

FROM THE SCIENCE OF COMPLEXITY TO CHRISTIAN FAITH

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Thesis

We humans, with the gift of a soul,
may learn from recent advancements
regarding natural complexity
to deepen our faith in Jesus Christ...

Outline

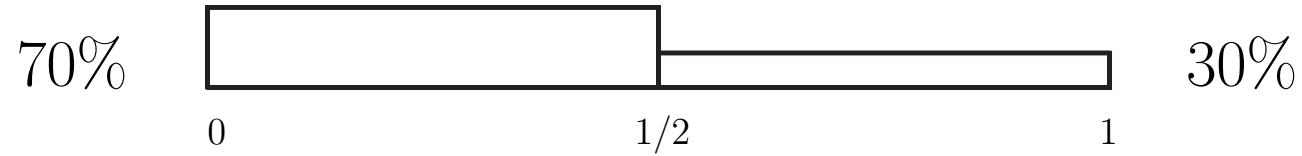
Lessons from turbulence

Lessons from chaos

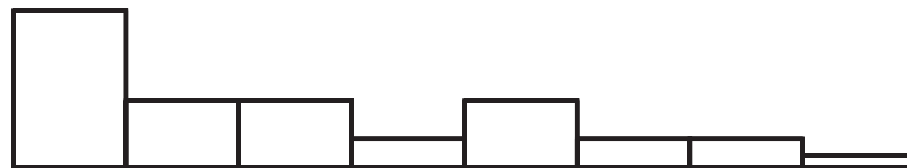
Lessons from normality

Lessons from turbulence

A game for kids

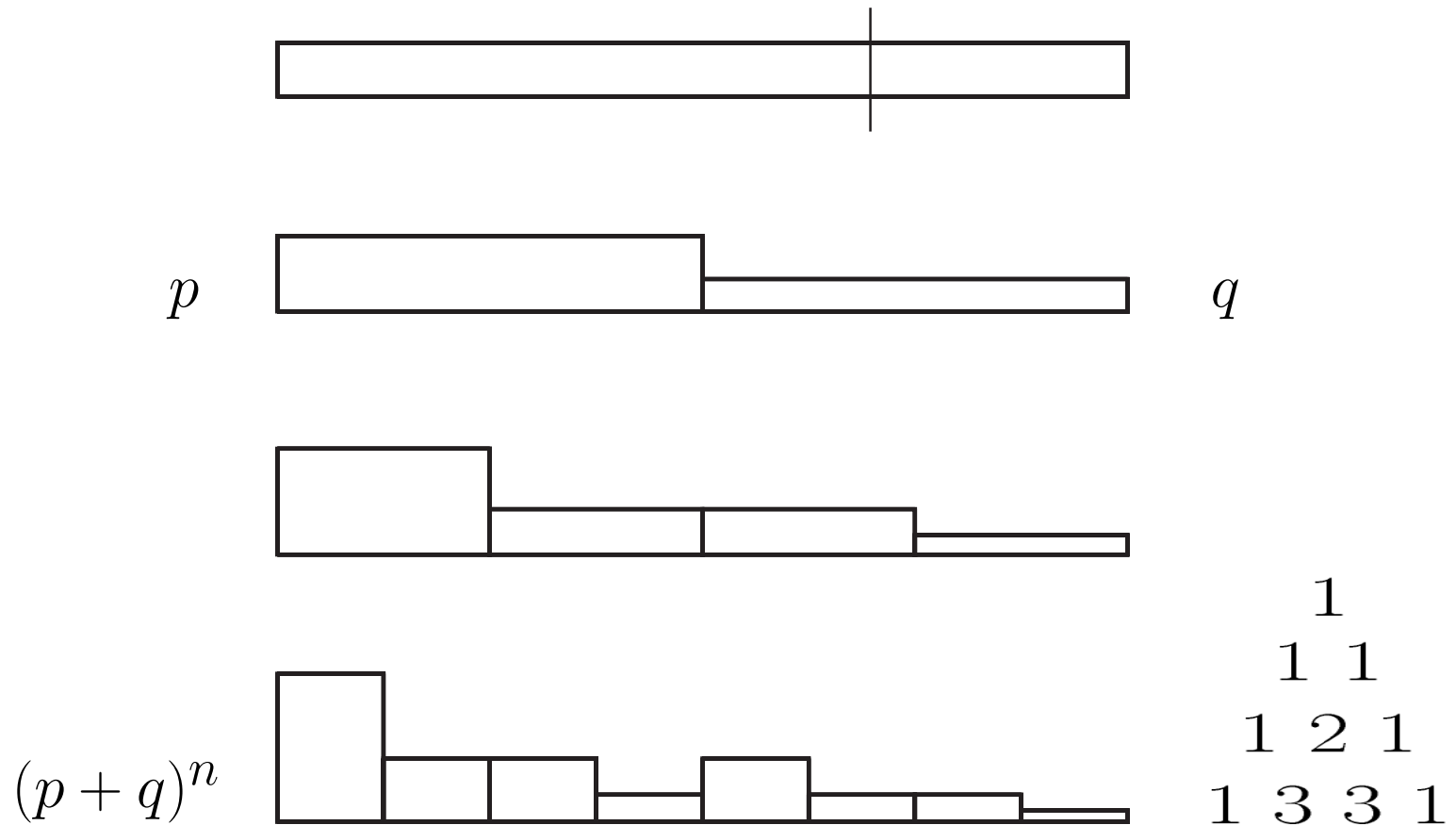


$$h = (0.7 \cdot 2)^3$$



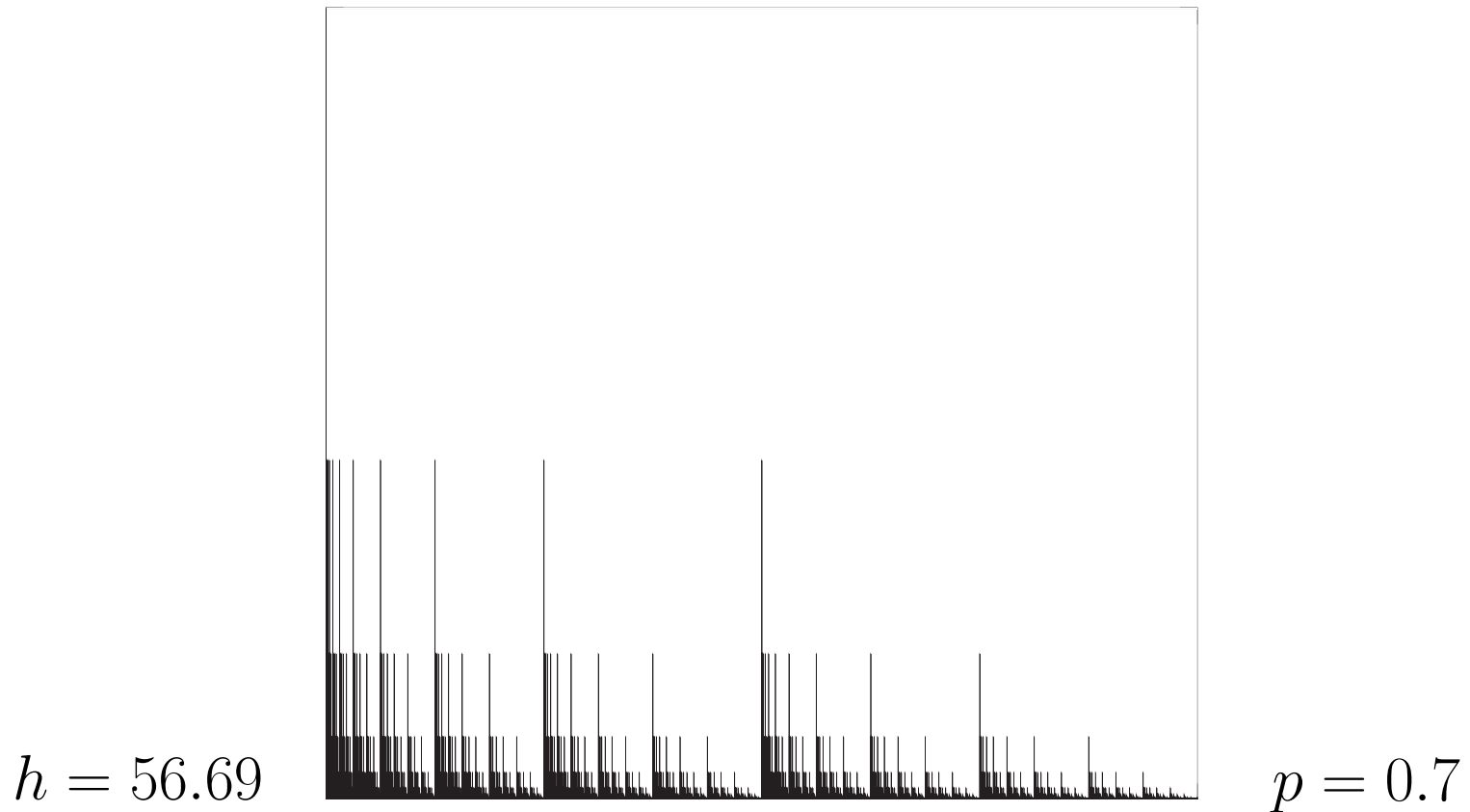
$\vdots$

A multiplicative cascade



...binomial expansion and Pascal's triangle

After $n = 12$ levels

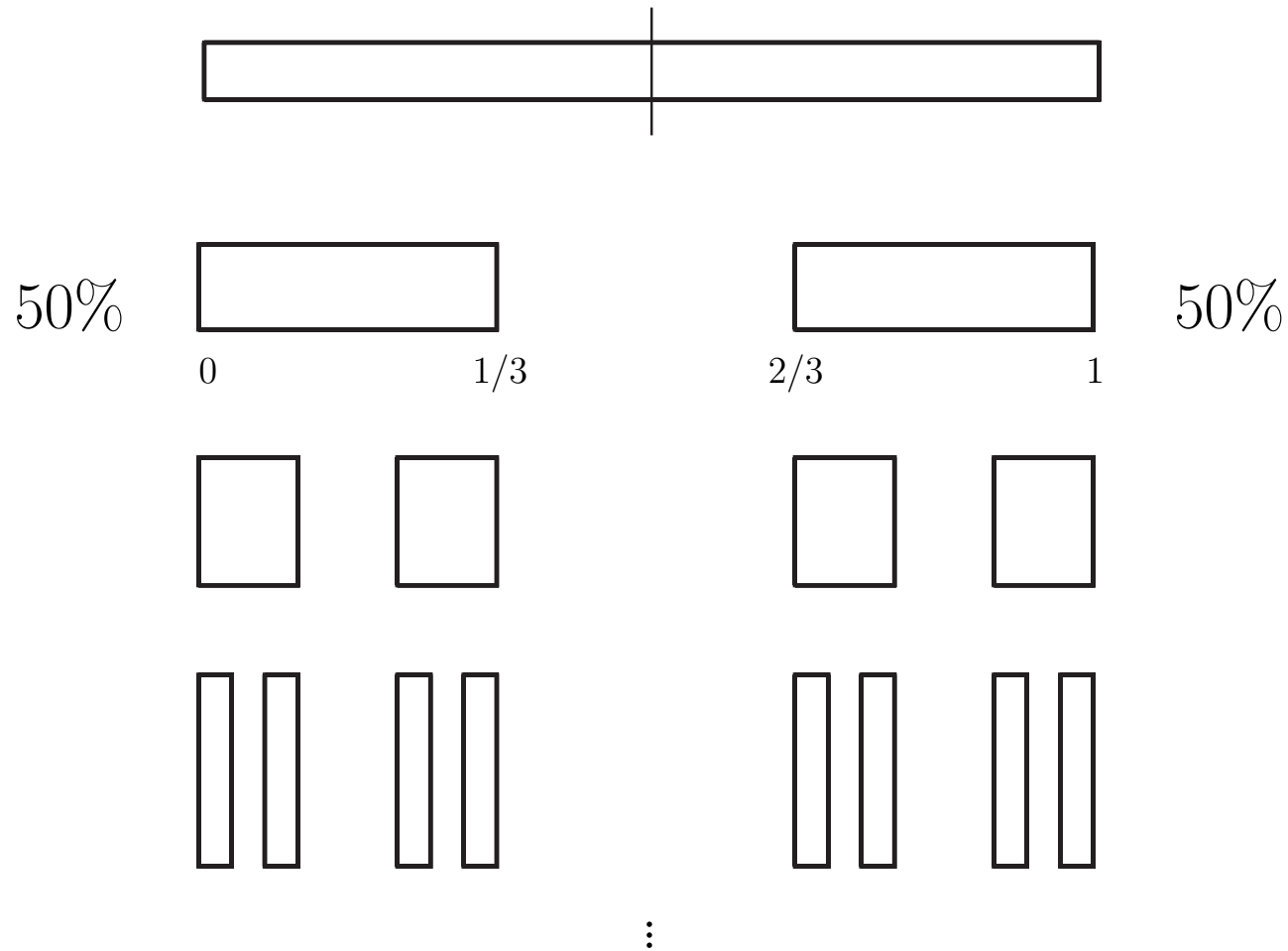


a **multifractal** measure:

