Comparing the Evolution of Fractal Encodings of Daily Streamflow and Temperature as a Tool to Assess Climate Change

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Abstract

The fractal-multifractal method (FM), a geometric approach based on the transformation of multifractal measures via fractal functions and requiring few geometric parameters, has recently been shown to produce faithful encodings of geophysical records. It is shown that such a procedure (and its variants): (i) closely represents daily streamflow and temperature records at the Sacramento River (Freeport), with maximum cumulative errors that are always less than 2.5% over a period of fifty years, and (ii) yields FM geometric parameters that allow visualizing the dynamics of both processes. A classification of FM parameters, based on clustering techniques, and a comparison between the attributes of streamflow and temperature is then presented in order to assess potential climatic trends and changes.

Introduction

Understanding, archiving and forecasting geophysical information is crucial in order to make proper planning and design decisions, including modern issues regarding climate change and its impact. For these tasks, however, water resources researchers are faced with rather complex data sets containing intricate details, which demand special attention, as it happens with a host of hydrological patterns.

Since the last couple decades, a fractal geometric methodology, the fractal-multifractal approach (FM) has been proposed by Puente (1996) in order to capture the complexity of observed patterns, as they are, and not only some of its key statistics. Instead of thinking that a given data set is a realization of an stochastic process, whose histogram, autocorrelation function, and distribution of inter-arrival times may be fitted, such an approach assumes that records are instead a fractal transformation of a multifractal measure. Studies have shown that such deterministic notions are indeed useful for encoding rainfall events lasting few hours, (e.g., Puente, 1996; Obregón et al., 2002, Huang et al., 2013), and for representing daily rainfall and daily streamflow sets, gathered over a year (e.g., Maskey et al, 2015, Puente et al., 2015).

The present work shows faithful FM encodings of yearly streamflow and water temperature sets gathered at the daily scale, and presents a methodology for classifying both sets, based on the FM parameters. For this purpose, a total of 51 years of both records gathered at the **Sacramento River** near Freeport (USGS station 11447650) were analyzed.

The Fractal-Multifractal Approach

Original approach

Figure 1 shows the construction of a derived measure dy (adding up to one) transforming a *multifractal* measure dx through a *fractal* interpolating function $f: x \to y$ (Barnsley, 1988; Puente, 1996). This example is done iterating three simple affine maps:

$$w_1\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0.24 & 0\\0.07 & \mathbf{-0.76}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix},$$

$$w_2\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0.36 & 0\\1.79 & \mathbf{0.18}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}0.24\\-0.68\end{pmatrix},$$

$$w_3\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0.40 & 0\\-0.01 & \mathbf{-0.27}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}0.60\\1.28\end{pmatrix},$$

according to proportions **49**-**35**-16%, yielding a fractal function that passes by {(0,0), (**0.24**, **-0.68**), (**0.60**, **1.28**), (1,1)}. The FM parameters are marked in **bold** and include the vertical scalings in the matrices.

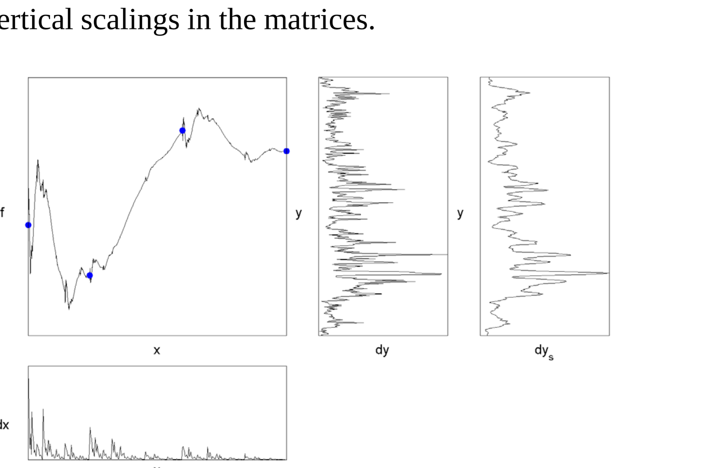

Figure 1. The original FM approach: from a multifractal dx to a projection dy, via a fractal interpolating function $f: x \to y$. dy_s is a smoothed version of dy.

