1. Jesus, the hypotenuse, the only way to the Father
Carlos E. Puente

Cover. It is a great joy for me to return to Rome to share this series of talks about what, I think, may be the key role of science in the New Evangelization.

I wish to thank Father Rafael Pascual for his continued friendship and support and, in particular, for making this event possible. I would like to also acknowledge Father Pedro Barrajón for his constancy throughout the years.

Before I start, I would like to extend my gratitude to all of you for being here at Regina Apostolorum. As we celebrate the year of the faith, I sincerely hope that you will find these talks edifying.

Today’s presentation is entitled Jesus, the hypotenuse, the only way to the Father.

Yes, I do realize that this title is a bit strange, but you shall see here, in the very spirit of Fides et ratio by Pope John Paul II, how such an assertion is true, as it will be explained step by step, in a logical progression.

Page 2. The thesis of this work is that we humans, with the gift of a soul, may learn from recent advances regarding natural complexity in order to find the peace of Christ and share the good news in a novel way.

Page 3. As a reminder, here is a right triangle having legs $a$ and $b$ and a hypotenuse $c$, satisfying the celebrated Pythagorean theorem, $a$ squared plus $b$ squared equals $c$ squared.

As is seen, this triangle has two equal legs, and if they are one unit long each, the hypotenuse would measure the square root of two, which, represents the shortest distance from the bottom left to the top
right. This is true despite our inability to know the precise meaning of the dot-dot-dot here, as the number is irrational and has an infinite expansion, without repetition.

**Page 4.** As you shall note, this talk contains a few simple games that illustrate how fragmentation happens. Here is the first one.

This is a **game for kids**, one easily understood molding modeling clay. Drawn here, on top, is a bar of clay as it comes out of the box. The game starts cutting the bar by a given factor, say 70% from the left, as shown by the vertical line.

**Page 5.** Then, the game continues, piling up the largest piece to the left and enlarging the second piece, also to the left, so that they make two contiguous uniform bars with equal horizontal lengths.

Clearly, the first piece is higher than the original bar, and the second piece is lower.

**Page 6.** The game continues repeating the same process on each piece, using the same proportions.

At the next level, there are four rectangles, whose masses are, from left to right, 70% of 70%, or 49%; 30% of 70%, or 21%; 70% of 30%, or the same 21%; and 30% of 30%, which gives 9%. Clearly, 49 + 21 + 21 + 9 gives a 100%, in virtue of the well known principle of “conservation of modeling clay,” something that does not work well if there are little kids at home.

**Page 7.** The next level contains eight pieces, and the most massive rectangle continues to grow in height. As the base of such a piece is half of a half of a half, or one eighth, and as the area equals 0.7 cubed, the height gives 1.4 cubed, which is 2.74 times higher than the height of the original bar.
Page 8. What the game produces may be calculated easily for arbitrary partitions $p$ and $q$.

At the first level of the game, beneath the initial bar, the masses are precisely $p$ and $q$.

Page 9. At the second level one finds, in order, $p$ of $p$ or $p$ squared, $q$ of $p$, $p$ of $q$ and $q$ squared, which is no more than the familiar expansion of $p + q$, all squared.

Page 10. At the next level one gets $p + q$, all cubed, for the masses from level to level are found just multiplying, by $p$ to the left and by $q$ to the right.

Page 11. As may be noted, all is related to the famous Pascal’s triangle shown here and the associated binomial theorem expanding $p + q$ to a power $n$, and the game defines a properly named multiplicative cascade.

Page 12. Here is what happens when the game is repeated twelve times when $p$ equals 0.7. We obtain 2 to the twelve, 4,096, rectangles with very small bases equal to one over 2 to the twelve, and the original bar is broken into many thorns, which prick us if we touch them from above.

As the vertical scale increases dramatically due to the successive pile-ups, the diagram, that would have been 1.4 to the twelve or 56.69 units in height, is compressed here so that it may fit on the page.

As is seen, the thorns are ordered in layers according to the expansion of $p + q$ to the twelve and Pascal’s triangle. The highest thorn occurs once and contains $p$ to the twelve of the mass. The smallest rectangle on the right happens also once and is almost invisible as it has only $q$ to the twelve, that is, 0.3 to the twelve, of the mass.
Then, the object has twelve large thorns with masses $p$ to the eleven times $q$, twelve small thorns (also invisible) with $p$ times $q$ to the eleven, sixty-six thorns with $p$ to the ten times $q$ squared, and so on.