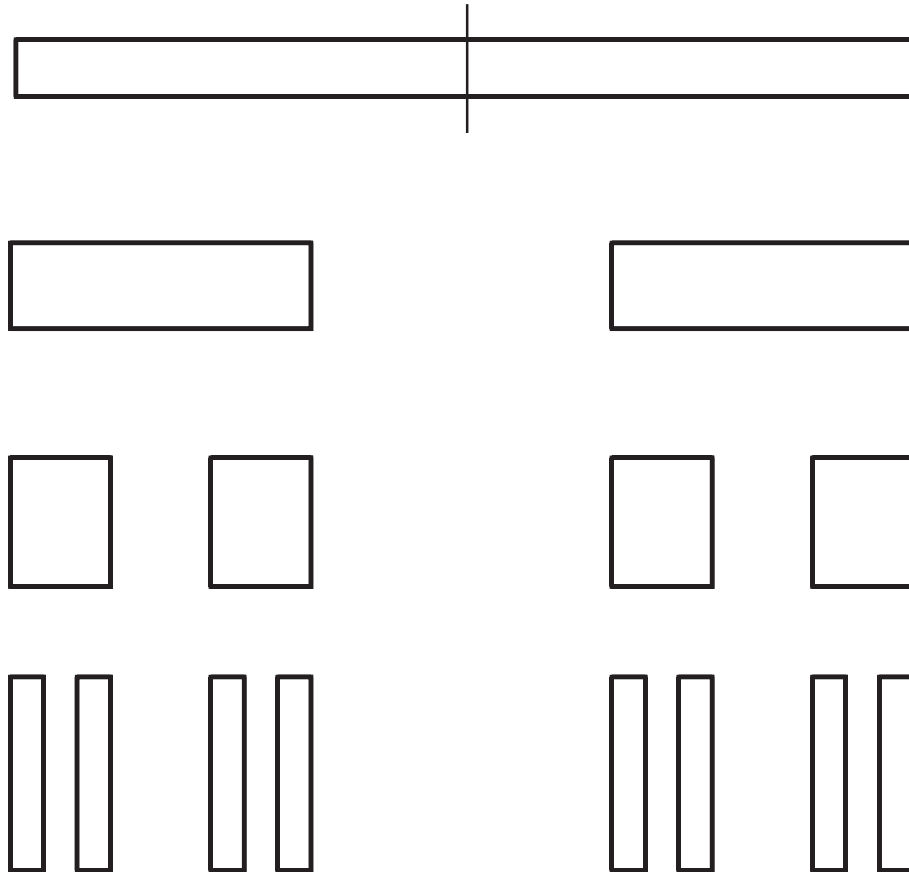
intertwined layers of **thorns** having distinct densities

the support of each layer is fractal **dust**

Another game for kids



Another divisive cascade

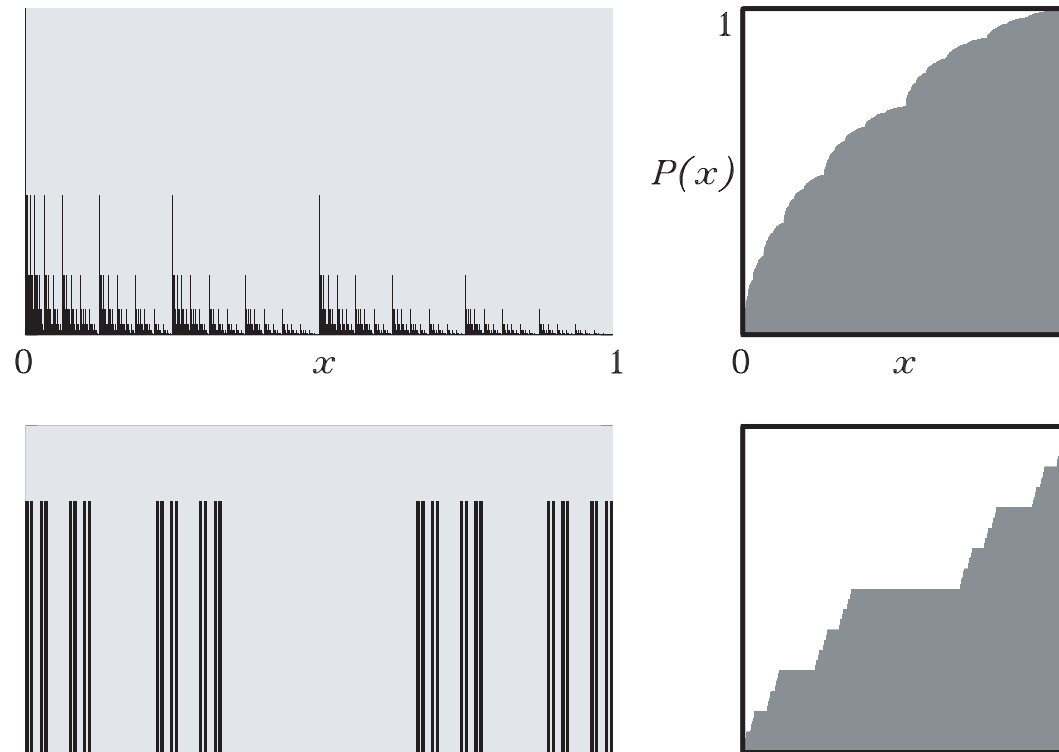


disjoint equal **thorns** over **dust**

reproduces the layers of first cascade by varying the hole size:

the two cascades are intimately **related** to one another

Cumulative modeling clay

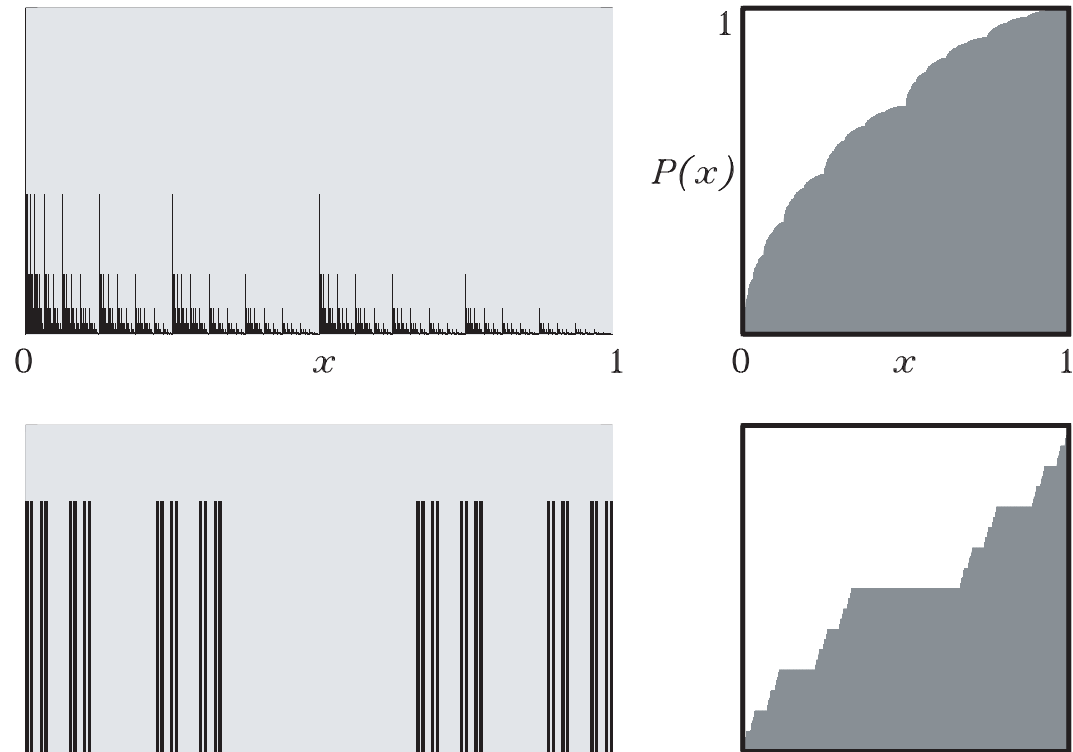


based on the dynamics of the games:

notches above, $P(1/2) = 0.7$, $P(1/4) = 0.49$, etc.

plateaus below, $P(1/3) = P(2/3) = 1/2$, etc.

Devil's staircases

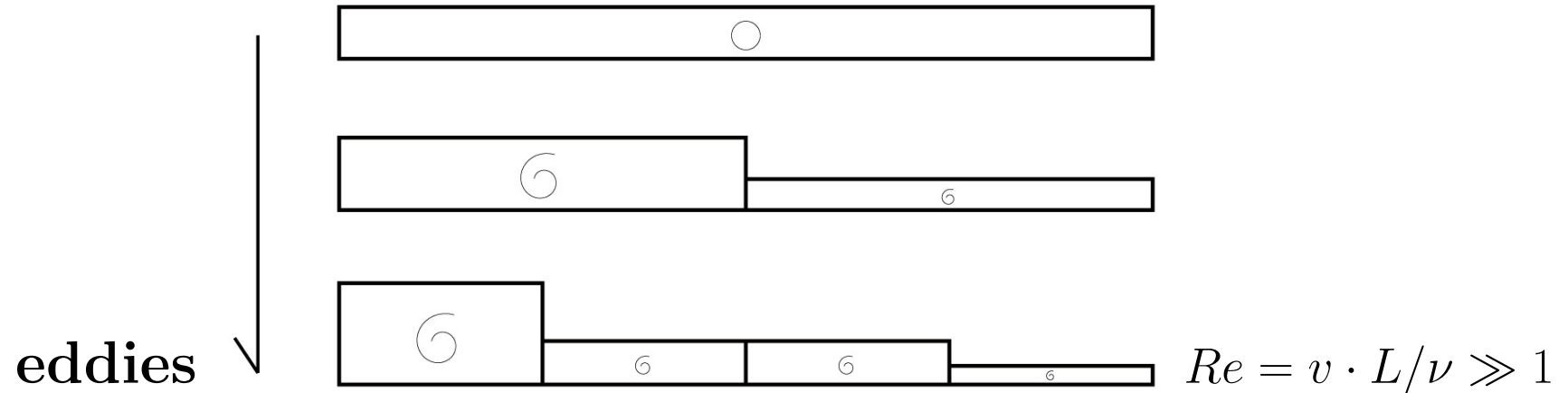


the profiles are **locally flat** and have no tangents
at the end, they always have **maximum lengths**:

$$d\{(0, 0), (1, 1)\} = 2, \quad p \neq 1/2, \quad h \neq 0$$

Fully developed turbulence

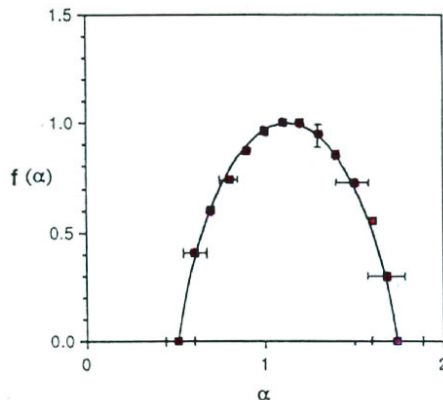
(Meneveau and Sreenivasan, 1987)



layers in one-dimensional turbulence as in first cascade

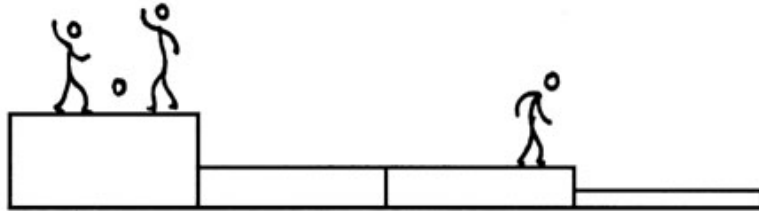
dissipation: atmospheric, boundary layer, wake of a cylinder...

$$p = 0.7$$



universal

Our turbulent times



competition

inequalities

cynicism



discrimination

forced equality

distrust

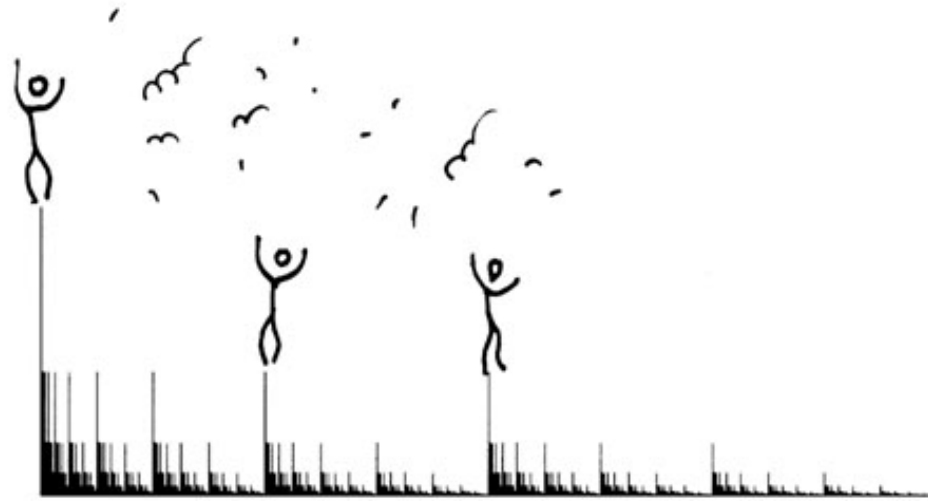
selfish postures and actions

2/3 of the world live under **poverty**

6,000 kids **die** a day for lack of **water**

violence and **terror...**

An optimal system?

