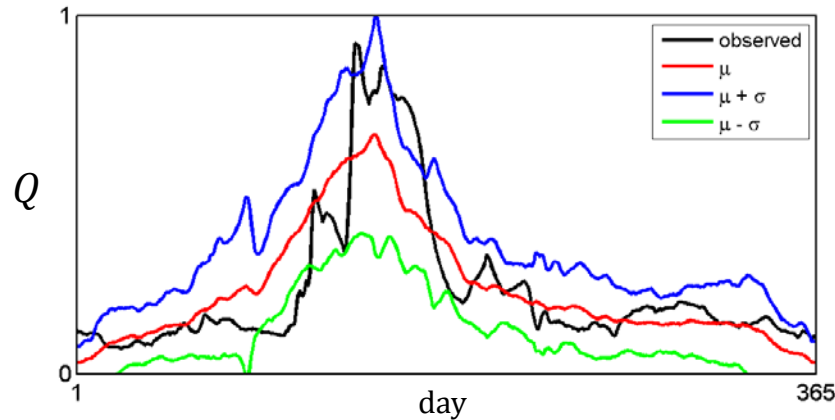
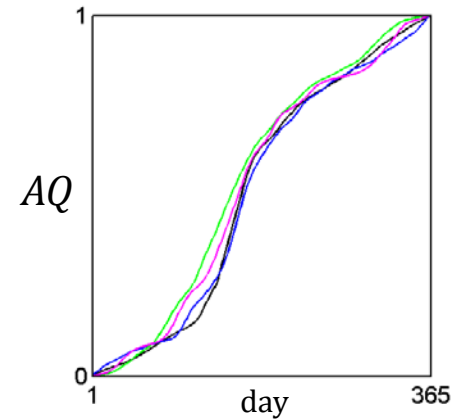
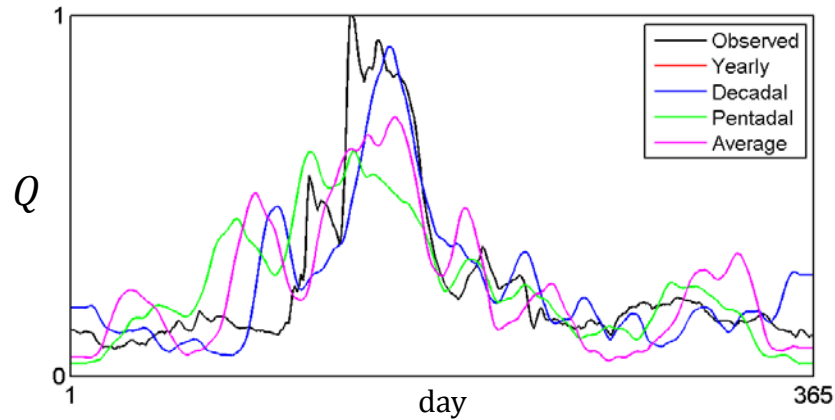
# Classifying streamflow in Sacramento

