DETERMINISTIC GEOMETRIC MODELING
OF PRECIPITATION PATTERNS

Carlos E. Puente
Department of Land, Air and Water Resources
University of California, Davis

Andrea Cortis
Earth Sciences Division
Lawrence Berkeley National Laboratory
A Platonic approach to complexity

\[ w_1(x, y) = (x/2, x + z \cdot y), \quad w_2(x, y) = (x/2 + 1/2, 1 - x - z \cdot y) \]

iteration of simple maps produces a “wire” from \( x \) to \( y \)
shadows yield complex sets, \( dy \), based on a multifractal, \( dx \)
patterns appear to be random but they are not...
Some sample shadows
(Puente, 2004)

varying the height of a middle point by which a wire passes sets have autocorrelations and spectra as found in natural data
sets with diverse shapes and statistics may be found
all sets are fully characterized via few “geometric” parameters

More sample shadows
(Puente, 2004)
Yet more Platonic designs
(Cortis et al., 2008)

adding a nonlinear cosine perturbation on y component...
iterating 4 maps maintaining data’s moments and multifractality geometric model also preserves spectra and chaotic nature of data
Real and simulated rainfall
(Puente and Cortis, 2009)

above: real data at La Honda, California
below: simulations via Cantorian $dx$’s
Extensions to two dimensions: wires from 1D to 2D

(Puente, 1994)

Mappings:

\[
w_n \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_n & 0 & 0 \\ c_n r_n^{(1)} \cos \theta_n & -r_n^{(2)} \sin \theta_n & 0 \\ k_n r_n^{(1)} \sin \theta_n & r_n^{(2)} \cos \theta_n & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} e_n \\ f_n \\ g_n \end{pmatrix}
\]

more parameters, but similar...
Deterministic precipitation patterns in space
(Puente, 2004)

from single wires obtained iterating two linear maps
Pollution dynamics and predictions
(Puente et al., 2001)

Parameters:

Predictions:

the geometry of pollution may be captured via successive wires such allowed computing reasonable predictions from trends...
Simulations over two dimensions

From a **3D wire**: scalings, $0.75 \pm 0.05$, $-0.9 \pm 0.08$ angles, set $\pm 20$

From a **4D surface**: scalings, $\approx 0.59, 0.54 \pm 0.03$ angles, set $\pm 5$
variable heights, weights
Inverse problem via modified particle swarm  
(After Juan Luis Fernández Martínez)

<table>
<thead>
<tr>
<th>Set</th>
<th>Model</th>
<th>$ob \leq 0.01$</th>
<th>$0.01 &lt; ob \leq 0.02$</th>
<th>$0.02 &lt; ob \leq 0.03$</th>
<th>$0.03 &lt; ob \leq 0.04$</th>
<th>$ob &gt; 0.04$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston</td>
<td>3p</td>
<td>5</td>
<td>52</td>
<td>29</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4p</td>
<td>10</td>
<td>85</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>5p</td>
<td>13</td>
<td>63</td>
<td>9</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>Data 1</td>
<td>3p</td>
<td>16</td>
<td>21</td>
<td>54</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4p</td>
<td>10</td>
<td>71</td>
<td>13</td>
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<td>1</td>
</tr>
<tr>
<td></td>
<td>5p</td>
<td>14</td>
<td>76</td>
<td>7</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

From 100 random initial conditions

Data 1 is synthetic via a 3p projection
A storm in Boston revisited
Cumulative distributions

$3p, 5 \text{ par, } 0.0096$


$3p, 5 \text{ par, } 0.0194$
A storm in Boston revisited

Data sets

3p, 5 par, 0.0096

3p, 5 par, 0.0194
A storm in Boston revisited

Scatter

3p, 5 par, 0.0096

3p, 5 par, 0.0194
A storm in Boston revisited
Cumulative distributions

4p, 8 par, 0.0078

4p, 8 par, 0.0082

4p, 8 par, 0.0087

4p, 8 par, 0.0212
A storm in Boston revisited

Data sets

4p, 8 par, 0.0078

4p, 8 par, 0.0082

4p, 8 par, 0.0087

4p, 8 par, 0.0212
A storm in Boston revisited

Scatter

4p, 8 par, 0.0078

4p, 8 par, 0.0082

4p, 8 par, 0.0087

4p, 8 par, 0.0212
A storm in Boston revisited
Cumulative distributions

**5p, 11 par, 0.0063**

**5p, 11 par, 0.0077**

**5p, 11 par, 0.0080**

**5p, 11 par, 0.0218**
A storm in Boston revisited

Data sets

5p, 11 par, 0.0063

5p, 11 par, 0.0077

5p, 11 par, 0.0080

5p, 11 par, 0.0218
A storm in Boston revisited

Scatter

5p, 11 par, 0.0063

5p, 11 par, 0.0077

5p, 11 par, 0.0080

5p, 11 par, 0.0218
A Synthetic data set
Cumulative distributions

3p, 5 par, 0.0012

3p, 5 par, 0.0202
A synthetic data set

Data sets

3p, 5 par, 0.0012

3p, 5 par, 0.0202
A synthetic data set

Scatter

3p, 5 par, 0.0012

3p, 5 par, 0.0202
A synthetic data set
Cumulative Distributions

4p, 8 par, 0.0053

4p, 8 par, 0.0067

4p, 8 par, 0.0079

4p, 8 par, 0.0209
A synthetic data set

Data sets

4p, 8 par, 0.0053

4p, 8 par, 0.0067

4p, 8 par, 0.0079

4p, 8 par, 0.009
A synthetic data set

Scatter

4p, 8 par, 0.0053

4p, 8 par, 0.0067

4p, 8 par, 0.0079

4p, 8 par, 0.0209
A synthetic data set
Cumulative Distributions

5p, 11 par, 0.0062

5p, 11 par, 0.0073

5p, 11 par, 0.0087

5p, 11 par, 0.0205
A synthetic data set

Data sets

\textbf{5p, 11 par, 0.0062} 

\textbf{5p, 11 par, 0.0073} 

\textbf{5p, 11 par, 0.0087} 

\textbf{5p, 11 par, 0.0205}
A synthetic data set

Scatter

5p, 11 par, 0.0062

5p, 11 par, 0.0073

5p, 11 par, 0.0087

5p, 11 par, 0.0205
Conclusions

• The Platonic ideas may be used to simulate a host of rainfall patterns, both in time and in space.

• These geometric notions may ultimately provide a parsimonious deterministic “language” of natural complexity.

• The inverse problem for a given set is, for us, difficult to solve.

• But if the inverse problem is solved, the ideas may perhaps provide new vistas to understand rainfall dynamics.