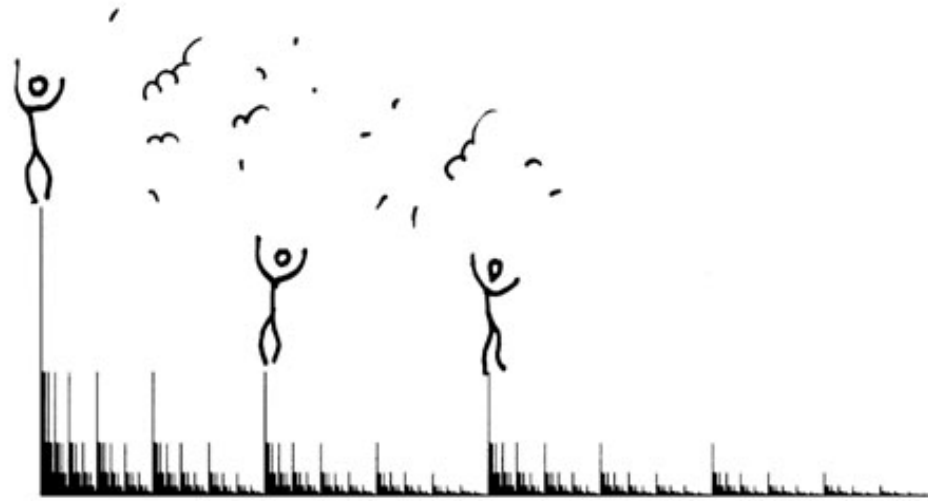


$$p = 0.7$$

$$n = 20$$

5, 10, 20 and 40% largest thorns have
57, 70, 84 and 95% of the mass

An optimal system?



$$p = 0.7$$

$$n = 20$$

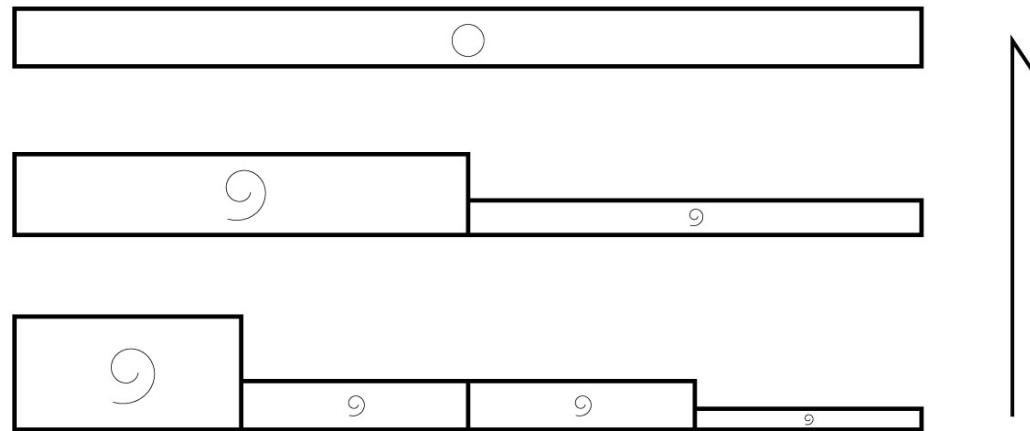
5, 10, 20 and 40% largest thorns have
57, 70, 84 and 95% of the mass

this is quite close to **USA**: 59, 71, 84 and 95%

Moral: as both cascades **dissipate** the energy,
if we ride them, they lead us to “bite the dust”

Common sense solution

reverse the direction of the cascade to **mend** what is broken:



live at low Reynolds numbers in order to avoid **violence**

“cut mountains and fill valleys” to restore **unity**:

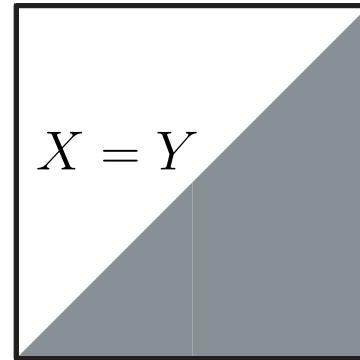
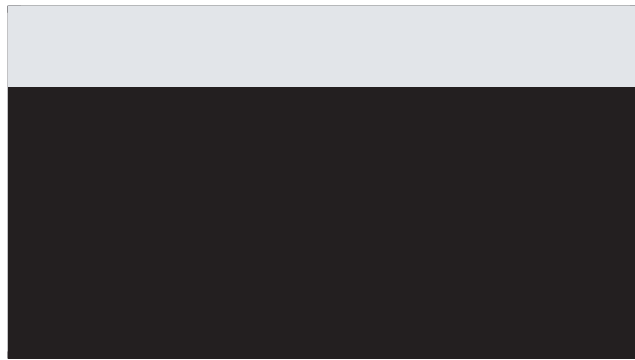
$$1 = 0.999\dots$$

The only possible solution

based on simple **geometry**:

$(p + q)^0$

50-50



$d = \sqrt{2}$

No holes

a **straight** and **solid** condition without **thorns** and **dust**

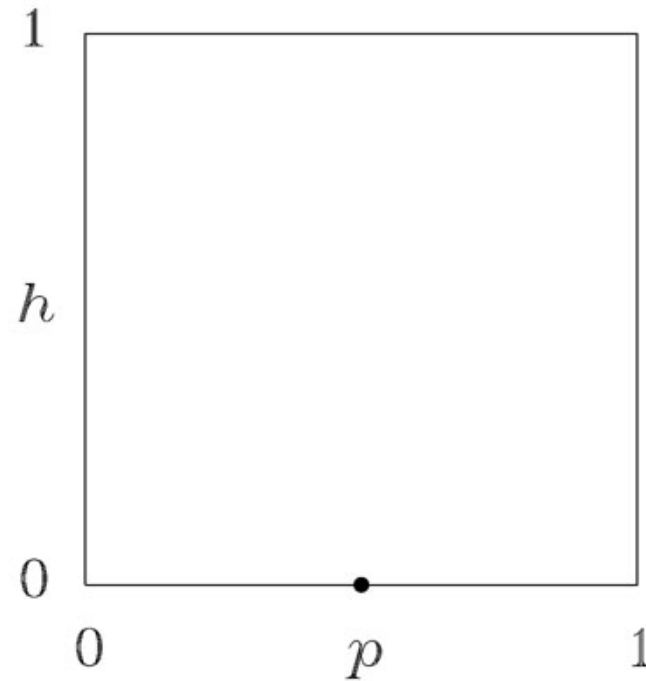
the cumulative mass is the **one** to **one** line

the **hypotenuse** is the pathway of **peace**

division travels by the **legs**

Moral: $X = Y$ represents and provides the **root** of **love**

Equilibrium is an improbable point



it is **hypocritical** to judge if we are “far away” from the **point**
a convex surface is found for a **finite** number of levels:

balance is found easily aided by **gravity**

in truth there is **darkness** between **6** and **9**...

Our options

equilibrium

turbulence

calmness

violence

rectitude

wickedness

reconciliation

separation

integration, /

division, \$

unity

dust

$1 = 0.999 \dots$

$2/3 = 0.666 \dots$

completeness

emptiness

life

dissipation

PATHWAYS

Two options before us
two pathways ahead,
the one is the longest
the other straight.

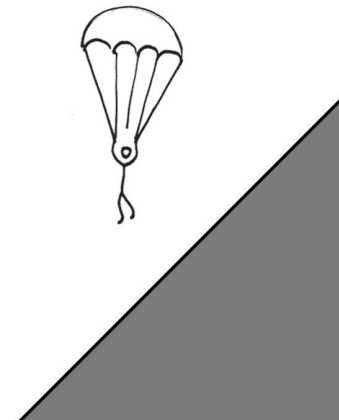
We journey directly
or go by the legs,
we follow intently
or end up in pain.

By walking the flatness
or hiking the spikes,
we travel in lightness
or take serious frights.



The incentive is unity
and the call proportion,
the key is forgiveness
and the goal true notion.

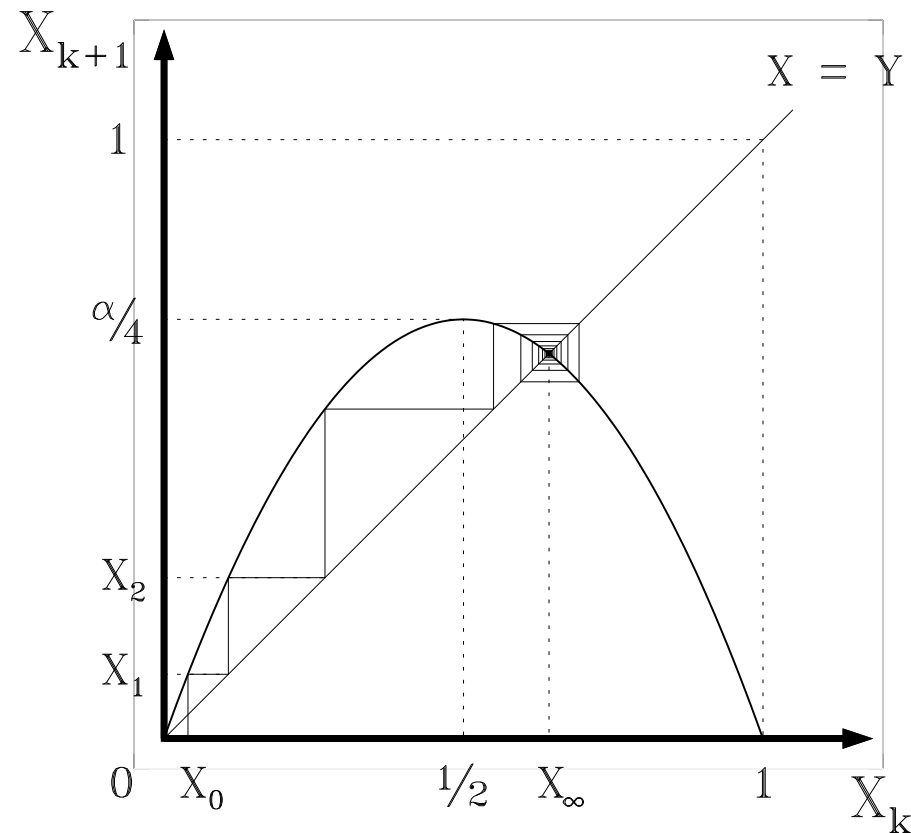
In wandering wickedness
there is never fruit,
but in ample humbleness
one encounters the root.



Lessons from chaos

The logistic parabola

$$X_{k+1} = \alpha X_k(1 - X_k), \alpha \in [0, 4]$$

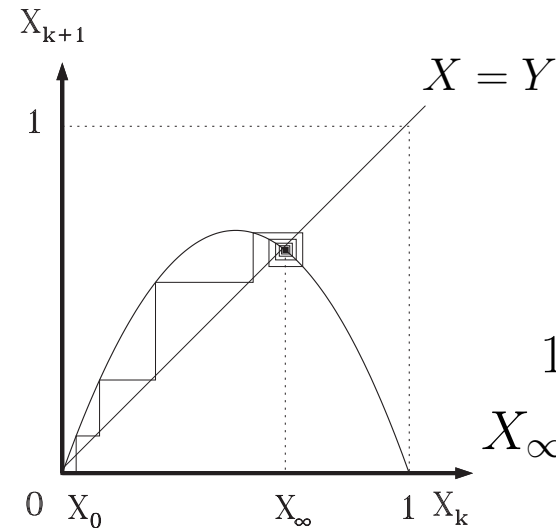
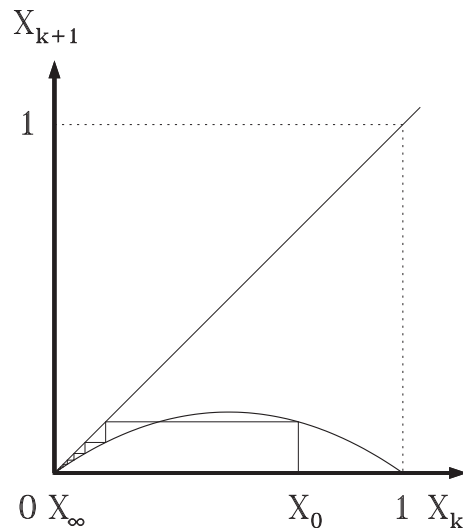


...a population of “rabbits” as a function of time
different cases depending on the parameter α :

From the origin to bifurcations

$$0 < \alpha \leq 1$$

$$X_{\infty} = 0$$

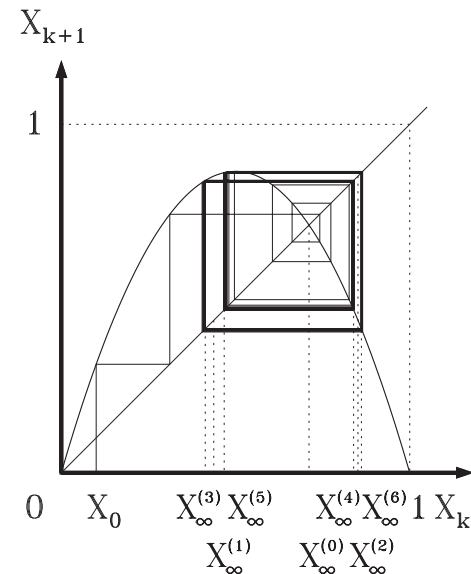
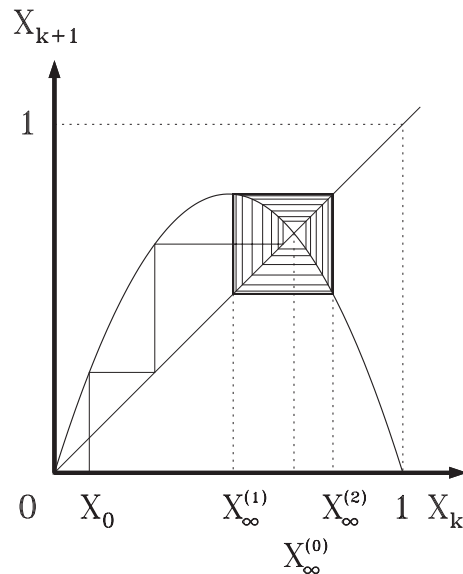