Generalization of FM approach

Instead of a function appearing as an stable set from the iterations, simple maps may be used to generate other kinds of attractors made clouds of points, which transform an input multifractal into a FM derived measure (Huang et al., 2013). Figure 2 shows how a pattern dy (also adding up to one) is found transforming an input dx through successive iterations of two affine maps (via proportions **66**-34%), whose end points are {(0,0), (**0.81**, **1.74**), (**0.46**, **0.43**), (1,1)}:

$$w_1\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0.81 & 0\\1.00 & \mathbf{0.74}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix},$$

$$w_2\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0.54 & 0\\1.22 & \mathbf{-0.65}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}0.46\\0.43\end{pmatrix},$$

where **bold** face values are, once again FM parameters: the end points of the maps, the vertical scalings in the matrices, and the proportions used in the iterations.

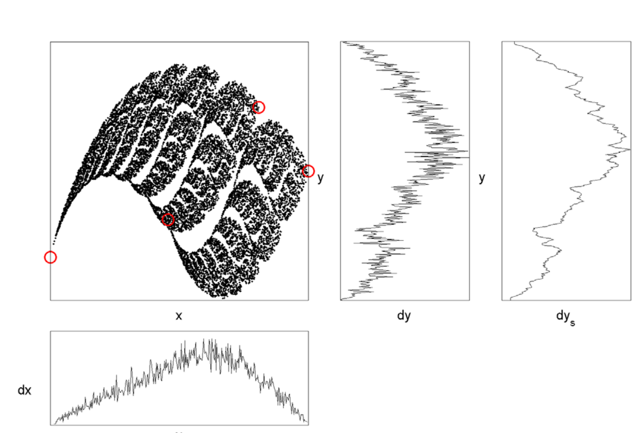

Figure 2. A generalized FM approach simple maps that overlap over x: from an input texture dx, to an output projection dy, via a "leafy" attractor. dy_s is a smoothed version of dy.

FM Encodings of Streamflow and Water Temperature

Streamflow

Water temperature

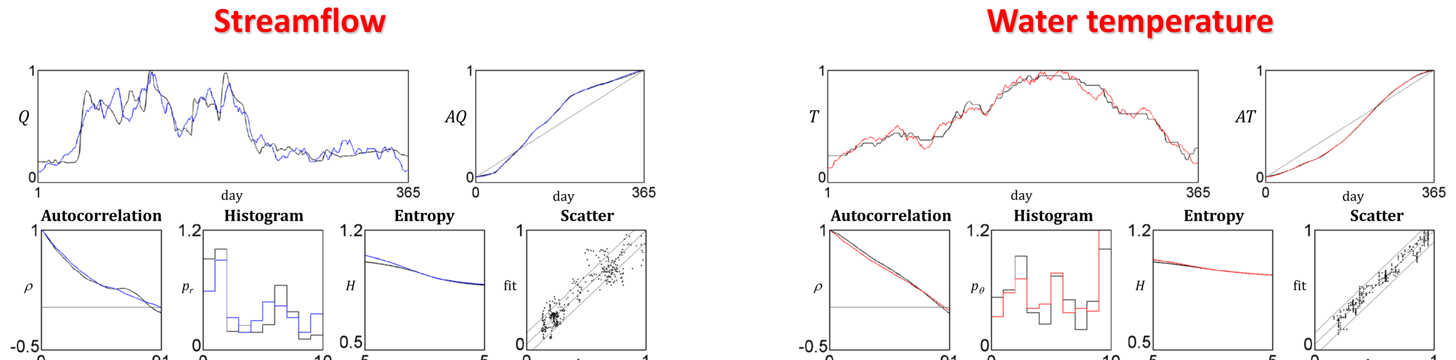

Figure 3. Normalized **streamflow** and **water temperature** sets (black) for Oct 1973-Sept 1974 and Jan 1967-Dec 1967, and FM representations duly smoothed (blue for streamflow, red for temperature), followed by accumulated sets (top) and key statistics of the FM fits (bottom).