As may be seen, the layers of thorns are finely intertwined and increase in density as we enter into Pascal’s triangle from both sides.

When the game is played many more times, the additional fragmentation gives rise to infinitely many layers of thorns of infinite sizes that, by lacking any cohesion due to the presence of gaps, are supported by a disperse collection of points that have the structure of dust.

As ultimately there are infinitely many such dusts, one per inside layer, the fractured and thorny object generated by this game is known as a multifractal.

Certainly, walking on this object is quite a challenge, for to visit someone at the same level of mass requires going up and down many times, as thorns, for all levels, end up separated by holes.

To fully appreciate the empty structure on every layer of the first game, it is pertinent to introduce another game for kids. This one is also played with modeling clay, but instead of cutting the original bar by its $70\%$, it is done by the middle, piling up left and right so that there is a hole of size one third by the middle, as shown.

As before, this game progresses repeating the same idea: dividing each piece and piling up left and right in the same propor-
tions. For each level, this process creates a collection of equally-sized rectangles that, by construction, never touch, hence defining a rather efficient way to produce perfect spaghetti.

**Page 20.** Clearly, this simple game is another multiplicative cascade that eventually generates thorns of equal size emanating from a collection of points that, by being separated by gaps, have, once again, the structure of dust.

**Page 21.** It happens that by varying the size of the hole, say, from one third to an arbitrary size $h$, this construction adjusts the topological sparseness of the layers in the first game. While the denser layers require the propagation of smaller holes, those that are more disperse correspond to larger holes.

**Page 22.** The moral of the story is that the two games you have learned, although seemingly different, are, at the end, intimately related to each other. Both are divisive cascades and the second lives inside the first, on each one of its layers.

**Page 23.** To further appreciate the games, and as they give rise to thorny objects, containing nothing individually while growing to an infinity that can not be drawn, it is convenient to consider their accumulated masses from the beginning, zero, to a point $x$ that varies from the beginning to the end, or from zero to one. As such, the two cascades, on the left, yield the accumulated objects shown on the right, where $C(x)$ is the total amount of modeling clay from zero to $x$.

**Page 24.** The curious profiles on the right are easily found following the dynamics of the games, as follows.
For the first game, we obtain a cloud profile, like the one produced by an explosion, that contains a great multitude of horizontal-vertical **notches**. The most notorious happens when \( x \) equals one half and has a height of 0.7, and such is the case as from the beginning to the middle of the spiky object there is, by construction, 70\% of the mass.

**Page 25.** As seen, there is a notch at \( x \) equals one quarter with height 0.49, which is the aforementioned 70\% of 70\% of the mass, and so on.

**Page 26.** For the second game, we find a host of **plateaus** that correspond to the successive holes on such a cascade. Clearly, the largest one happens from one third to two thirds and has a height of one half, as half of the mass was piled up to the left.

**Page 27.** Then, there are two plateaus with length one ninth –a third of a third– and heights a quarter and three quarters, and so on.

**Page 28.** As is seen, the accumulated sets are “mathematical monsters” with many points where tangents cannot be defined. While the first profile has no **derivatives** at any point, the second one does not have them at the extremes of the (infinitely many) plateaus.

As there are notches and plateaus everywhere, both accumulated objects turn out to be **locally flat**.

**Page 29.** As a consequence, their distances, from bottom left –zero, zero– to top right –one, one–, are, at the end, equal to **two** units –one horizontal plus one vertical.

Remarkably, walking such profiles requires doing infinitely many horizontals and verticals, which yields a **maximal length** of 2.

It happens that such a property is universal, for when imbalances
$p$ or holes $h$, no matter how small, propagate, such define thorns and dust that give rise to accumulated objects containing notches or plateaus everywhere.

**Page 30.** The same happens when combining the games, yielding cascades with imbalances and holes, and also when chance is used to define variable imbalances and holes from level to level.

**Page 31.** As the jagged profiles given by the cascades are locally flat everywhere, if one were to parachute on them, one would believe to have landed on flat ground.