$$1 < \alpha \leq 3$$

$$X_{\infty} = (\alpha - 1)/\alpha$$

$$\alpha = 3.2$$

$$\text{period } 2$$



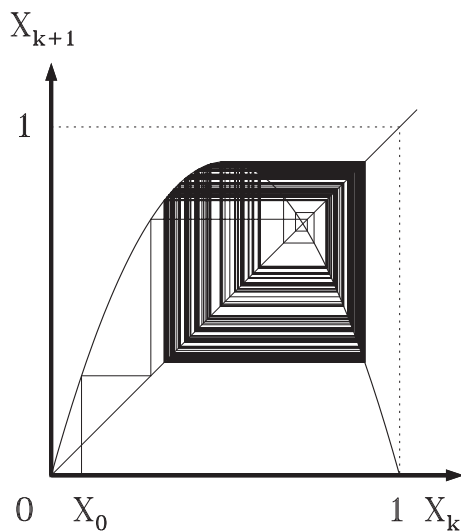
$$\alpha = 3.46$$

$$\text{period } 4$$

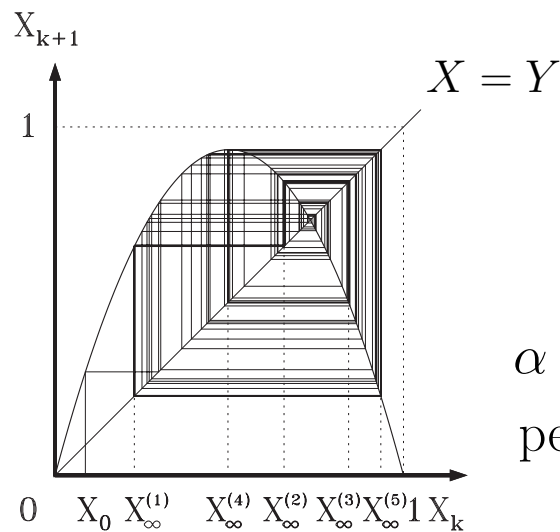
Periodicities and chaos intertwined

$$(\alpha > \alpha_\infty \approx 3.5699)$$

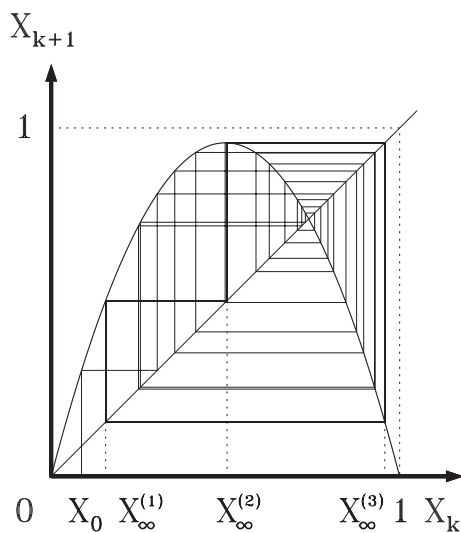
$\alpha = 3.6$
aperiodic



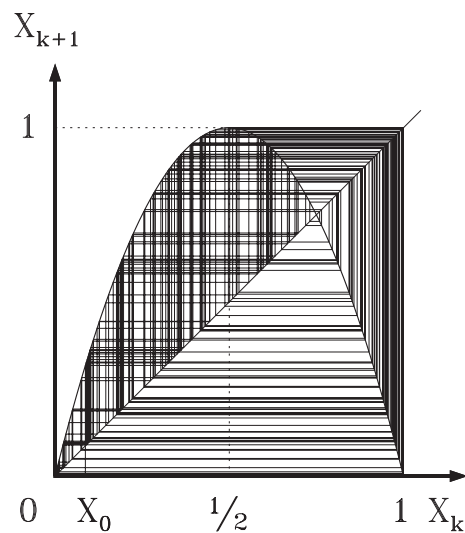
$\alpha = 3.74$
period 5



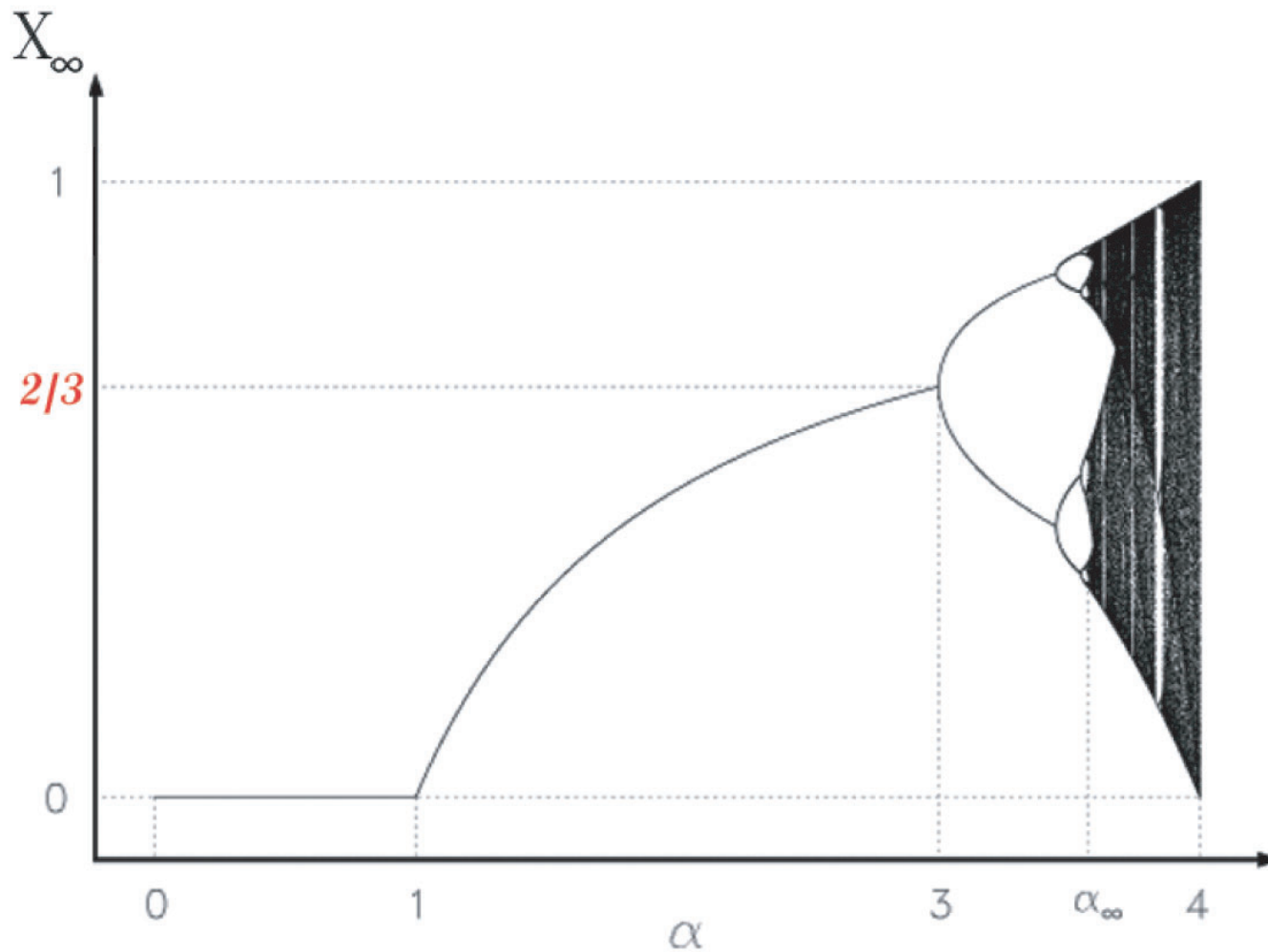
$\alpha = 3.83$
period 3



$\alpha = 4$
aperiodic

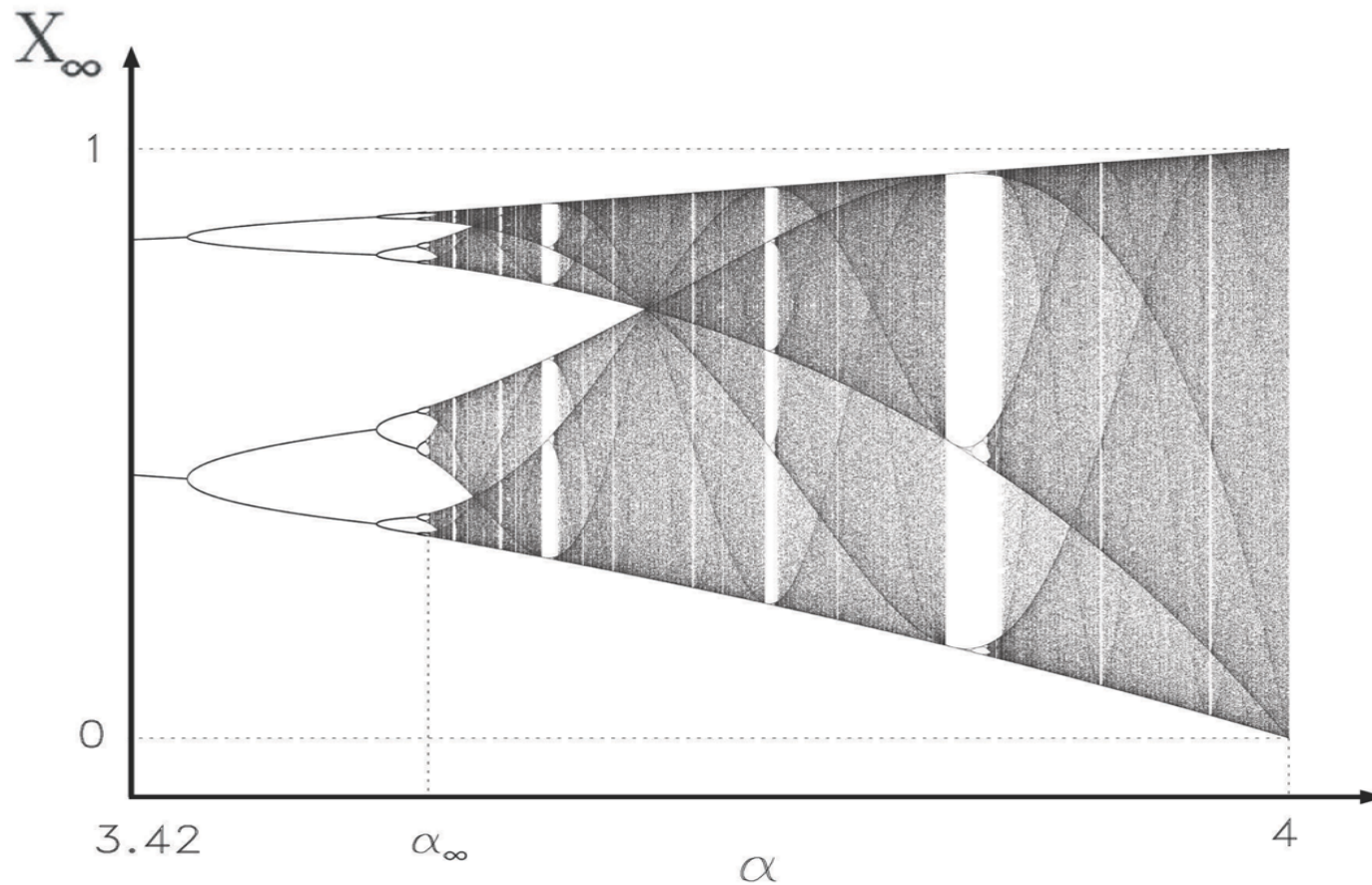


The diagram of bifurcations



the **tree** contains periodic behavior, for any natural number

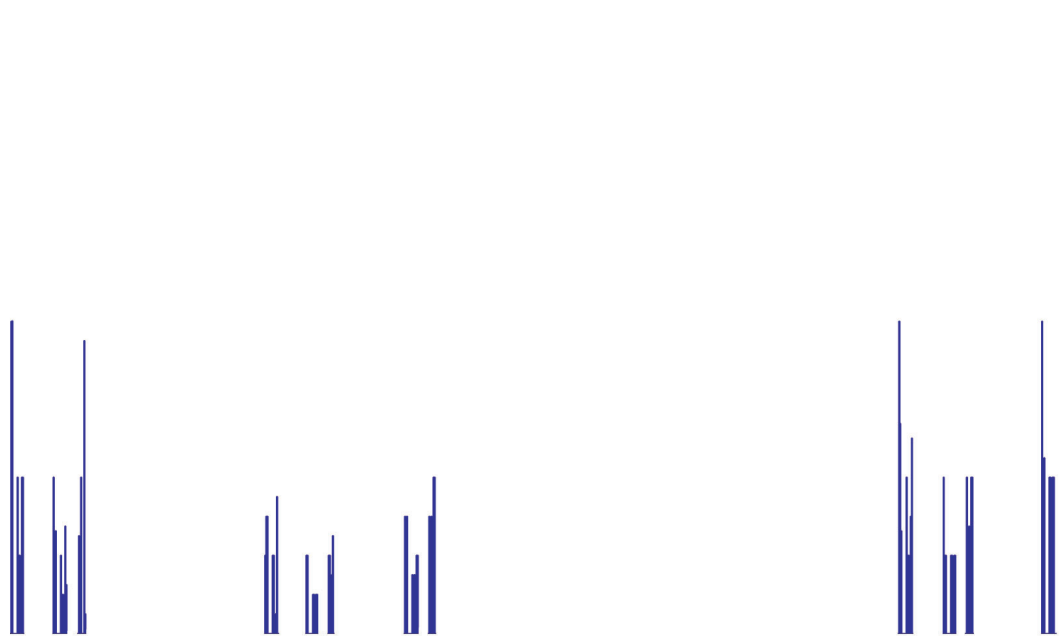
Dust galore and self-similarity



after α_∞ the **dusty** strange attractors are common
in the periodic “**buds**” there are small copies of the tree

Thorns galore

the chaotic tree contains many **multifractal** measures
they combine imbalances and holes, as in α_∞ :