Evolution by classes and transition matrices



# Predicting streamflow in Sacramento

A successful year, via trends and FM parameter sensitivity

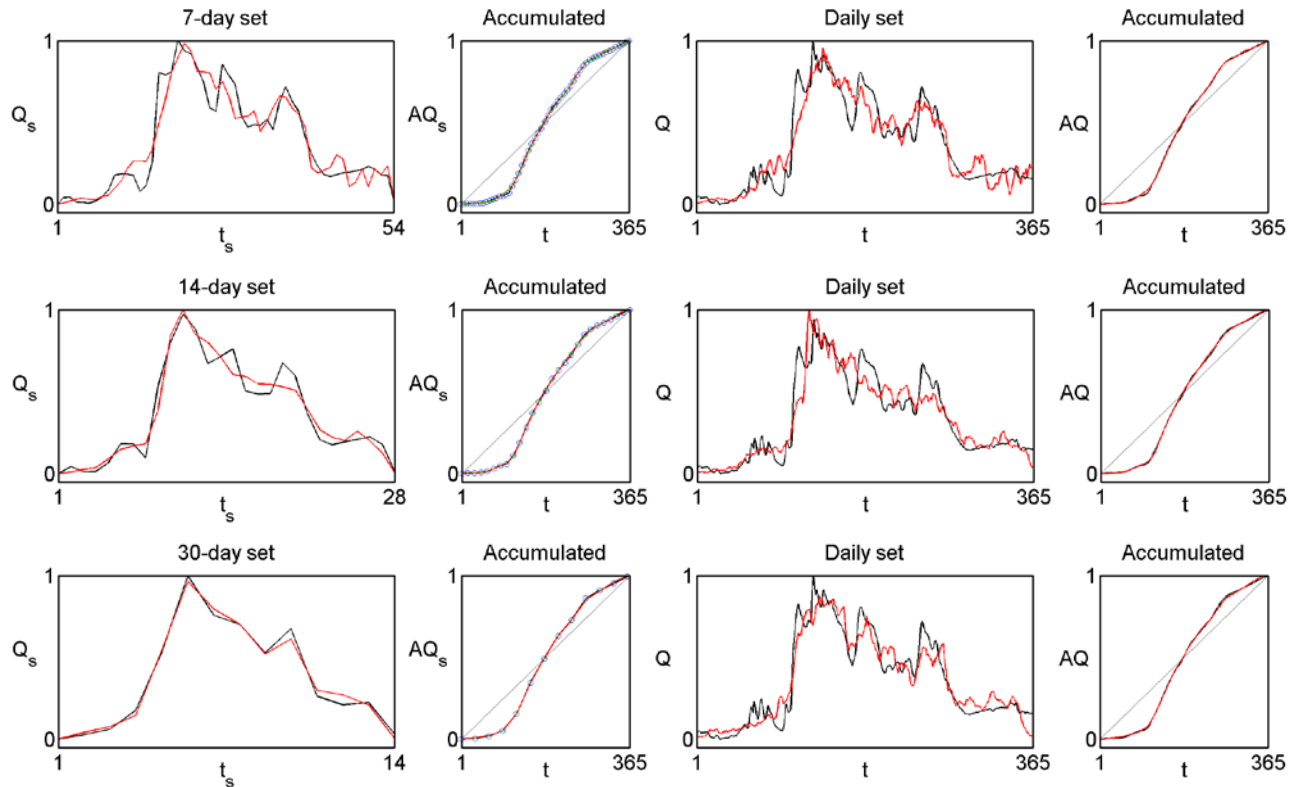


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# Downscaling streamflow in Sacramento

Every 7, 14 and 30 days



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# Outside the cave

- The platonic ideas may be used to model a variety of data sets, both in time and in space.
- The notions provide a compact deterministic language for some features of natural complexity.
- Perhaps the ideas may help classify patterns in different locations and also discern structural changes of qualifiers of climate change as a function of time.
- The notions may conceivably provide an alternative way to evaluate future scenarios if trends in geometric parameters may be established.

# Potential collaboration

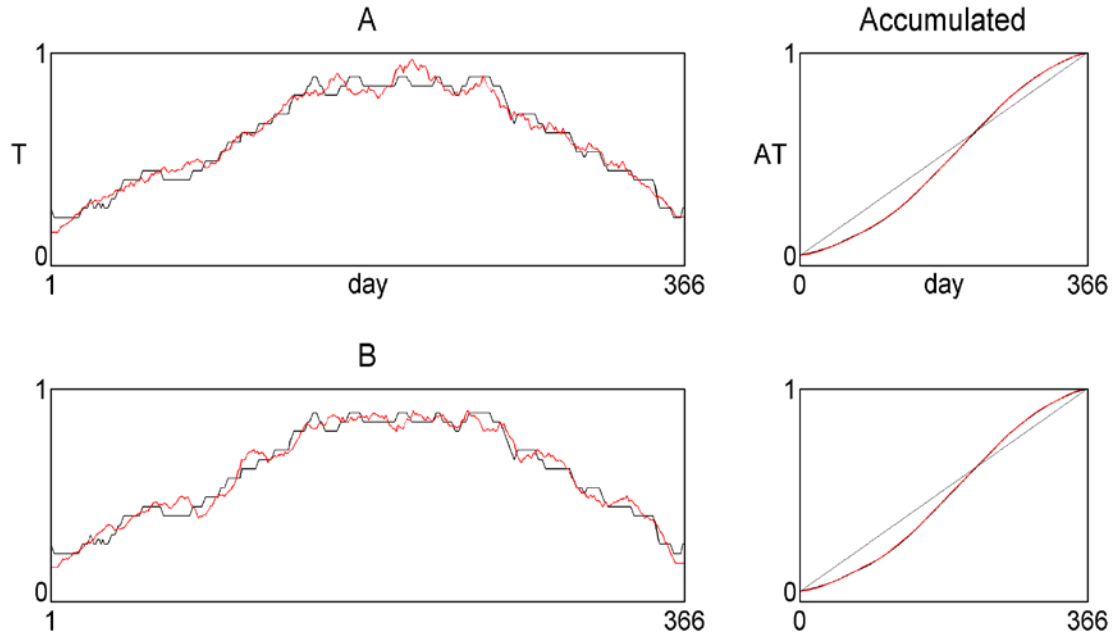
- Studying the dynamics of rainfall/streamflow/temperature at a few locations within the state and at various time resolutions, aiming at finding possible holistic predictions.
- Assessing the effects of climate change studying the structure of rainfall based on a FM geometric study at distinct locations and scales. Are distributions changing? Are those predictable?
- Analyzing distinct design scenarios generating FM synthetic rainfall/streamflow/temperature sets and/or downscales.
- Classifying and comparing the geometric structure of records to establish improved regional understanding. Can intensity-duration-frequency curves be parameterized via FM codes?