**Page 32.** Because of this clear deception, and due to the fragmentation of the games, such profiles are appropriately known in physics and mathematics as **devil’s staircases**.

**Page 33.** It happens that the first game for kids is related to the way turbulence happens in nature, the very common process that scares us while riding an airplane.

When the inertia of a fluid –given by the product of its velocity, $v$, and a characteristic length, $L$,– subjugates the fluid’s cohesion –given by its viscosity, $\nu$,– that is, when the Reynolds number, $Re$, shown here on the right, is sufficiently large, the fluid breaks into an irreversible chain of **eddies**, which divide into eddies, that divide into eddies, and so on.

**Page 34.** Those inwardly rotating elements, traveling from more to less pressure –or from plus to minus– as in hurricanes, carry with them unequal amounts of energy that remarkably correspond to the layers of the first cascade, when the imbalance $p$ equals precisely 70%.

Turbulence turns out not to be predictable, however, for nature’s
eddies, from step to step, are not always higher to the left, but rather happen left or right as guided by “chance.”

**Page 36.** Inescapably, when the scale of those eddies becomes sufficiently small, the energies carried by them **dissipate** in the form of heat. Even though the higher the Reynolds number the longer the process lasts, a natural cascade—as opposed to what may be done mathematically—turns out to be finite.

Notably and as reported by Meneveau and Sreenivasan, observations for several flows, both natural and in the laboratory and including atmospheric turbulence, boundary layer, and wake of a cylinder, yield layers of energy along one dimension that are just permutations of what is produced by the first game for kids.

**Page 37.** So that you may fully appreciate the goodness of the **universal** fit found by such investigators, here is shown the relationship between the magnitudes of the layers and their respective densities. While the squares denote the observations of turbulence, the parabola corresponds to the 70-30 cascade with densities that increase as we enter Pascal’s triangle from both sides.

**Page 38.** As the increase in entropy in turbulence happens universally via a simple cascade, common sense suggests we may employ such a process, and also the one defining its layers, to study how we humans create our own **turbulence**.

After all, all of us, from Afghanistan to Zimbabwe, are confronted by “inertial forces” that break our “internal cohesion” and, when such happens, for sufficiently high Reynolds numbers, such produces our turbulent behaviors, ultimately leading to violence. For, even if we want to deny it, many times we make mistakes and break what we should not repeating the same error again and again.
Page 39. In this spirit, while the first game may be used to vividly describe the proliferation of inequalities generated by our competitive and preferential instincts and leading to the marked cynicism of modern life, the second cascade may be employed to represent the appalling effects of discriminations and their related distrust and fear that result while imposing “equality” by force.

Page 40. Note how these simple ideas and their associated diagrams properly reflect not only the political systems that have governed the world, but also, more importantly, our own selfish postures and actions—What else could be the cause for us playing silly games?

Page 41. For, as seen vividly in the rotating negative spirals, these simple notions sadly express why the “third world” made of two thirds of the people in the world, that is 0.666... of all, live in poverty; why 6,000 kids die every day due to lack of water; and why we have been living for a really long time in an era of violence and terror.

Although the drawings here may appear funny to some, notice that they are not, as relationships—friendships and marriages—fail by either one of those cascades or their combinations.

Page 42. As history has proven that the second game does not work due to its conspicuous emptiness and falling walls, it is relevant to ask—even if such a question is inadequate or politically incorrect to some— if the globalization of the first cascade is the solution to the problems we face.

In this sense, it is useful to remember Pascal’s triangle to perform some simple calculations.

If the imbalance $p$ is set at 70%, as in nature, and if $n = 20$ levels of the cascade are considered, one may study where the modeling
clay is located. As such, the 5, 10, 20 and 40% of the largest thorns contain, in order, 57, 70, 84 and 95% of the mass.

**Page 43.** Sadly, these numbers adjust the skewed wealth distribution of the most powerful nation on earth, the **United States**, by the end of the 20th century, circa 1998, for the richest there had, for the same percentiles, 59, 71, 84 and 95% of the resources.

**Page 44.** This is an undesired coincidence that provides however a truthful warning and a clear **moral**. If imbalances continue their propagation, as they appear to do in the twenty-first century, the laws of physics and common sense assure us that energies shall **dissipate** and that we all shall “bite the dust.”

