A merciful lesson from a chaotic fig tree

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Cover. As we continue this series of lectures FROM MODERN SCIENCE TO GOD'S MERCY, as inspired by the Jubilee of Mercy, I would like to thank you very much for your presence.

This presentation is entitled **A merciful lesson from a chaotic fig tree**. Here it is shown how a recently discovered chaotic fig tree, and other such trees, provide relevant lessons to increase our faith, in consonance with what is written in the Bible.

May the Holy Spirit to be present during this lecture and may the Immaculate Virgin Mary pray for us all.

Page 2. The thesis of this work is that we humans, with the gift of a soul, may learn from recent advances regarding the geometry of natural complexity, in order to fully grasp the boundless nature of God's mercy and take on the mission to share the good news in a renewed fashion.

Page 3. As may be readily verified, and to set some connections for later, the fig tree is an important and mysterious symbol in Holy Scripture. It appears first in the fall of Adam and Eve, in the way they covered their nakedness using fig leaves. Throughout the Old Testament, it is a consistent symbol, together with the vine, for the behavior of the people of Israel, for they could only enjoy such desirable fruits if they would follow God's law. The fruit is also a notorious healing symbol, as prophet Isaiah administered a poultice of figs in order to cure the ailing good King Hezekiah, after the sun reversed and retraced its course.

Page 4. In the New Testament, the first mention of the tree is in the story of Nathanael, one of the twelve disciples, who first said "what

good can come from Nazareth" and then yielded to Jesus Christ only because He told him that He saw him under a fig tree.

Such a symbolic tree also appears prominently in the parable of the barren tree, one placed in the middle of a vineyard and given another year to produce fruit and in the –odd and seemingly out of character– cursing and killing by Jesus of a fruitless fig tree that, we are told, bore no fruit because it was not in season.

Additionally, the fig tree is also mentioned in Jesus' eschatological discourse, when He explains that such a tree, with a tender branch and budding leaves, would signal His return, a parable that ends with Jesus' mighty words that heaven and earth will pass away but not His words.

This presentation aims to show how modern science –and in particular chaos theory– provides suitable elements that allow us to further understand these passages, leading to urgent pleas to conversion and relevant reminders about us needing to be vigilant, while we await Jesus' return loving God and one another.

Page 5. Hailed as one of the most important scientific achievements of the 20th century, chaos theory provides poignant symbols and precise semantics.

The prototypical equation used to illustrate the well-established theory is the *logistic map*: X k plus 1 equals *alpha* X k times one minus X k.

Here, X may represent the size of a normalized *population* from 0 to 1 of, say, rabbits; k and k plus 1 are subsequent generations; and *alpha* is a *parameter* that may take values from 0 to 4.

Page 6. The logistic map gives rise to a symmetric **parabola** obtained by plotting the quadratic equation one generation against the

next.

As is seen here, the graph peaks by the middle at a height of *alpha* quarters and exhibits an increase if X k is small, but a decrease if it is large.

The name of the map reflects the fact that such a behavior is "logical" in many situations regarding resources: if there is a little of something there is a tendency to increase, but if there is too much, then the trend is to decrease.

Page 7. Aided by the one-to-one line $Y = X$, we may compute a sequence of *iterations* of the map starting at an arbitrary location, X zero, until it converges, following the vertical-horizontal lines, to the *ultimate fate* of the population, X *infinity*.

As is seen here for a value of *alpha* equal to 2.8, the population increases steadily and then spirals into the non-zero intersection between the parabola and the line.

Page 8. But the ultimate X *infinity* is not always the same and depends exquisitely on the parameter *alpha*, as summarized next.

Page 9. When the parabola is *below* the line $Y = X$, that is, when *alfa* is less than one as shown here for *alpha* equal 0.7, the ultimate X *infinity* equals **zero**.

Regardless of the initial population size, repeating the dynamics, following the vertical-horizontal lines, leads to the same destination: the extinction of the rabbits. *Zero* is hence termed a *stable fixed-point attractor* for the process.

Page 10. When the parabola crosses the line and *alpha* is less than 3, X *infinity* is given by the non-zero intersection between the line and the parabola, as illustrated in motion before. Now, starting with

a small number of rabbits does not lead to their extinction, but rather to a constant population size equal to *alpha* minus one over *alpha*.