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# Encoding water temperature in Sacramento

Calendar year 1968



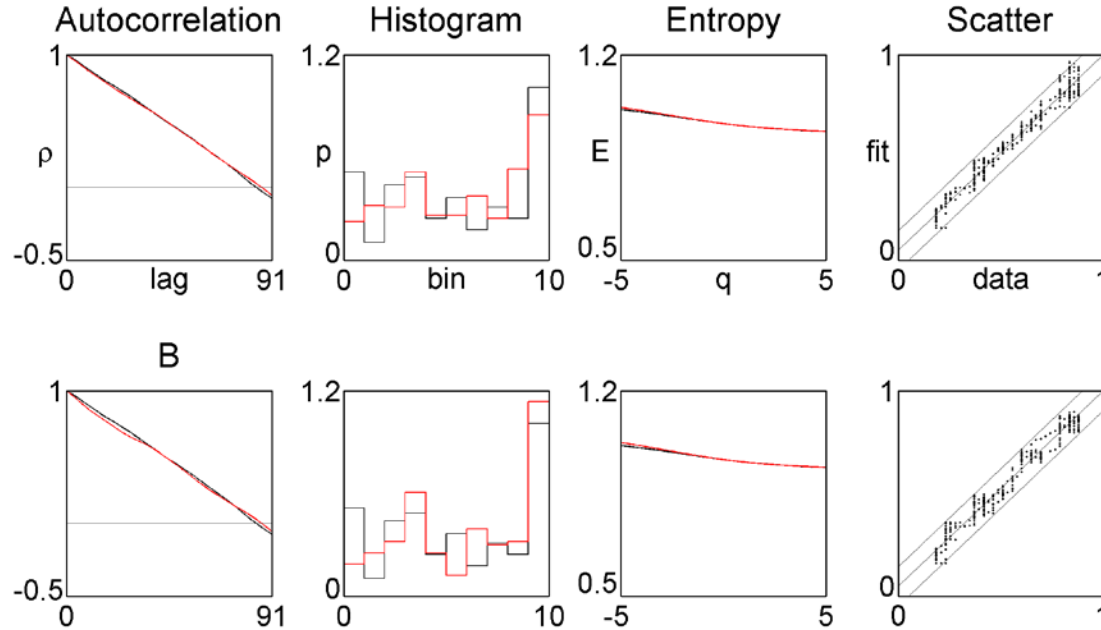
A: wire 2 maps, B: "leaf" 2 maps

RMSEAR 0.23%, 0.21%; MAXEAR 0.61%, 0.44%

compressions greater than 52:1

# Encoding water temperature in Sacramento

Calendar year 1968