FM Encodings (cont.)

As illustrated in Figure 3, streamflow and water temperature sets at the Sacramento River, for 51 years each from 1961-2012, have been successfully encoded via the FM approach, as in Figures 1 and 2 respectively. While Figure 4 (left) shows faithful encodings of streamflow records over the years - with root mean square accumulated errors **RMSEAR** (optimized and in percent) and maximum accumulated errors **MAXEAR** of , in order: (0.8 ± 0.3, 1.7 ± 0.5), Figure 4 (right) shows the goodness of water temperature FM fits having even smaller values of **RMSEAR** and **MAXEAR** of (0.35 ± 0.05, 0.74 ± 0.08).

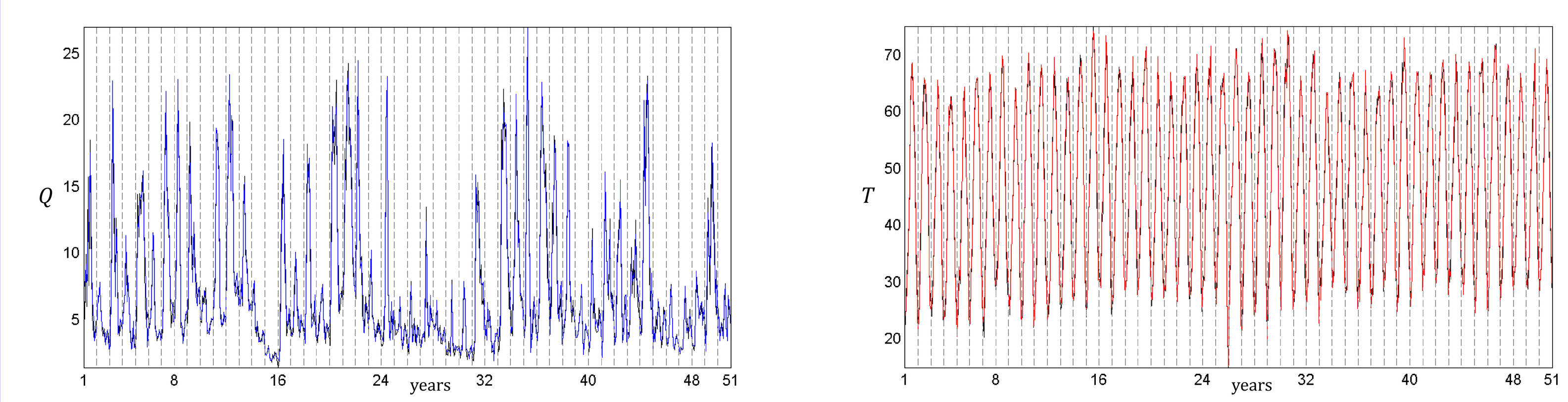

Figure 4. Overall fittings of **streamflow** (left), upgraded with constant base flow, in 100,000 cfs, starting from Sept 1961 - Oct 2012, and **water temperature** (right), with constant minimum temperature, in 10°C for the period Jan 1962 – Dec 2012.

Dynamics of the Two Processes

Figure 5 illustrates the dynamics of two processes in terms of the evolution of FM parameters as function of time.