The origin, which previously attracted all the dynamics, now repels, and this happens as the slope of the parabola at the origin is too steep, that is, it is more than 1 in magnitude.

Page 11. But this is not all that may occur. When *alpha* exceeds 3, what happened to the origin also happens to the non-zero intersection between the line and the parabola: it no longer attracts but rather repels, because the slope of the tangent there is too steep.

As shown when *alpha* equals 3.2, zero repels and although the lines appear to point to the non-zero intersection, such also repels and the iterations do not settle into a single point, but instead end up oscillating indefinitely between two values X *infinity* one and X *infinity* two, making up a square, and hence defining a so-called *periodic attractor*.

Page 12. If *alpha* is further raised to 3.46, oscillations between two populations are no longer achievable, but instead the dynamics settle into a repetitive pattern every four generations: X *infinity* three to X *infinity* four to X *infinity* five to X *infinity* six and back, making up a shape with eight segments that travels in and out as shown.

Page 13. In a surprising manner, this trend continues as *alpha* increases and the logistic map defines increasing **bifurcations** that encompass all powers of 2. This happens as *alpha* reaches a limiting *alpha infinity* value equal to approximately 3.5699.

Page 14. After *alpha infinity*, the dynamics may be either repetitive, that is, *periodic*, or *non-repetitive*, that is, aperiodic.

As an example of the latter, when *alpha* equals 3.6 and 4, the itera-

tions yield uncountable "*strange attractors*" that do not settle into any repetition. As such sets also have the property that nearby initial points end up diverging from one another, they are also known as **chaotic** attractors.

Page 15. When *alpha* equals 3.74 or 3.83, the dynamics result in oscillations that repeat every five and three generations, as shown. As if by magic, the curvature of the parabolas is such that the vertical-horizontal lines synchronize to yield eventual repetitions.

Remarkably, there are many such periodic oscillations for great many periods after *alpha infinity*, all intertwined with chaotic behaviors.

Page 16. When all is computed, the **diagram of bifurcations**, shaped as a **tree** when rotated 90 degrees counterclockwise, summarizes the incredible behavior of the deceptively simple logistic parabola.

Here in the horizontal is *alpha* and in the vertical X *infinity*.

For *alpha* between 0 and 1, there is the extinction of the rabbits in the straight root of the tree. For *alpha* between 1 and 3, there is the fixed point main branch given by the equation *alpha* minus 1 over *alpha*, leading to the upper tip at two thirds. Right after, there are repetitions every two generations, then every four, and quickly every power of 2, drawing bifurcating branches up to *alpha infinity*. Then, there are chaos and repetitions intertwined in the tail of the diagram.

Page 17. Notably, the diagram contains periodic behavior –as seen in white bands– that encompasses all natural numbers and great many non-repetitive behaviors, something impossible to foresee prior to the advent of modern computation.

Page 18. Here is observed in more detail the tail of the diagram.

Page 19. As is seen, strange attractors, plotted as vertical collections of dots, are very common after *alpha infinity*. Such aperiodic sets turn out to define **dusty** attractors, as there is empty space between their points, as indicated by distinct shades on the vertical lines.

Page 20. As is observed, within all periodic white bands, there are noticeable "**buds**" like the one here in the middle of period 3.

Page 21. When such a bud is magnified, one surprisingly notices that it includes a small copy of the shoot of the original tree.

Page 22. And as in the infinitely many buds of all periods such a generic repetition happens, this remarkable fact leads to the conclusion that the bifurcations diagram exhibits utter **self-similarity** –without the root– *ad infinitum*, something never seen before the sprouting of the chaotic tree.

Page 23. The diagram of bifurcations includes also a multitude of **multifractal** thorns that correspond to **spiky** distributions of dusty sets within the **chaotic tree**.

The first such a set, shown here, occurs precisely at *alpha infinity*, when there are infinitely many bifurcations. A histogram of the dynamics at such a location exhibits imbalances in the spikes and holes between them, which may be defined via a cascade that includes both features, as introduced during our first encounter.