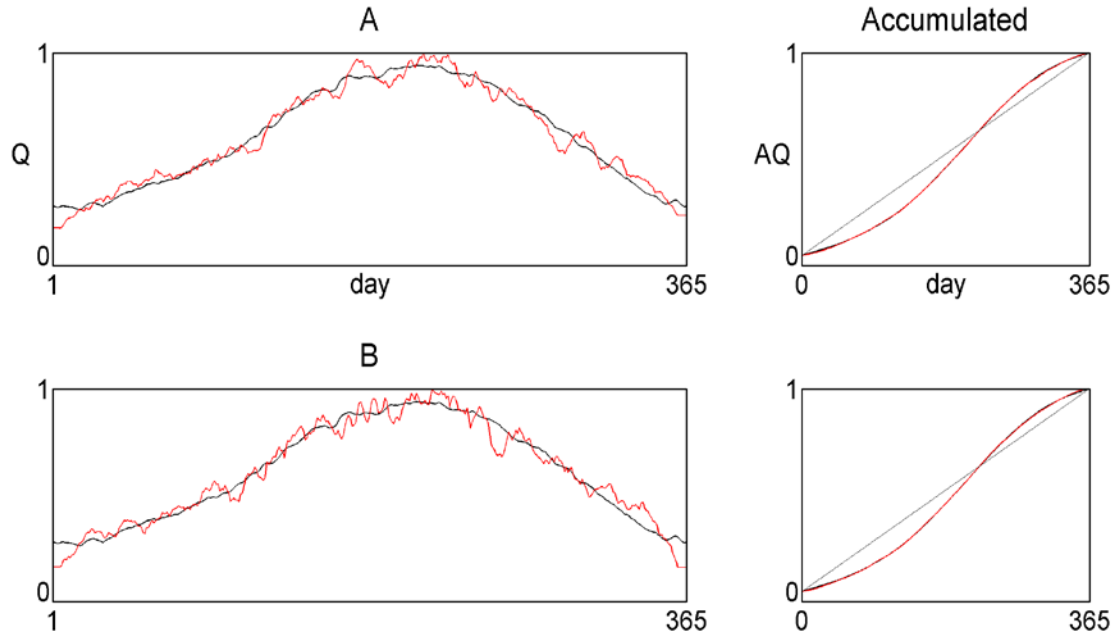


A: wire 2 maps, B: "leaf" 2 maps

NSD 96.7%, 96.4%; NSA 99.8%, 99.5%; NSE 99.1%, 98.0%

# Encoding water temperature in Sacramento

Calendar decade 1972



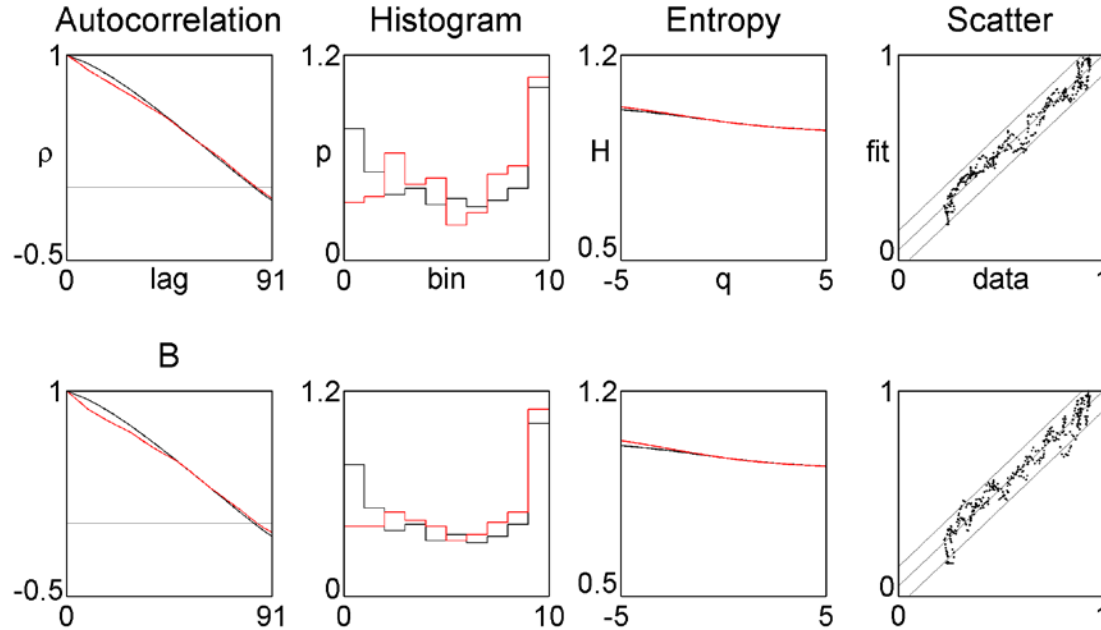
A: wire 2 maps, B: "leaf" 2 maps

RMSEAR 0.23%, 0.27%; MAXEAR 0.58%, 0.67%