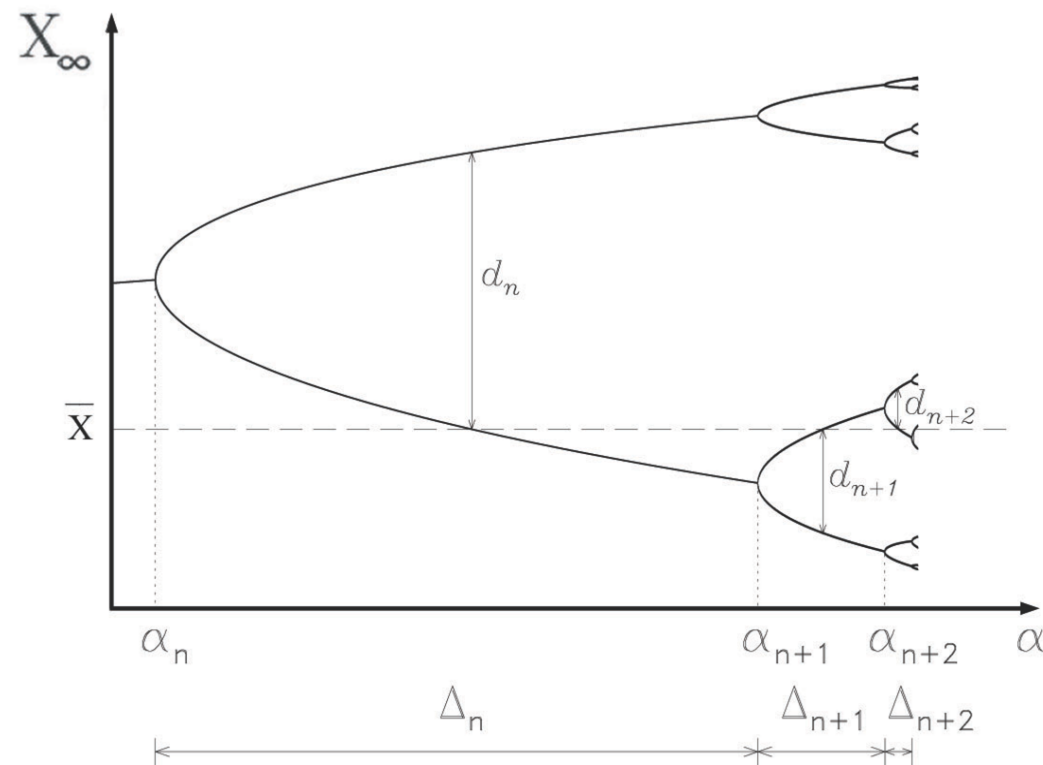


they happen at accumulation points of buds for all periods

Universality on the fig tree

(Mitchell Feigenbaum, 1978)

bifurcations happen in an **orderly** fashion:



$$d_n/d_{n+1} \longrightarrow \mathcal{F}_1 = -2.50 \dots$$

openings

$$\Delta_n/\Delta_{n+1} \longrightarrow \mathcal{F}_2 = 4.669 \dots$$

durations

Universality on other chaotic trees

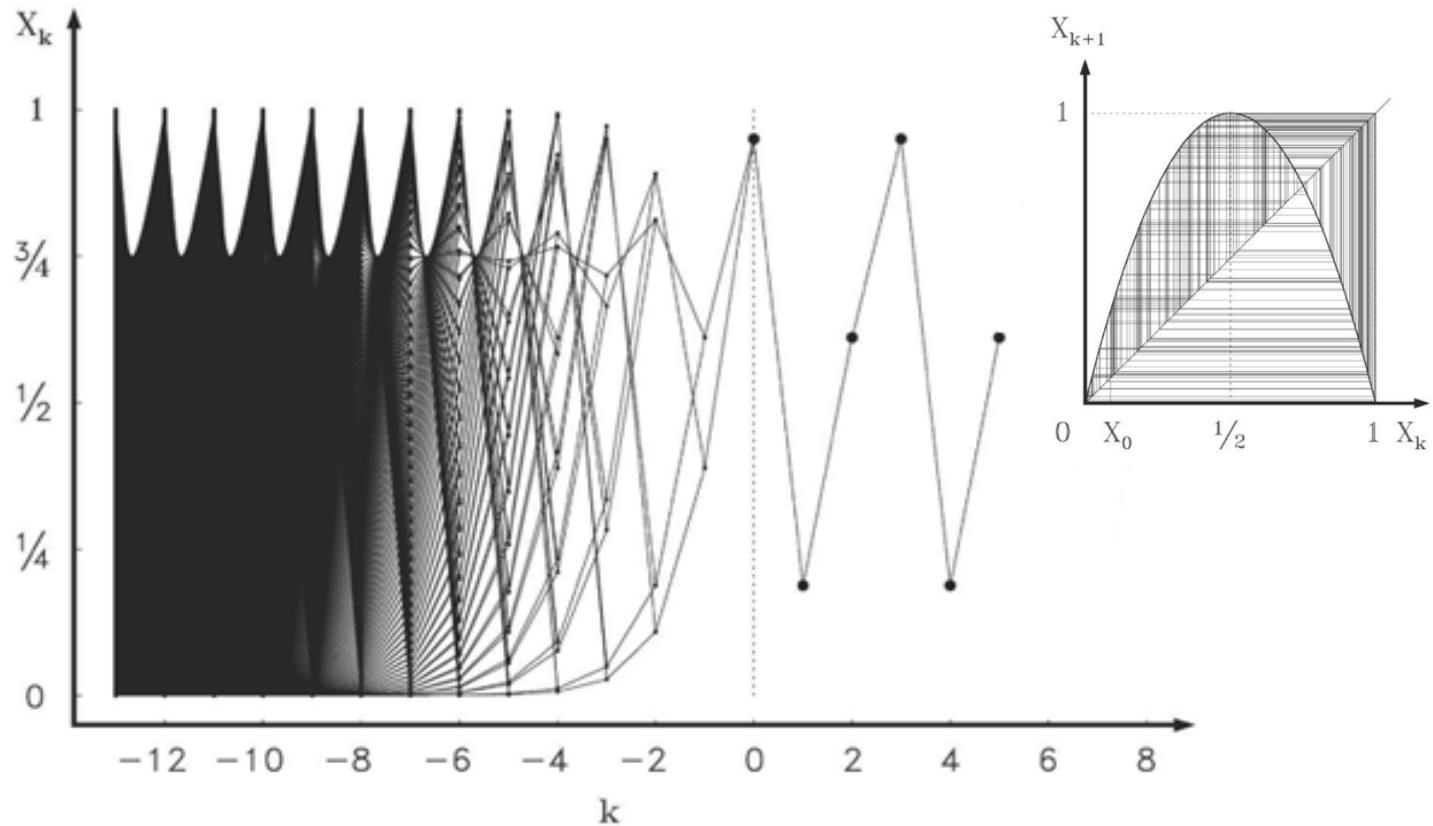
straight root, “tender branch,” branches, and dust (“fig leaves”)



relevant in Ecology, Chemistry, Physics, Economics...
the dynamics of **convection** are reproduced if α is **heat**

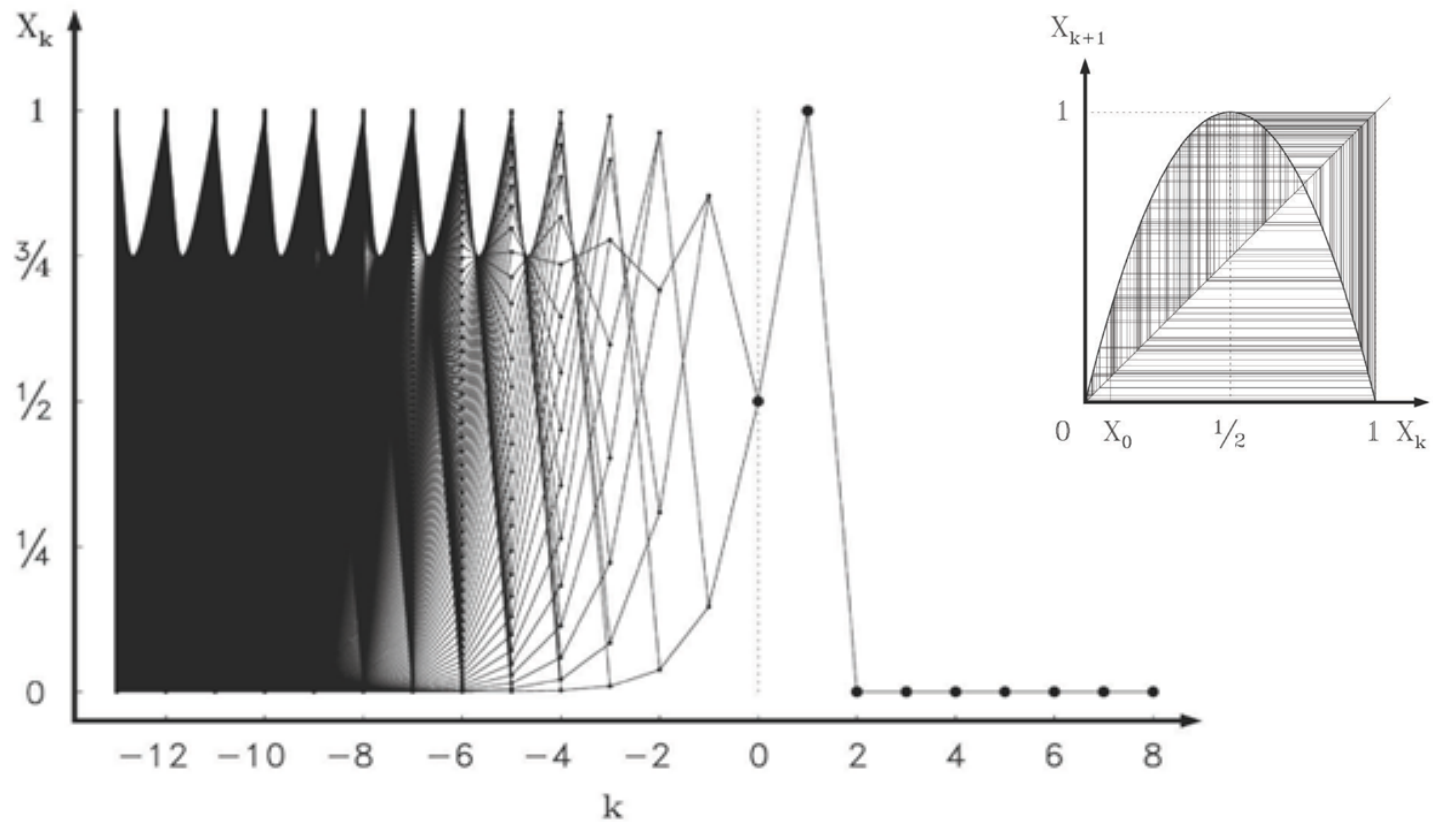
In the plenitude of chaos

not all wanders on a dusty strange attractor when $\alpha = 4$,
there are also **oscillations** forever, for any period:



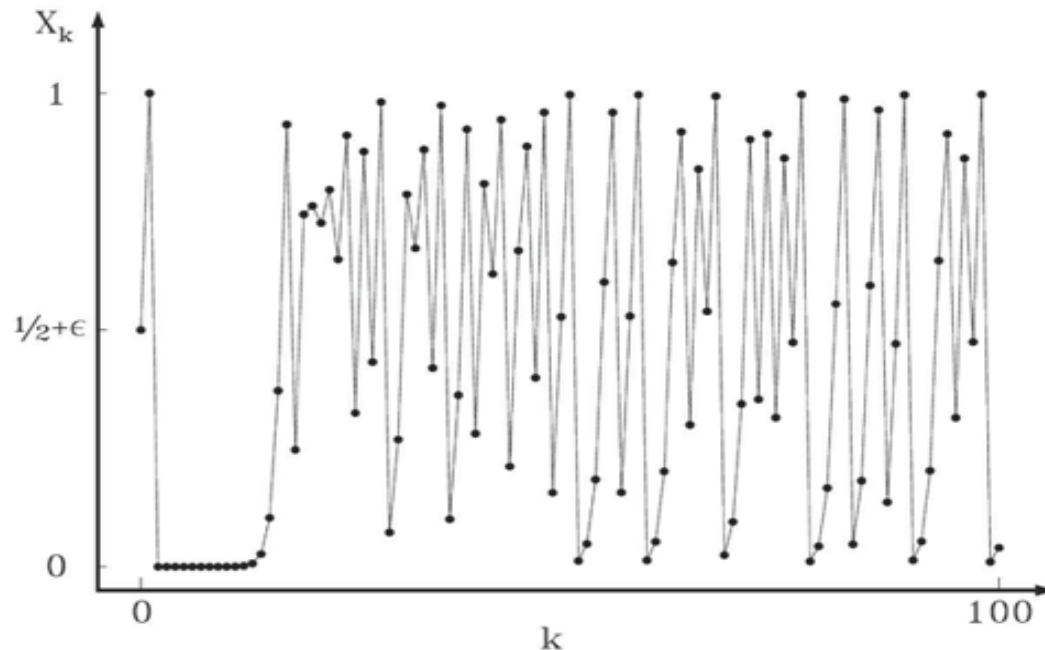
The vital escape to the origin

there are dynamically improbable points that return to the **root**:
the **pre-images of zero** find the way despite the **heat**:



More common sense

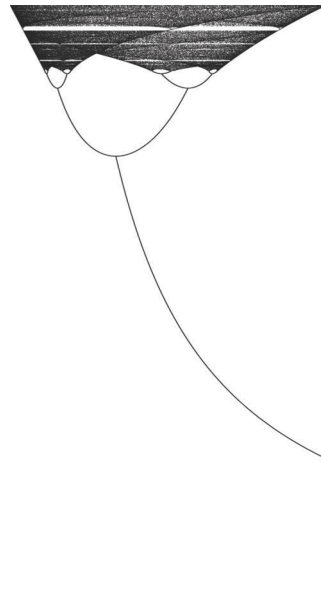
it is better to avoid **chaos** and its **infernal** turbulence:



come down the tree lowering your **heat** to find the **root**
abandon yourself to the threshold, $X = Y$, to reach the **Origin**
move away from non-linearities ($\alpha > 1$) to escape the **dust**

A scientific prophetic tree?

an allegorical **fig tree** without fruit, consistently cursed:



this tree and other **trees** have tender branches and many buds

in truth the ax is at the **root** of the trees

we may curse the fig tree as **He** rebuked the wind

More options

simple

complex

order

disorder

peace

chaos

to diminish

to augment

obedience

rebelliousness

below $X = Y$

above $X = Y$

blessings

curses

to rest

to wander

heaven

hell

A MODERN FIG TREE

Foliage of disorder
trapped in empty dust,
jumps astir forever
enduring subtle thrust.

Crossing of the outset
leaving faithful root,
looming tender offset
failing to yield fruit.

Cascade of bifurcations,
increasing heat within,
inescapable succession
of branches bent by wind.

Sprouting of dynamics
attracted to the strange,
o infinity reminding
at the origin: the flame.