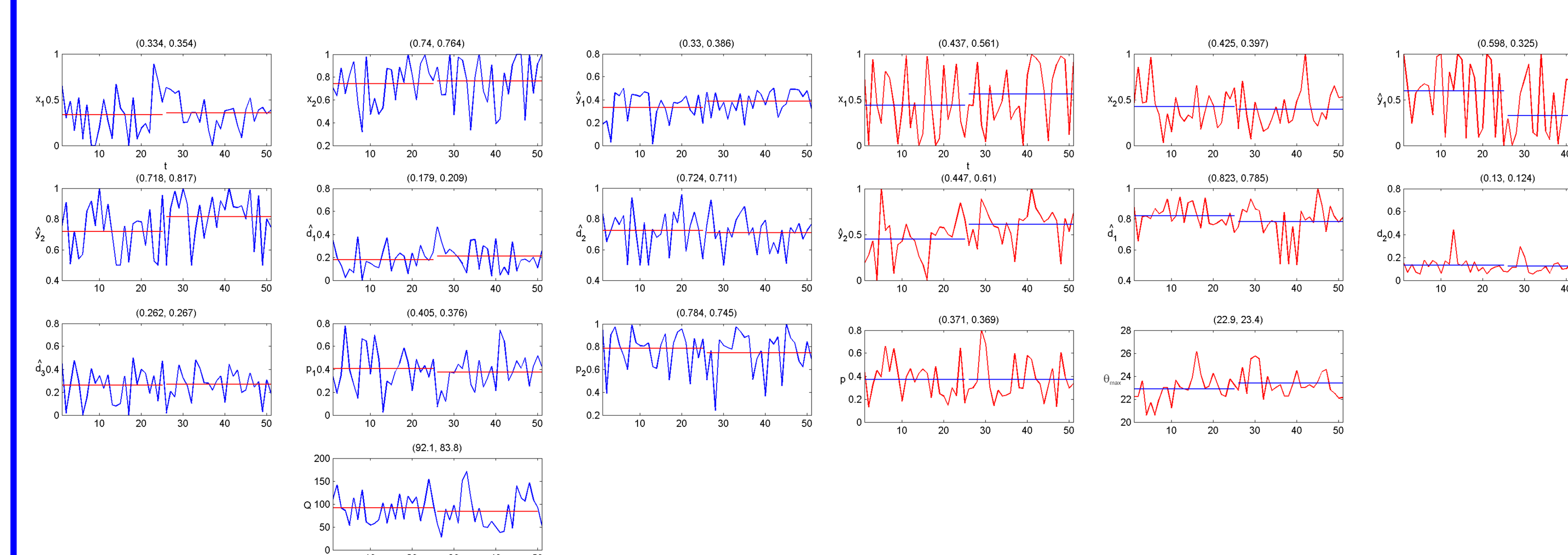

Figure 5. Evolution of parameters for **streamflow** (left) and **water temperature** (right). Values on top of each frame correspond to average parameter values over half of the sets, and are plotted in red or blue. The last blocks include the total flow volume and the average yearly water temperature.

Geometric Classification

The geometries of the two records available are classified based on k-means clustering of FM parameters aiming at the investigation of the dynamics of the processes. Employing such classifications, Markovian matrices for both sets may be established so that regimes of each processes may holistically be elucidated. Figure 6 shows the FM centroids for ten classes each for both processes, Figure 7 portrays the time evolution by classes of the two processes and the obtained Markov matrices are included in Figure 8.

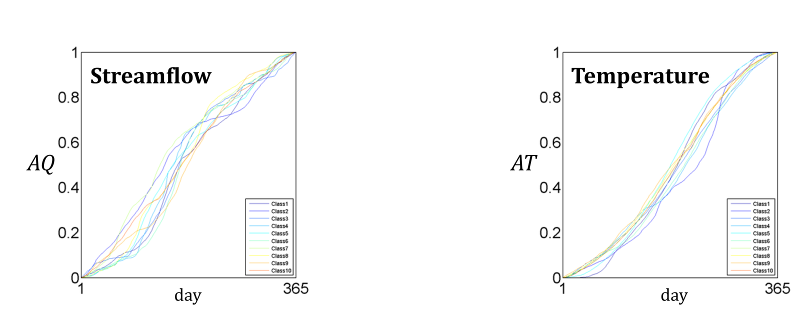

Figure 6. Centroids of FM classification of accumulated **streamflow** and **temperature** for the period considered.