Page 24. Remarkably, there are **thorns galore** by the ends of all white bands of buds corresponding to the infinitely many periods encountered in the tail of the diagram, and the object would prick us at infinitely many places.

Page 25. Believe it or not, the tree of bifurcations is also known as the Feigenbaum tree, or the "fig tree" in German, in honor of Mitch Feigenbaum, who proved, for the first time, some **universal** properties about the object.

It happens that the bifurcations occur in an **orderly** manner both in their **openings** and **durations**.

As is seen here, the X bar equal one half line crosses the bifurcations in an alternating fashion, and the openings *d sub n*, from such a line to the other side, decrease such that the ratio from bifurcation to bifurcation tends to the number *F sub 1*, -2.50, the first Feigenbaum constant.

Similarly, the duration of the bifurcations *delta sub n* decreases quickly and the quotient from one bifurcation to the next tends to the number *F sub 2*, 4.669, the second Feigenbaum constant.

These assertions show that there is order in the pathway to chaos via bifurcations, but such does not imply that chaos itself is ordered and predictable.

Page 26. Feigenbaum's numbers turn out to be **universal**, as they hold for countless equations that give rise to other chaotic trees.

The iterations of functions with one peak, as the ones depicted here and related to the simple equations shown, always yield, by increasing *alpha*: a straight root, a "tender branch," bifurcation branches, and, in an intertwined fashion, periodic branches and the dusty foliage of chaos, which then, figuratively, corresponds to "fig leaves." In all cases, the openings and durations of bifurcations –and also within all their buds– happen at the speeds given by *F sub 1* and *F sub 2*.

Page 27. These ideas turn out to be relevant in several fields of science, including biology, ecology, chemistry, physics, and economics.

Page 28. Among the most pertinent results are discoveries made and inspired by Albert Libchaber's work on how **convection** happens, that is, how the heating of fluids leads to turbulence.

Surprisingly, it turns out that *alpha*, at the very pace of the second Feigenbaum constant, yields the **heat** that needs to be added to a fluid, whether liquid helium, mercury, or water, so that qualitative changes are observed.

If little heat is added to a fluid, its internal temperature does not even change and such corresponds to the root. But if enough heat is progressively added, first the fluid becomes steadily warmer, as in the main branch, and then it exhibits temperature variations at it starts to move in response to the additional energy, as in the bifurcations.

The first bifurcation corresponds to a case where the warmer fluid, close to the source, becomes less dense and rises to be replaced by a colder fluid that, being more dense, sinks, making up a repetitive cycle. As *alpha* keeps increasing, new repetitive stable behaviors settle in response to the additional heat and such happen at the pace given by *F sub 2*, and if the heat is increased a lot, it is best not to put a finger inside the fluid!

This we all know, sometimes from experience, but there is even more.

Page 29. To appreciate other subtleties of chaos, it is pertinent to study in some detail what takes place in the **summit of chaos**, when α takes on its maximum value of 4 and the logistic parabola ranges from 0 to 1.

At such a maximum "heat," the dynamics wander on a compact strange attractor that, as shown, appears to encompass the entire interval from 0 to 1.

Page 30. However, this is not true, as unstable extensions of the

Feigenbaum tree, shown here below, need to be excluded.

For instance, the value of X equal to $3/4$, corresponding to the extension of the original tender branch in the non-zero intersection of the line and the parabola, cannot be in the ultimate attractor. Starting at such a value returns four times three quarters times one quarter or three quarters –the same value forever– yielding a repetition every generation which is clearly not chaos.

Values corresponding to any period need also be excluded, as the ones that repeat say every three generations, as shown next.

Page 31. As indicated in this diagram, not only do the three repeating values need to be excluded, but also all other points within the domain that are in the past of such values, that is, their pre-images.

Here k denotes a generation, with zero and positive values indicating eventual repetitions –high, low, medium, high, low, medium, etcetera– and with negative values denoting points that are not yet on the periodic attractor.

This diagram turns out to be easily computed as there are two values in the past for any single value in the present. As such, the highest repeating value shown in generation zero is associated with two pre-images found reading the parabola backwards –just drawing a horizontal line at such a value and finding the intersections, as illustrated on the right– , and if such a process is repeated on each point, it yields a tree in powers of 2, a countable collection of points that hence cannot be part of the strange and chaotic attractor.