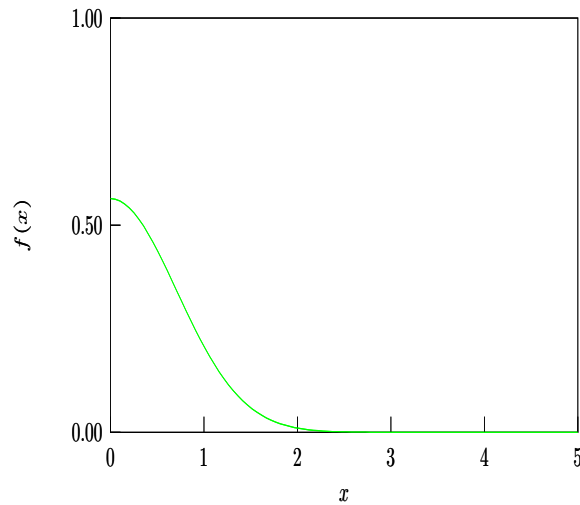


Lessons from normality

Bells and power-laws

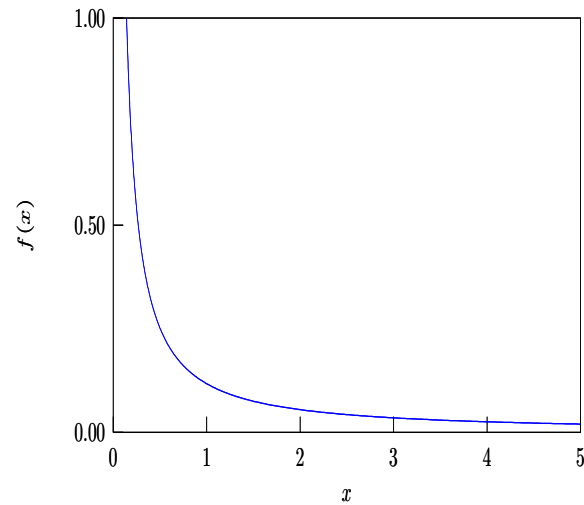
Gaussian

$$f(x) \sim e^{-x^2/2}$$

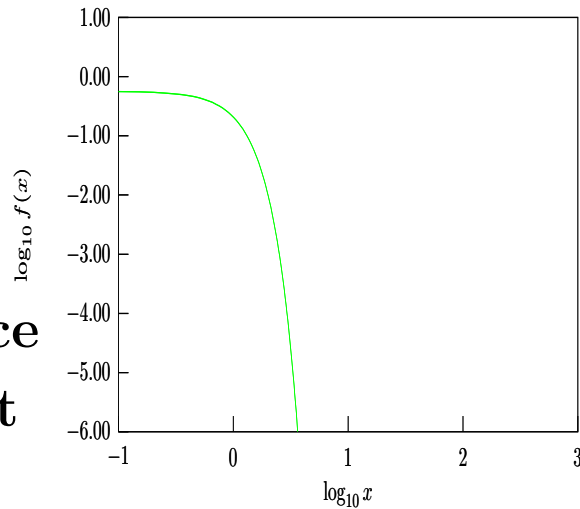


Pareto

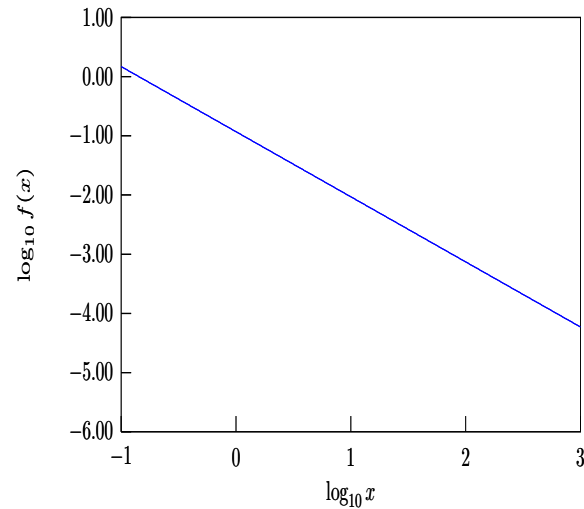
$$f(x) \sim 1/x^{a+1}$$



**independence
central limit**

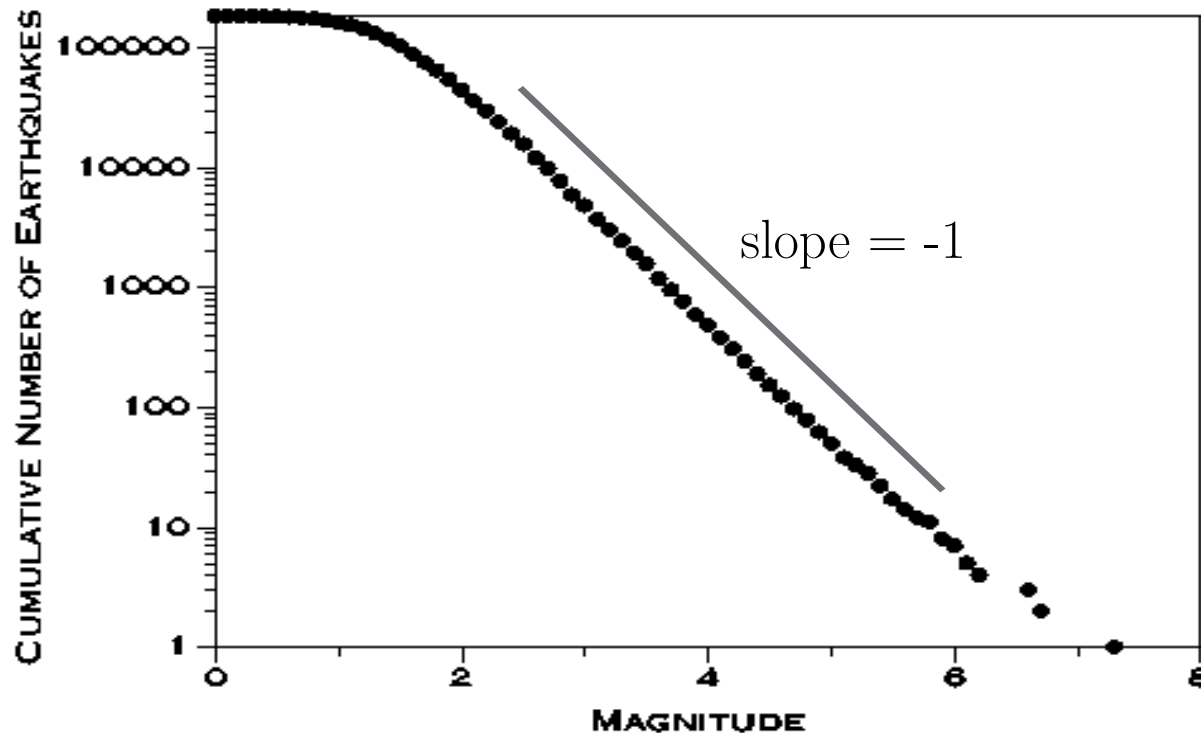


**without scale
heavy tails**



Power-laws and natural complexity

$$P[X \geq x] \sim x^{-c}$$



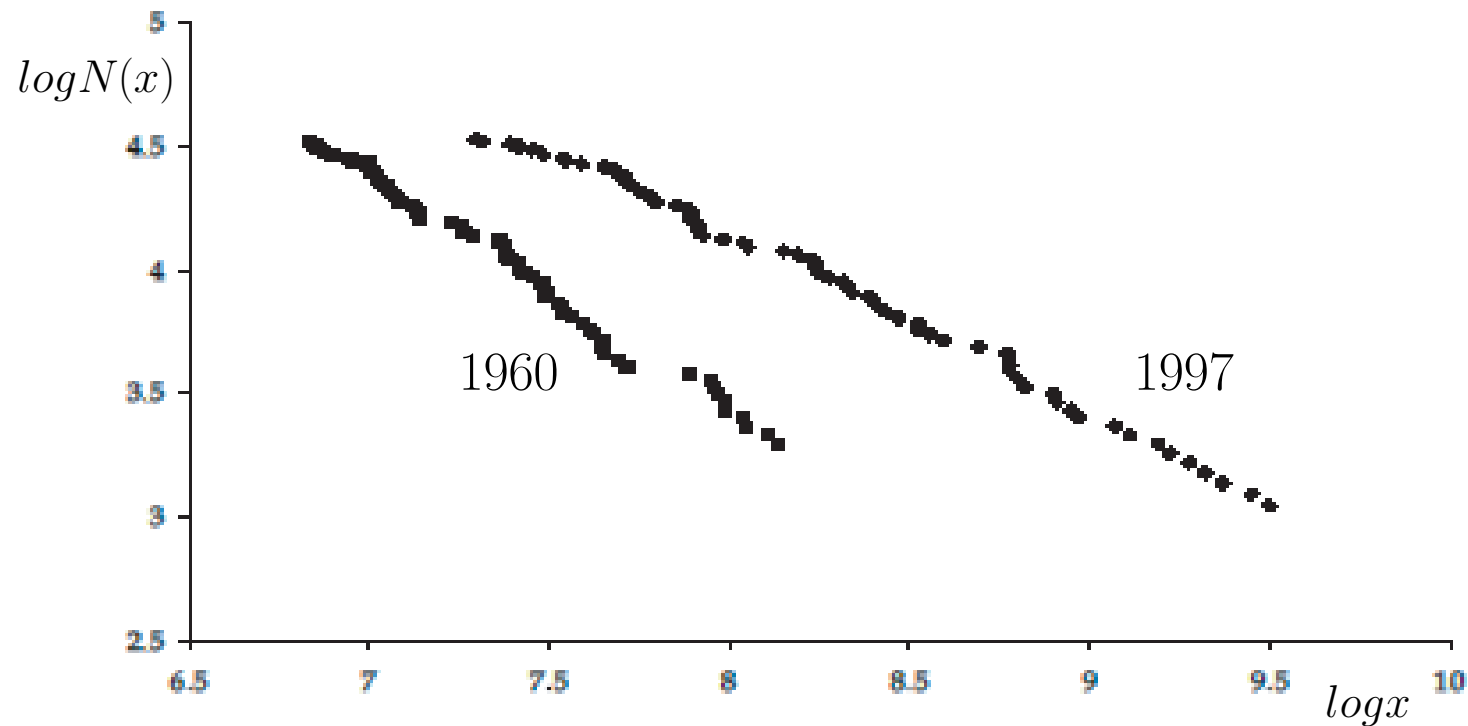
earthquakes, avalanches, hurricanes, floods,
volcanic eruptions, forest fires

utter violence...

Power-laws and inequalities

(Vilfredo Pareto, 1896)

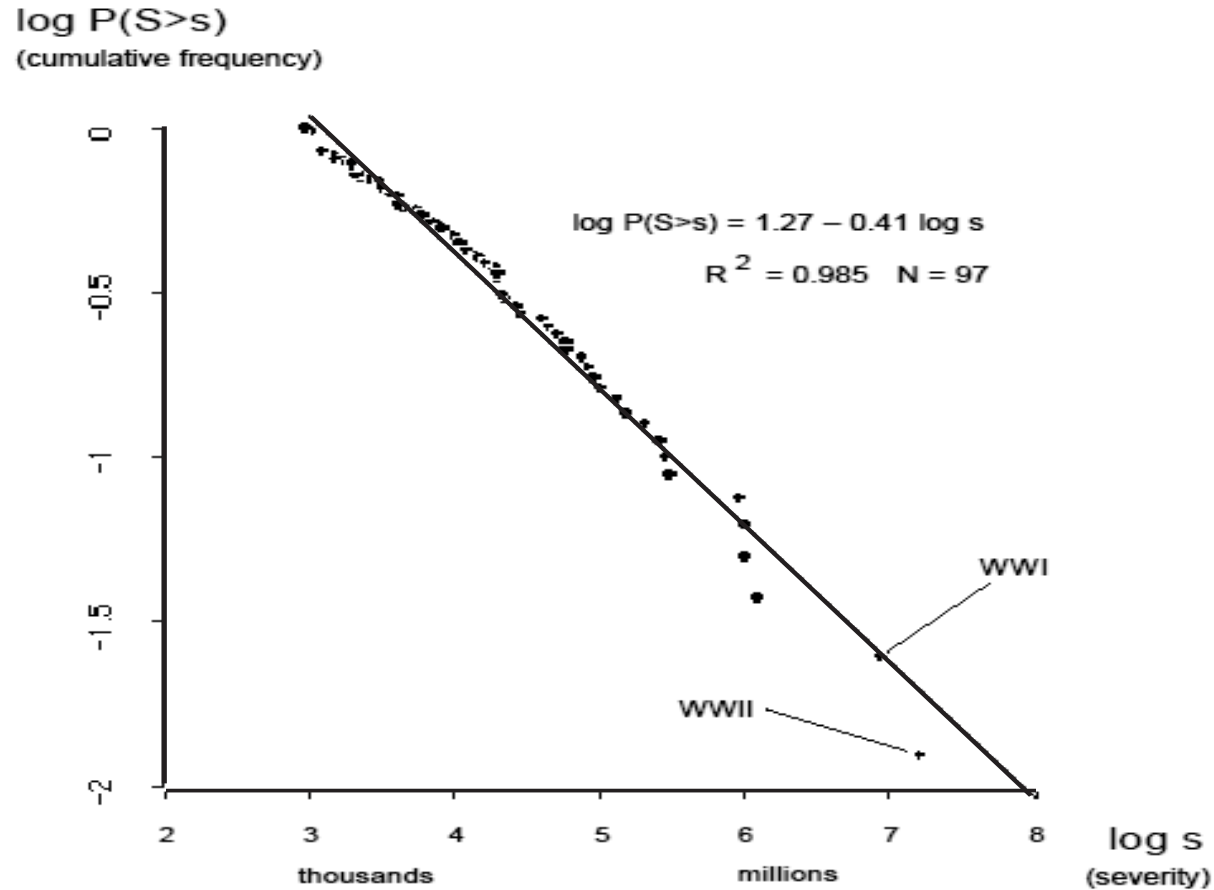
$$\log N(x) = \log A + m \cdot \log x$$



30th to 85th percentiles of world income distribution
(After Di Guilmi et al., 2003)

Power-laws and wars

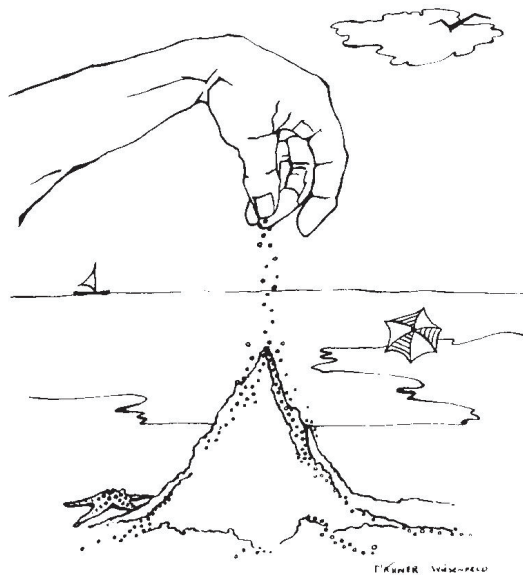
(Lewis Fry Richardson, 1960)



(After Cederman, 2003)

A bit more of common sense

as **violence** generates power-laws, it is relevant to study how such are produced in order to find **harmony** and **peace**



(After Bak, 1996)

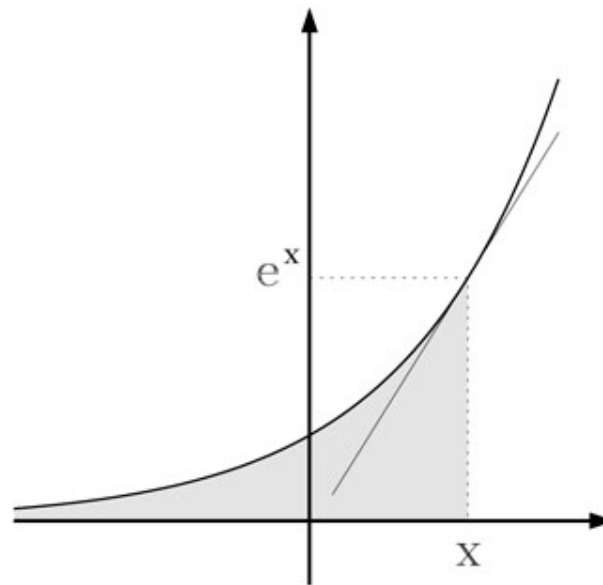
- multiplicative cascades
- preferential attachments
- highly optimized tolerance
- self-organized criticality...

there is no “**invisible hand**” but ours:

“humans without scale” abuse their power and yield power-laws
power-laws express three **negations**...