compressions greater than 52:1

# Encoding water temperature in Sacramento

Calendar decade 1972

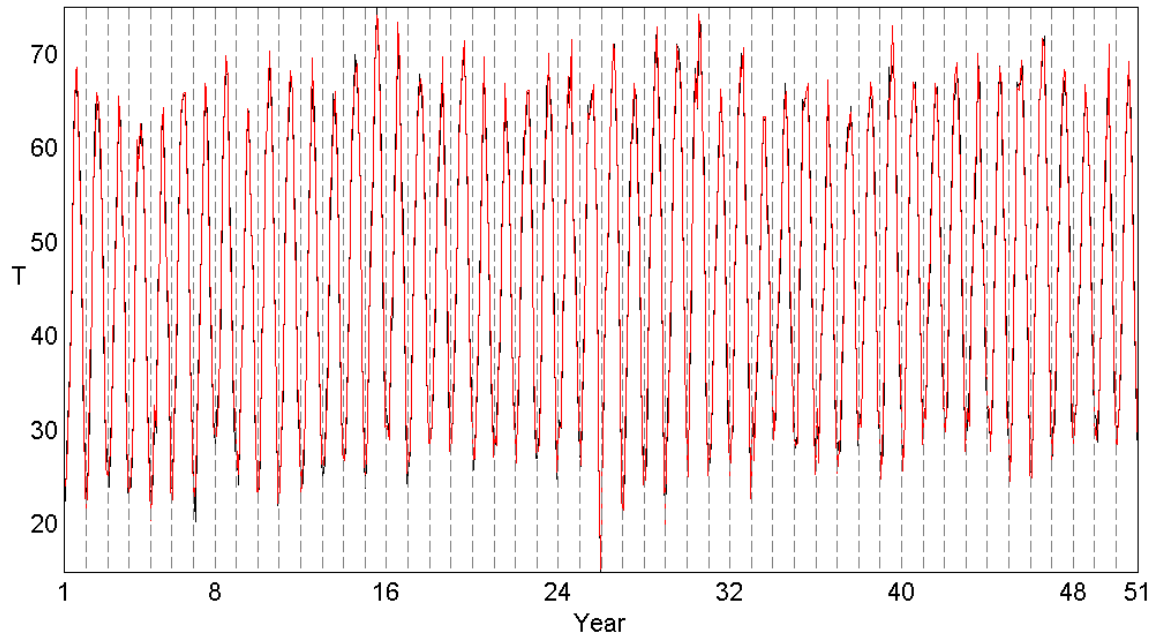


A: wire 2 maps, B: "leaf" 2 maps

NSD 96.7%, 95.3%; NSA 99.2%, 98.3%; NSE 98.4%, 94.8%

# Encoding water temperature in Sacramento

By year 1962-2012