Page 32. At the end, many similar "trees," containing a countable number of points and corresponding to the values of any **period**, need to be excluded from the interval from 0 to 1: two of period two, three of period three, four of period four, and so on.

Page 33. As a consequence, the strange attractor is a rather disperse and **dusty** uncountable set, as it is equal to the whole interval from 0 to 1 minus the infinitely many trees branching densely in the interval, like the one here that has pre-images all over the interval from 0 to 1, just 13 generations to the past.

Page 34. All this turns out to be very relevant because there is a collection of initial values that altogether **escape** the dreadful consequences of jumping forever in high heat, for such return dynamically –even if improbably– to the **root** of the tree.

These are the **pre-images of zero** that find their way despite the most implacable **convection**, which, by arriving precisely to the **middle** at generation zero next go to **one** to finally rest at the **origin**.

Page 35. Note how this process paints a remarkably consistent, even if inverted, **square root**! Just twist up...

Page 36. Observe how these observations remind us of the concept of *purgatory* and the associated parable of the weeds, for the wheat in the pre-images of zero is finely surrounded by undesired behaviors that unfortunately end up forever in fire.[1]

Note how the ideas also remind us of the three exalted friends of prophet Daniel dancing in the highest heat without consequences and the protection of the prophet himself in the lions' den.[2]

Page 37. As Jesus spoke in parabolas, for such and parables are the same word in many languages, these notions suggest, I think, some

[1] Mt 13:24-30
[2] Dn 3:1–92, Dn 6:2–24

plain common sense reflections.

For instance, these ideas reaffirm that it is best to avoid **chaos** and its truly **infernal** turbulence, because missing the **middle point**, the point, even by a small distance "epsilon," has, as shown, tragic consequences.

As the mid value is not a half, the next value is not one but something less and the following point is not zero but more, and although the dynamics remain close to zero for a while, the original small error amplifies into the unpredictability of chaos that, in all probability, misses the origin.

For the famous "**butterfly effect**," expressing the incredible sensitivity of chaotic dynamics to the start of the iterations, does not provide us with good options at all and instead leaves us irremediably trapped in an empty strange attractor in which, as some prophets have clearly explained, we find no rest.[3]

Page 38. As such, we may learn from these notions, and not only in relation to sharing our "rabbits," that our best choice is for us to come down the chaotic tree, just as the little man Zacchaeus did, and straighten out our errors, that is, our sins, hence diminishing our intrinsic **heats** to find the straight **root** and accept God's salvation.[4]

Page 39. As convergence to the root is associated with little parabolas that do not cross the line, these ideas also invite us to abandon ourselves to the chosen threshold, $Y = X$,[5] that is, to **Jesus Christ** himself, seen geometrically in his cross X and his crucified silhouette Y, who we are told is the **door** and the **narrow gate** –and how narrow it is in the summit of heat–,[6] so that we may arrive to the love of the **Father** in the gentle state X *infinity* equal to zero,

[3] Jb 40:12–13, Mi 7:17, Hos 8:7
[4] Lk 19:1–10, Mt 18:3–4
[5] Mk 8:34–35, Lk 9:23–26
[6] Jn 10:9–11, Mt 7:13–14

which consistently denotes the saintly **Origin**, now with a capital O representing His halo of perfection.[7]

Page 40. To say it yet in another way, the precise symbols from chaos theory remind us that it is best to move away from the implicit nonlinearities that happen when *alpha* is greater than 1 and into mild slopes at the origin, for amplifying without proportion takes us into the shoot of the tree, thorns, and eventually into **dust**, which represents death.[8]

For this is what is prescribed regarding our actions, even if some chaotic features are wired into a healthy brain and even if it appears, falsely, that we may arrive to heaven through other paths and without Jesus.

These ideas represent unexpected yet rather accurate modern connections between the geometric symbols and universal concepts of the science of chaos and our Christian faith that provide an unforeseen invitation to reconciliation and love, consistent with the very logistic of salvation in accepting Jesus Christ. However, the modern advent of the symbolic fig tree suggests even more.

Page 41. Could it be that the **scientific feigenbaum**, even if springing twenty centuries later, is prophetic? Could it be that in that God is giving us a **merciful** clue that illuminates the ancient Word?