The exponential function and love

“integration” without “differentiation” has a unique solution,
the **exponential** function, the inverse of the logarithm:



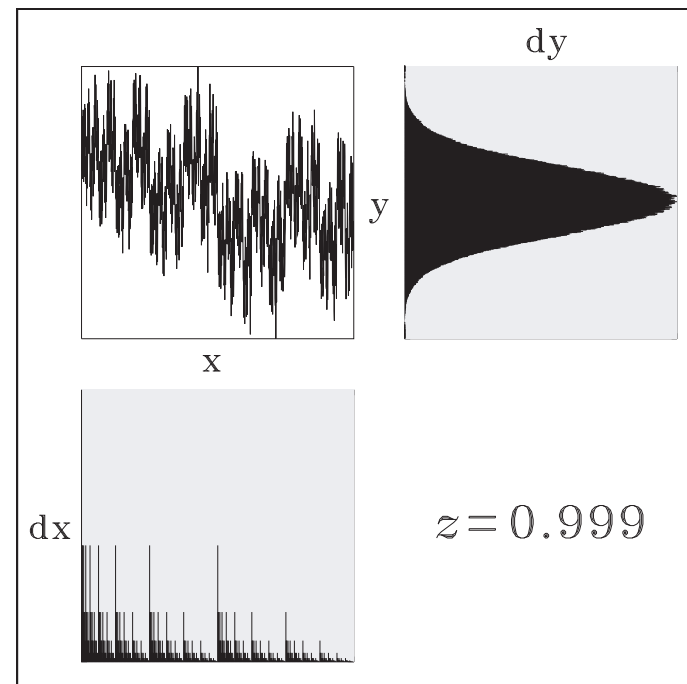
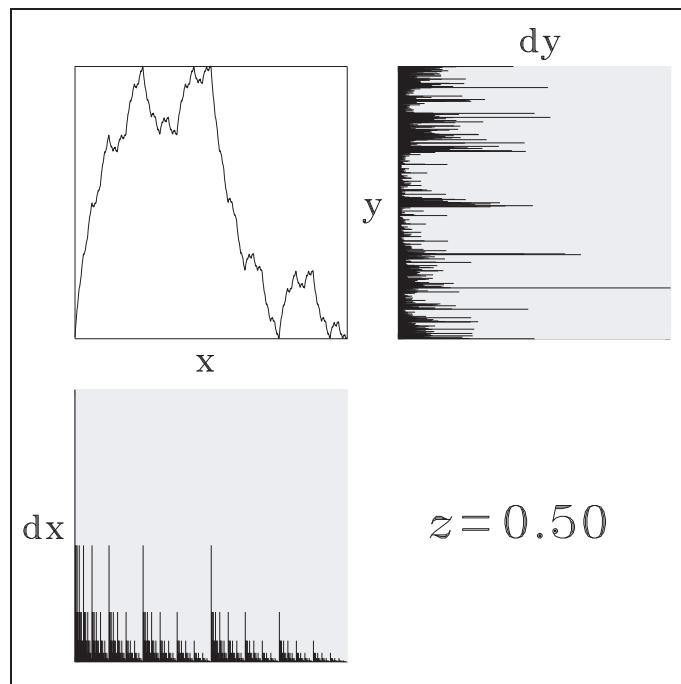
e denotes the supreme “power” of our communion with **Jesus**:

$$(1 + 1/x)^x \rightarrow e, \text{ when } x \rightarrow \infty, \text{ as in John 15}$$

A Platonic approach to complexity

$$w_1(x, y) = (x/2, x + z \cdot y), w_2(x, y) = (x/2 + 1/2, 1 - x - z \cdot y)$$

(Puente, 2003)

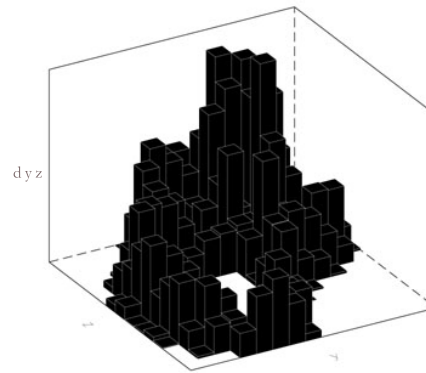
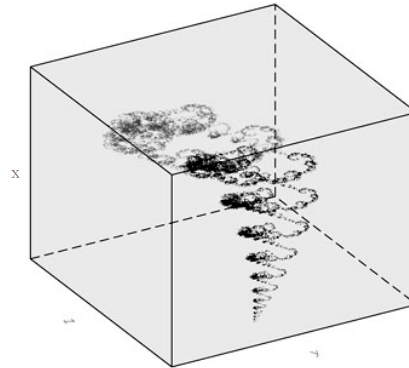


iteration of w_1 and w_2 produces a **wire** from x to y

left: geophysical sets based on a multifractal and without chance

right: from any spiky and dusty multifractal to a limiting **bell**

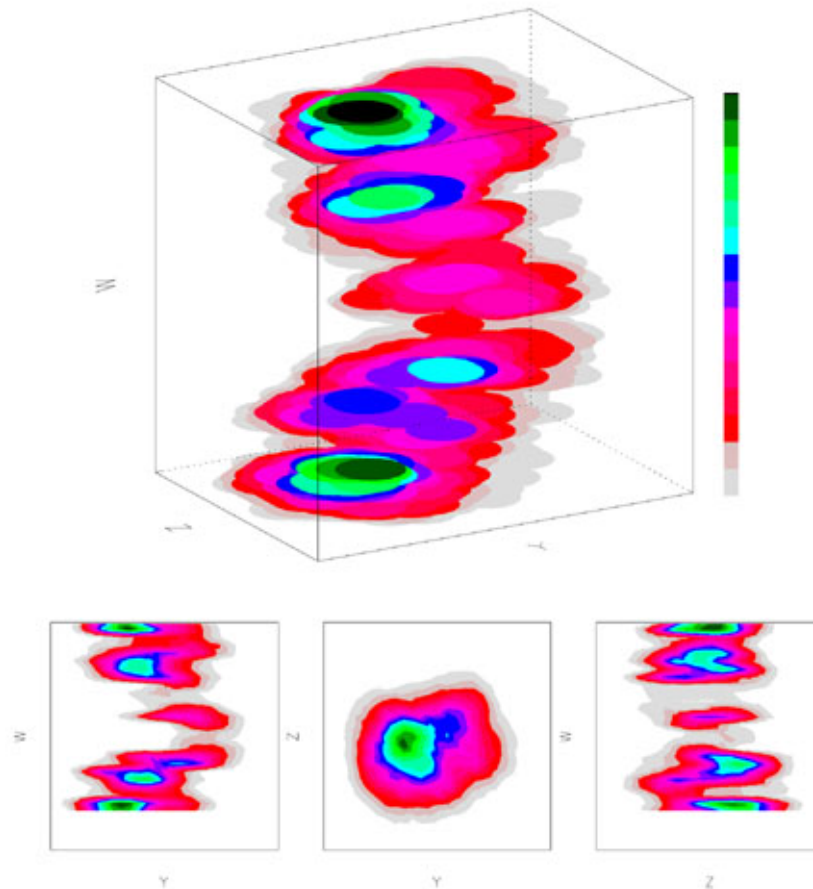
Extensions to higher dimensions



using:

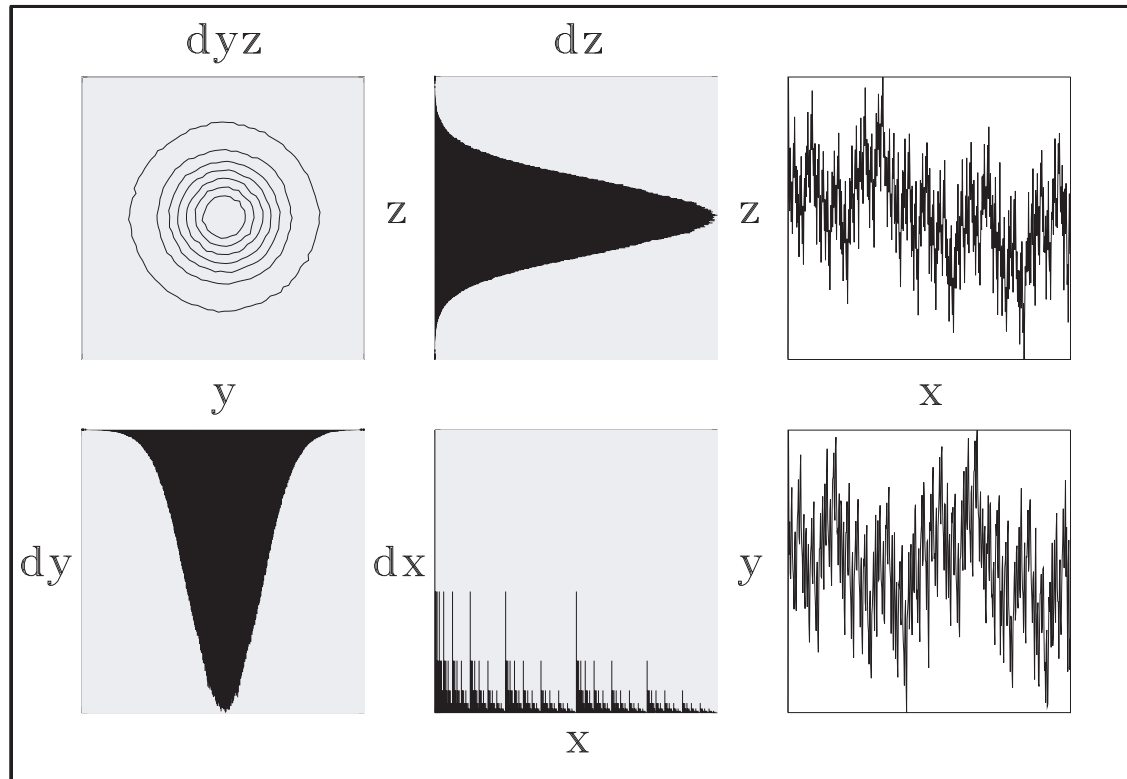
$$w_n \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_n & 0 & 0 \\ c_n & r_n^{(1)} \cos \theta_n^{(1)} & -r_n^{(2)} \sin \theta_n^{(2)} \\ k_n & r_n^{(1)} \sin \theta_n^{(1)} & r_n^{(2)} \cos \theta_n^{(2)} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} e_n \\ f_n \\ g_n \end{pmatrix}$$

Complex patterns in higher dimensions



a pollution pattern based on a wire in 4 dimensions
similarly for rainfall patterns and others...