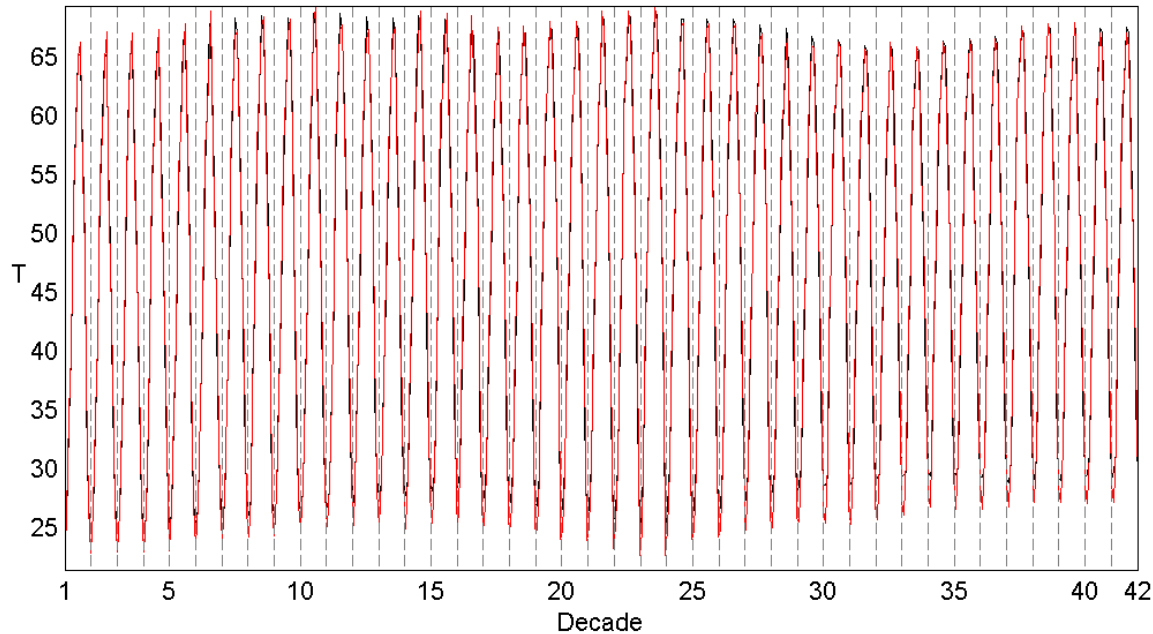


“leaf” 2 maps, scale in  $10 \times ^\circ\text{C}$

RMSEAR  $0.32 \pm 0.07\%$ ; MAXEAR  $0.72 \pm 0.15\%$ ; NSD  $92.1 \pm 3.08\%$

# Encoding water temperature in Sacramento

By decade 1971-2012

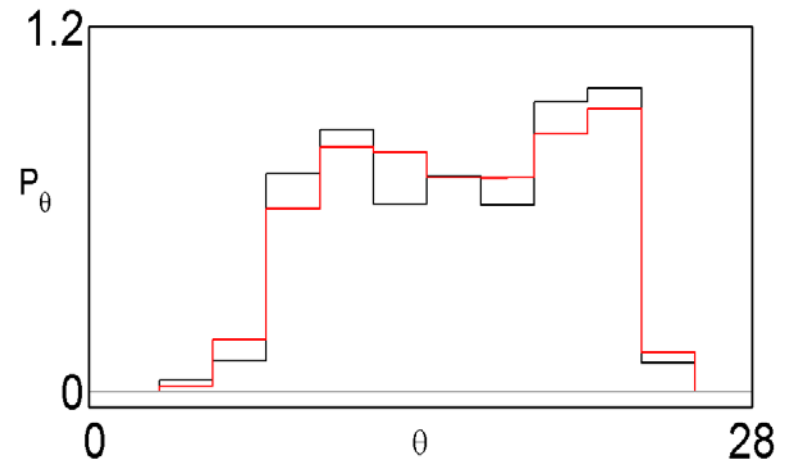
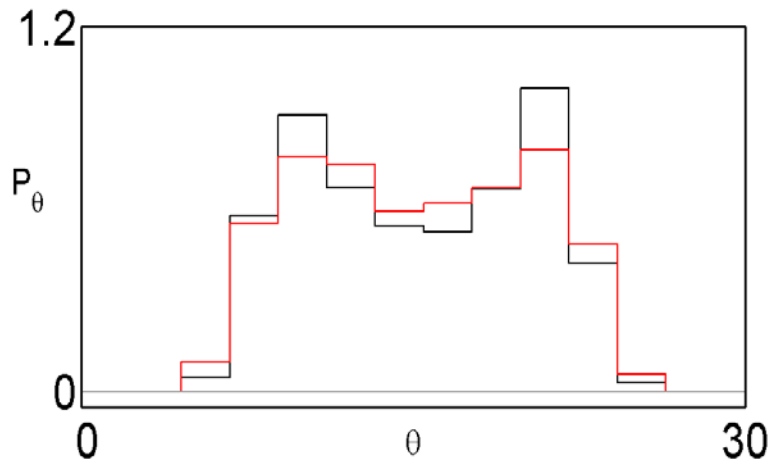