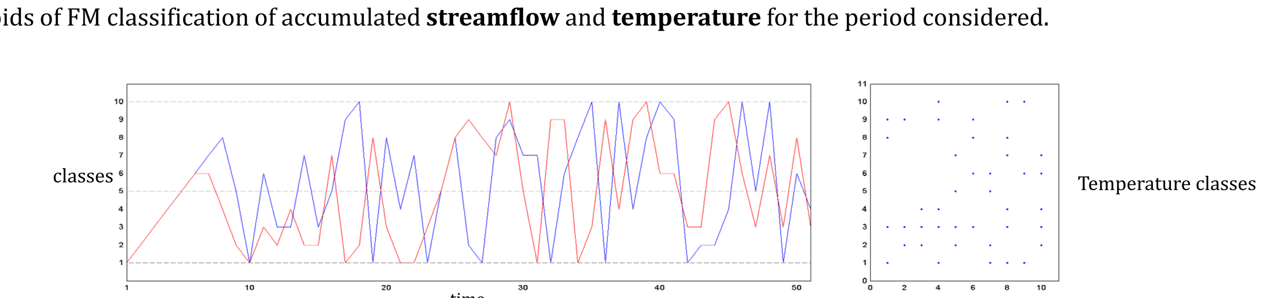

Figure 7. FM class evolutions for **streamflow** (blue) and **temperature** (red) over the period, followed by correlation plot between two attributes.

Geometric Classification (cont.)

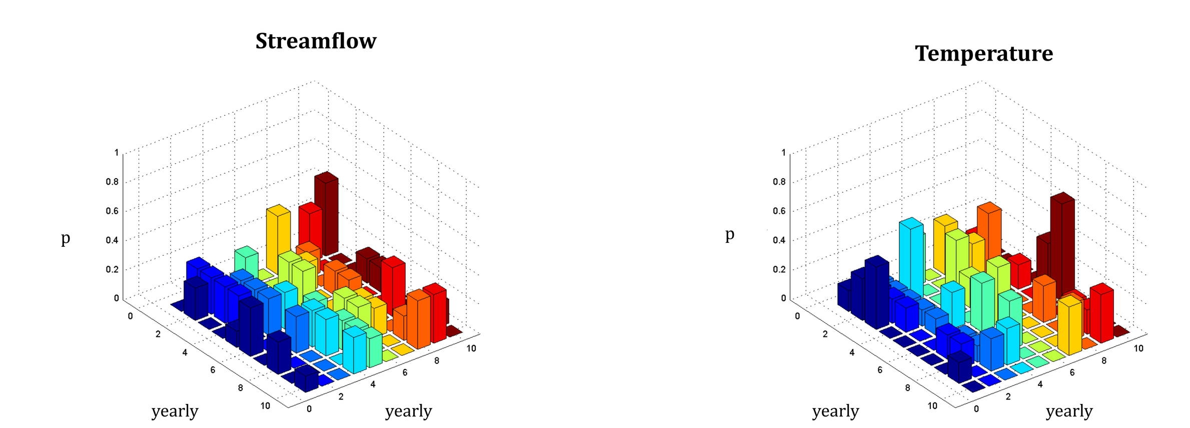

Figure 8. Markov matrices based on Figure 7: streamflow (left) and water temperature (right).

Discussion and Conclusions

Excellent FM encodings of both sets of records have been found via distinct variants of the FM approach. This has led to noticeable parameter variations for both processes as a function of times, in a manner that, parameter by parameter, exhibits little correlation. Despite such intrinsic changes year after year, suitable classifications of accumulated streamflow and accumulated water temperature have been found, with the temperature ones showing (as expected) less variation and hence more tightly bunched accumulated profiles. Although the classification yields an evolution in which both streamflow and temperature match for a few years, after a while both go up and down on their respective classes in a manner that appears to be unpredictable. Notwithstanding the fact that the streamflow records start in October and the temperature ones in January, the classification of parameters per year exhibit also a lack of correlation between the two processes.