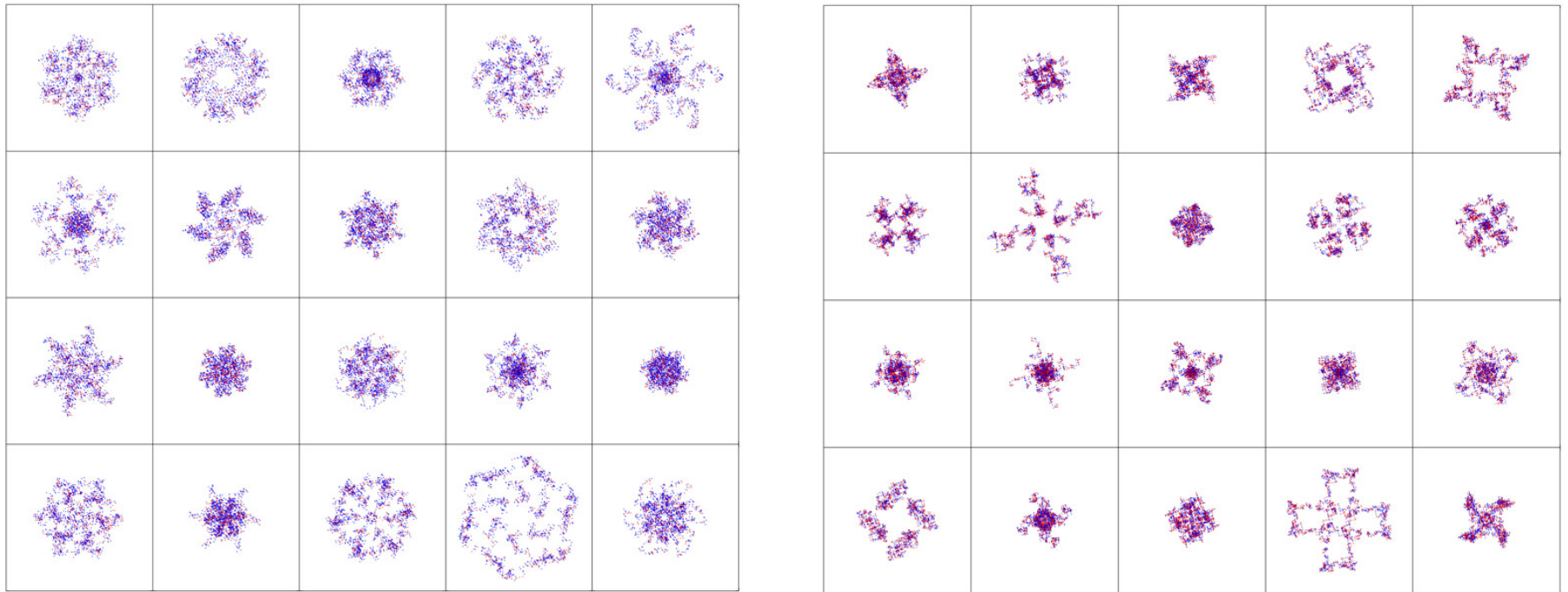
Bells in higher dimensions



from arbitrary **thorns** and **dust** to **harmony**
 or from **dissipation** to **conduction**:
 only if the transformation **fills-up** the space, $|r_n^{(j)}| = 0.999 \dots$

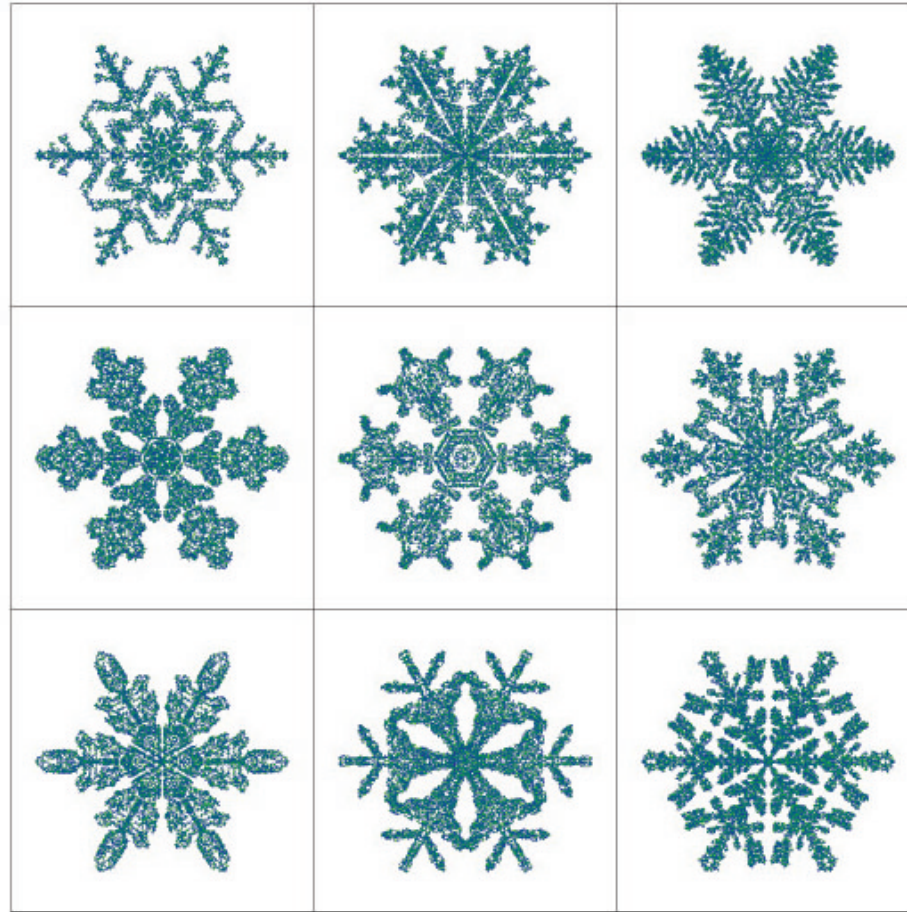
The exquisite decompositions of the bell

(Puente, 2003)



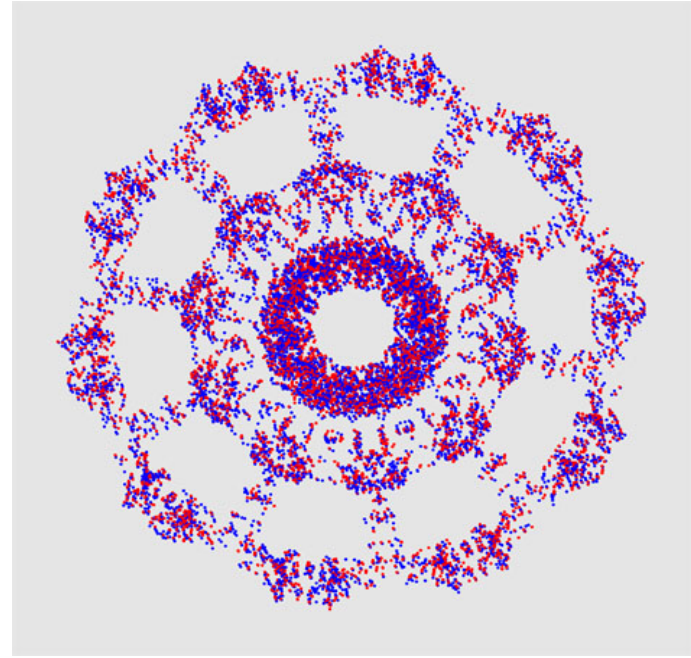
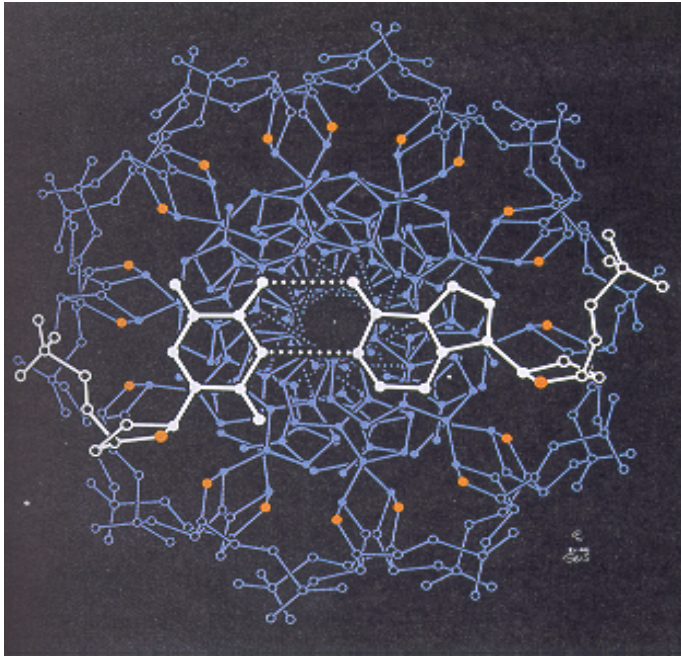
when lovely patterns are superimposed they form perfect circles
in the **limit** there is hidden **order** in chance
all is **beauty** if the angles are synchronized...

Ice crystals inside the bell



filling templates with iterations guided by fair coins
and using two functions with angles equal to 60 degrees

DNA, π and the bell

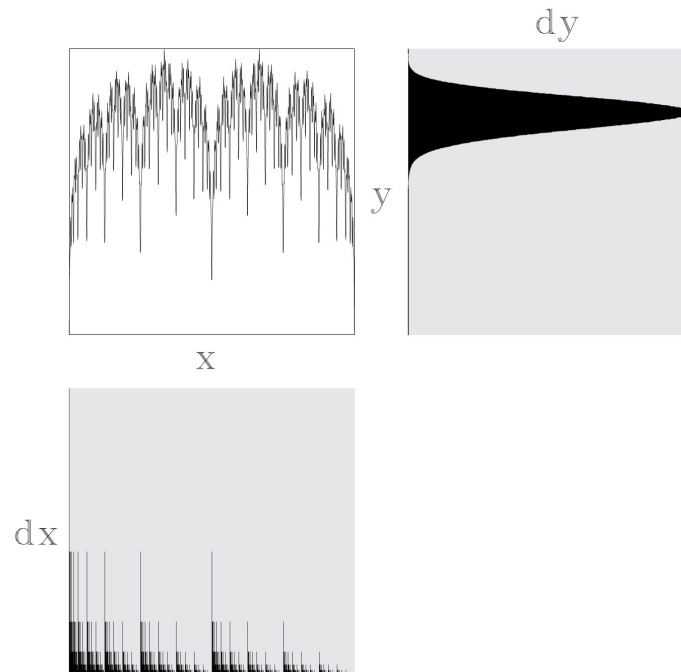


using two functions with angles equal to 36 degrees
and iterating them according to the **binary** expansion of π :
an improbable **design** that requires 40,000 bits!

A truly special case

$$w_1(x, y) = (x/2, x + z \cdot y), w_2(x, y) = (x/2 + 1/2, 1 - x + z \cdot y)$$

always positive, $z \rightarrow 1$

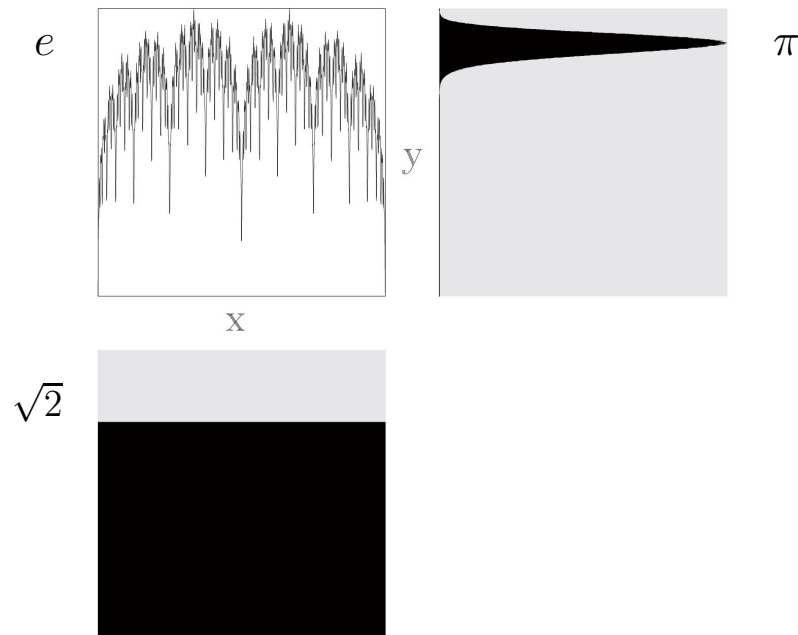


always a bell concentrated at **infinity**

the wire that is **maximum positive** filters any disorder:
where is, oh death, your victory?

Three majestic symbols

$w_1(x, y) = (x/2, x + z \cdot y)$, $w_2(x, y) = (x/2 + 1/2, 1 - x + z \cdot y)$
 the **limit** that fills-up space, $z \rightarrow 1$



the **Father** mighty in heaven
 the **Son** always perfect and positive
 and the **Holy Spirit** that comes from them

Yet more options

a bell

a power-law

meekness

stubbornness

normality

violence

conduction

dissipation

plenitude

loneliness

light

darkness

positive

negative

truth

lies

love

selfishness

THE AMAZING BELL

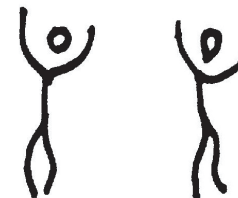
The bell peals silent, o o
reflecting its peace,
and inside it gathers
lovely masterpiece.

Symmetric pure beauty, o o
o mighty delight,
this limit in fullness
stores lifes designs.

Such vessel contains, o o
alephs of all tastes,
diatoms and crystals
including DNA.

But there is a case, o o
reason to this song:
the forward selection
that raises it all.

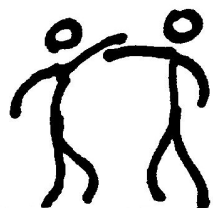
There is clear choice
that rotates the 8,
by loving sincerely
we surely converge.



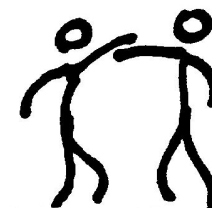
In consonance with Sacred Scripture:

simplicity is always much better than **complexity**
refrain from doing what is “**natural**” and do what is “**normal**”
if required, reclaim your inherent scale by **rectifying**:

avoid accumulating energies living in serene **equilibrium** $p = 1/2$
do not cross the chosen threshold remaining in the **root** $\alpha \leq 1$
maintain the central limit always in the **positive** case $z \rightarrow 1$



May the peace of the Lord
be with you all



Y = X

Y = X

is justice that illuminates,
is balance that fascinates:

Y = X.

Y = X

is the incarnate alliance,
is the established reliance:

Y = X.

Y = X

is true word that matures,
is a spiral that endures:

Y = X.

Y = X

is the precious resting place,
is the state of mighty grace:

Y = X.

Y = X

is smoothness that esteems,
is a hummingbird that gleams:

Y = X.

Y = X

is the short and precious root,
is the weaving of the truth:

Y = X.

Y = X

is a future that forgives,
is crowned science that is:

Y = X.

Y = X

is the ever tender tune,
is the impartial tribune:

Y = X.

Y = X

is all innocence that heeds,
is a garden with no weeds:

Y = X.

Y = X

is the simple clear sign,
is the majestic design:

Y = X.

Y = X

is independence that heals,
is matrimony that shields:

Y = X.

Y = X

is the real chaste embrace,
is the goodness of a yes:

Y = X.

Y = X

is a smile that edifies,
is a spin that rectifies:
Y = X.

Y = X

is all gentleness in us,
is the everlasting plus:
Y = X.

Y = X

is inspiration that calls,
is growing to be small:
Y = X.

Y = X

is the forgotten territory,
is the improbable story:
Y = X.

Y = X

is revelation that nests,
is surrendering the rest:
Y = X.

Y = X

is the dustless short incline,
is the faithful narrow line:
Y = X.

Y = X

is renouncing all spears,
is experiencing no fears:
Y = X.

Y = X

is the perennial giveaway,
is pure life with no decay:
Y = X.

Y = X

is the only perfect remedy,
is loving, even the enemy:
Y = X.