“leaf” 2 maps, scale in  $10 \times ^\circ\text{C}$

RMSEAR  $0.27 \pm 0.05\%$ ; MAXEAR  $0.56 \pm 0.08\%$ ; NSD  $96.01 \pm 0.83\%$

# Encoding water temperature in Sacramento

Two histograms 1962-2012

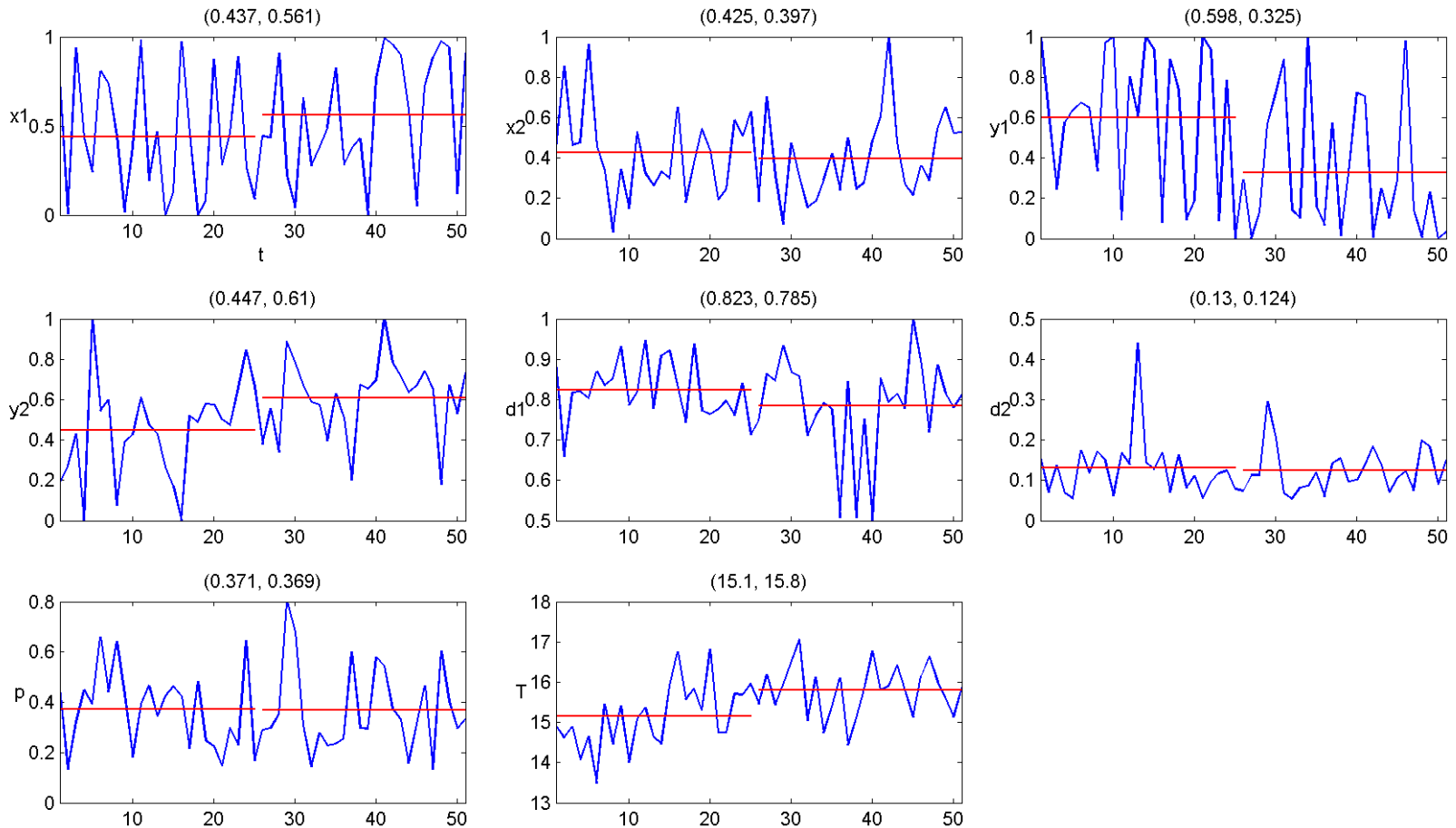


left: first 25 years, right: second 26 years

useful for climate change...