Page 42. As we recalled earlier, Jesus cursed an allegoric fruitless **fig tree** and it consequently dried up to the root.[9] Like the biblical one, the chaotic fig tree lacks any visible fruit and contains instead leaves of dust that properly remind us of the leaves Adam and Eve

[7] Mt 11:28-30
[8] Rom 5:12
[9] Mt 21:18–22, Mk 11:12–23, Lk 13:6-9

used to try to cover their shame and God's famous edict to Adam "dust you are and to dust you shall return."[10]

As such, this tree is consistently **cursed**, above the root, by the inherent disobedience of crossing the threshold $Y = X$.[11]

Page 43. As Scripture hints that we may see it for ourselves, this fig tree –and many other chaotic ones– has a tender branch (as curiously written in singular in the original Greek) and literally contains a multitude of bursting buds in its periodic bands.

As climbing the shoot of the trees symbolizes our progressive detachment from the root of goodness, and as the trees also contain multitudes of equally symbolic thorns,[12] it is sensible to ask ourselves –if we pay attention to Scripture– if these ideas are a preamble of a near summer, of a near return, as Jesus expressed it in His eschatological discourse.[13]

Page 44. For in the bifurcations diagram, it may also be appreciated why the ax lies at the **root** of the trees, as expressed by fellow hydrologist John the Baptist;[14] why it makes sense that the root is the logical location where Jesus saw the future apostle Nathanael, that is, "under the fig tree;"[15] and why **Jesus** said to his astonished disciples that they could also curse and kill the fig tree,[16] no doubt in the same way He rebuked the wind,[17] whose ruler, the devil, is potently symbolized by the ratio $2/3 = 0.666\ldots$, as found by the end of the first bent branch of the tree.

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[10] Gn 3:7, Gn 3:19
[11] Dt 30:15–20, Ps 37:22
[12] Mt 13:22
[13] Mt 24:32–35, Mk 13:28–31, Lk 21:29–33
[14] Mt 3:10
[15] Jn 1:45-51
[16] Mt 21:21
[17] Mk 4:39–41, Mk 16:17–18

Page 45. Although by divine design the notions herein, and other repeating chaotic calamities, such as wars and rumors of wars, famines, tribulations, and persecutions, do not allow us to set exact dates,[18] they help us, however, to sketch some **options** we have, which heighten the goodness of being vigilant and behaving properly, in preparation for a triumphant return.[19]

Such options are: order or disorder, simplicity or complexity, serenity or turbulence, peace or chaos,

Page 46. decreasing, as John the Baptist did when he knew about **Christ**, or increasing, believing that we are more than Him,[20] following God's way and encountering Him below $Y = X$ or choosing our way above $Y = X$, abandonment or selfishness,

Page 47. obedience or rebelliousness, to receive in consequence blessings or curses, and ultimately, to rest eternally in heaven or to wander forever in hell.

Pages 48-49. I would like to share now a song called, in German, **Feigenbaum's Parabel**, or the **Parable of the Fig Tree**:

Foliage of disorder
trapped in empty dust,
jumps astir forever
enduring subtle thrust.

Crossing of the outset
leaving faithful root,
looming tender offset
failing to yield fruit.

Cascade of bifurcations,
increasing heat within,

[18] Mt 24:36, Acts 1:6–7
[19] Mk 13:32–37
[20] Jn 3:30

inescapable succession
of branches bent by wind.

Sprouting of dynamics
attracted to the strange,
o infinity reminding
at the origin: the flame.

Could it be, oh my friends
that science offers a rhyme?
For a rotten tree foretells
the very advent of time.

Could it be, oh how plain,
that nature extends a call?
For old parable proclaims
the crux in growing small.

In the midst of chaos
there is a small gate
leading to fine rest.

In the midst of chaos
there are loyal paths
inviting to a dance.

On top of the fig tree
there is a key point
that runs to the core.

On top of the fig tree
there is a clear light
that averts a fright.

In the midst of chaos
there is leaping game
discerning the way.

In the midst of chaos

there is a fine well
watering the brain.

On top of the fig tree
there is a clean frame
that cancels the blame.