As illustrated in Figure 9 partitioning the sets in two groups of about 25 years, the analysis represents the varying histograms of both records very well. Such a fact and comparisons of records at distinct locations may be useful to elucidate climate change impacts.

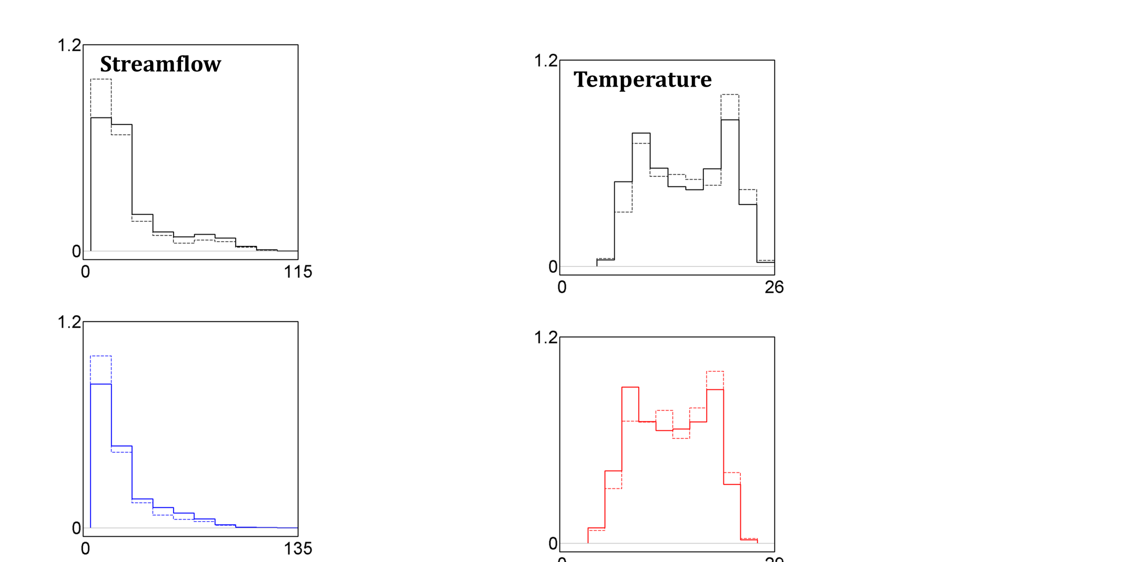

Figure 9. Histograms for both processes. Solid lines correspond to first 25 years and dashed to last 26 years. Top frames are observed records and bottom ones FM fits.

Ideas for Future Work

As local smoothing of the actual data does not affect the accumulated records greatly, we intend to study if FM encodings of increasing moving averages may result in more consistent encodings for the Sacramento River throughout the years and if such would lead to improved classifications of the records. As the notions lend themselves to comparisons between sites, other catchments within the Western United States and elsewhere shall be considered, aiming at finding plausible relations between all transition matrices obtained. As done earlier, it is envisioned that the analysis would also be carried at decadal scales in order to aim at predictions of records, which may reflect global climate change trends.

References

Barnsley MF (1988) *Fractals Everywhere*, Academic Press, San Diego, California.
Huang HH, CE Puente, A Cortis & JL Fernández-Martínez (2013) An effective inversion strategy for fractal–multifractal encoding of a storm in Boston, *Journal of Hydrology*, 496, 205-216.
Maskey ML, CE Puente, B Sivakumar & A Cortis (2015) Encoding daily rainfall records via adaptations of the fractal multifractal method, *Stochastic Environmental Research and Risk Assessment* DOI 10.1007/s00477-012-0617-6
Obregón N, CE Puente & B Sivakumar (2002) Modeling high resolution rain rates via a deterministic approach, *Fractals,* 10(3), 387–394.
Puente CE (1996) A new approach to hydrologic modeling: derived distributions revisited, *Journal of Hydrology*, 187, 65-80.
Puente CE, ML Maskey, B Sivakumar & A Cortis (2015) Predicting streamflow from fractal geometric encodings of yearly and decadal records. American Geophysics Union, Fall Meeting.