# Dynamics of water temperature in Sacramento

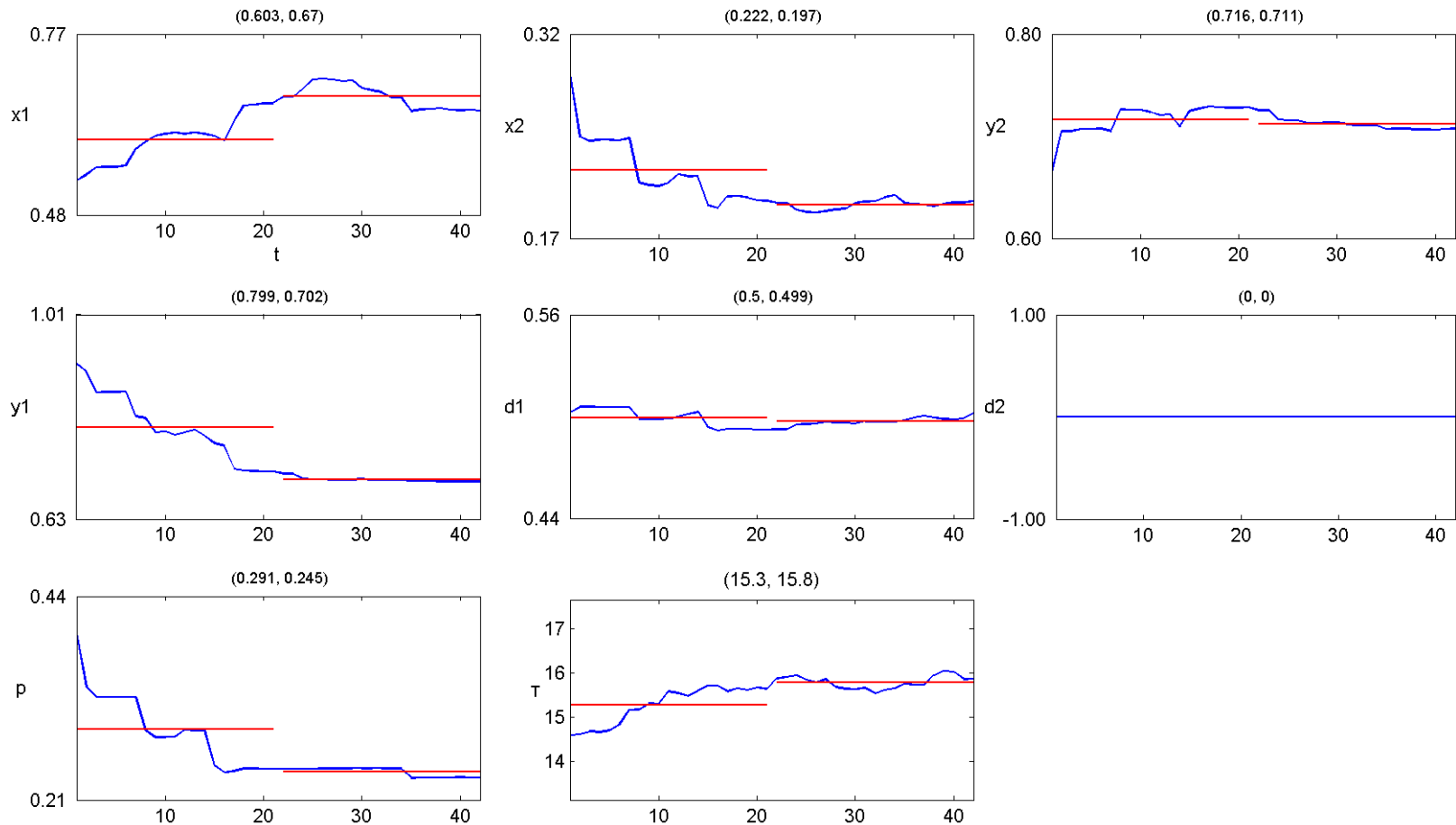
By year 1962-2012



inherent complexity in smooth temperature records...

# Dynamics of water temperature in Sacramento

By decade 1971-2012



smoother evolution maybe useful to elucidate climate change...