On top of the fig tree
there is mighty help
that shelters from hell.

Page 50. Now, very close to the end, I would like to invite you to rejoice at other coincidences of faith you may visualize based on the concepts of science in this lecture.

It happens that the four states in Jesus' famous parable of the sower may be seen in the Feigenbaum tree. While the three fruitless scenarios correspond to chaotic, periodic and fixed points within the shoot of the tree, the presence of fruit is associated with the root of the tree, that is, with those that listen to the word, understand the word and put it into practice.[21]

Jesus is clearly, and in a rather geometric way, "our shelter from the wind,"[22] from the devilish wind to be precise.

Page 51. Jesus' invitation for us to be part of his flock may be seen in several passages. For instance, "abandon yourself, pick up your cross and follow me,"[23] "whoever loses his life for my sake will save it,"[24] and "if a grain of wheat dies, it produces much fruit."[25]

Page 52. In the summit of chaos we may vividly differentiate the

[21] Mk 4:1-20
[22] Is 32:2
[23] Mk 8:34
[24] Mk 8:35
[25] Jn 12:24

ever intertwined wheat and chaff[26] and the corresponding gentle sheep and stubborn goats.[27] There, in the pre-images of zero and in the same protection of the friends of Daniel, we may appreciate why "though 1,000 fall at your side, 10,000 at your right, near you it shall not come," as the Psalmist proclaims.[28]

And in the dynamics of a strange attractor we may get a glimpse of how "God may imprison the haughty in the dust together," as He affirmed to Job as a rather hellish and lonely punishment in the hidden world, where the ones trapped there never find each other.[29]

Page 53. As the chaotic trees contain multitudes of thorns and no visible fruit, we may appreciate the curious but accurate quotation by Jesus when He said that "people do not pick **figs** from *thornbushes*."[30]

Also, as the notions hint to the root at zero as a precursor of infinity in heaven, such allow us to visualize the **power of sainthood** geometrically and in a consistent manner consistent with Jesus' assertion that when two or three are gathered in his name, He is there in their midst.[31] In love then, zero plus zero equals infinity, as one plus one is equal to one, as explained in the previous presentation.

Finally, it ought to be emphasized, to make it really clear, that in the summit of chaos it is indeed rather difficult to find the way to the root, for there chaos is everywhere. As such, the middle point, the point and rather small gate expressing God's infinite mercy via purification, may be called the **omega** point, the one at the end, for Jesus is the *alpha* and the *Omega*, the beginning and the end, with no end!

[26]Mt 3:12
[27]Mt 25:31-46
[28]Ps 91:7-8
[29]Jb 40:12-13
[30]Lk 6:44
[31]Mt 18:19-20

Page 54. Now to end, and giving you thanks for your attention, I would like to share a poem in two languages, one entitled **Le Plus Improbable**. I hope you like it...

In the highest ugly heat
sûrement misérable,
almost never a repeat
oh détour incroyable,
sad travel with no rest
dynamique indésirable,
ever close to the best
étrange état exécrable.

In the most hellish beat
également déplorable,
a silly and vain defeat
périodicité interminable,
also a spectacle disjoint
obstination réprochable,
an epsilon from the point
oh hasard inévitable.

In the real driving seat
chaudement guérissable,
surely a fantastic feat
oh purgation ineffable,
a way to erected root
oh victoire admirable,
faithful relief and fruit
oh miracle vénérable.

In the symbolic tree
égoïsme détectable,
thorns, dust in a spree
oh fractal pitoyable,
but in humble zero set

oh origine inégalable,
line cancels all debt
oh paradis inalterable.

In the unbent choice
oh l'amour véritable,
always a clear voice
oh poème formidable,
an appointed up lift
miséricorde verifiable,
deterministic a gift
oh le plus improbable.

Page 55. Thank you very much for your presence. I believe I have reminded you once again that there is only one merciful way that we ought to proclaim lovingly for the salvation of man.[32]

Page 56. For additional information, please refer to the book *The Fig Tree & The Bell* and, of course, to the Holy Bible.

[32]Jn 14:6, Phil 2:10, Phil 2:11, Lk 13:24, Jn 10:9, Jn 10:11, Mk 16:15-16