This warning certainly applies not just to a single country, but all over our world, as any nation’s wealth distribution may be adjusted via a multiplicative cascade that gives a devil’s staircase, even if it requires variable partitions from level to level.

**Page 45.** Based on these reflections, we may see that there is indeed a **common sense code for true peace**, and against violence, one easily defined by the following imperatives: run cascades in reverse to **repair** what is broken; live at low Reynolds numbers in order to avoid any **violence** and the anxieties of modern life; and, to quote ancient prophets,¹ “cut mountains and fill valleys” to restore **unity**.

**Page 46.** For both graphically and mathematically, reconciled unity is composed of infinitely many outward and loving spirals, traveling unnaturally from less to more or from minus to plus, that are opposed to the negative eddies (in polar coordinates) induced by the diabolic power of the air.²

¹Lk 3:4–6, Is 40:4–5
²Eph 2:2, Eph 6:12, Jn 12:31, Rv 12:9
For it is the devil himself, the “ruler of the power of the air” as well as the “ruler of this world,” who is, by his cascading division, our common enemy. He is the one who falsely whispers in our ears that death is victorious and that brotherhood is an unachievable utopia in this world, of which he is the prince of disorder.³

**Page 47.** Based on these observations, we may see by ourselves that there is a **unique geometric solution** (left) and an associated straight ramp of accumulated clay (right) –there is 50% of the mass from the beginning to the middle, 25% up to a quarter, and so on–, that reflect the fact that we should not play any divisive games, but instead do good to defeat evil.⁴

**Page 48.** Clearly, the key is in dynamically maintaining the original bar as it came out from the paradisiacal box of level zero, always practicing the proverbial 50-50 without exceptions, that is, without holes.⁵

**Page 49.** This means spiritually growing such that we fulfill the sanctifying power of zero that yields unity with God.

**Page 50.** For the uniform bar is the only **straight** and **solid** condition which, by not containing **thorns** or **dust**, we may walk safely without fear.⁶

**Page 51.** As we may see, once again by ourselves, this solution is none other than Our Lord **Jesus Christ**, “the way, the truth, and the life,” who, by maintaining the straight path and never playing false games, that is, by never lying and hence being truthful, kept

³ 1 Jn 3:8, Jn 8:44, Gn 3:14  
⁴ Rom 12:21  
⁵ Jn 13:35, Mt 5:44  
⁶ 1 Jn 4:18
always the essential energy, defeated death and is alive.\textsuperscript{7}

\textbf{Page 52.} As the accumulation of the uniform bar results in the \textbf{one-to-one} line –again, Jesus Christ in his relation to each one of us– and as such a ramp has a minimal distance of square root of two from bottom left to top right, we may see why the \textit{hypotenuse} of the triangle, while reflecting unitive and “radical” \textit{love}, is indeed the path of and to \textbf{peace}!

For the wholesome \textbf{truth} travels efficiently and justly with a slope of \textbf{one}, while the \textbf{divisive} diabolic games produce crooked devil’s staircases that are as long as the \textbf{legs} of the same triangle.

\textbf{Page 53.} The moral is then that we ought to humbly \textbf{rectify} when needed and \textbf{love} God and everyone in order to find true \textbf{joy}, as we have been told.

\textbf{Page 54.} For in the hypotenuse’s simple equation $Y = X$, we may see, once again, a crucified silhouette of the one on a positive cross, whose sacrifice represents the antidote of dust, which is dissipation, which is death.

\textbf{Page 55.} Although these connections may be surprising to some, even some who wholeheartedly work on the New Evangelization, they end up being very useful to explain the good news in the huge difference that exists between devil’s staircases and the divine ramp. For if we were to parachute on the hypotenuse...

\textbf{Page 56.} we would end up sliding to the bottom,\
\textbf{Page 57.} which is nothing else but the \textit{Origin}, with capital o, a consistent location with Jesus’ famous proclamation that nobody

\textsuperscript{7} Jn 14:6, Is 40:5, Mt 5:17
can arrive to the Father except through Him,\(^8\) that is, via the hypotenuse!

Notice that there is no other choice, as it is impossible to go off on a tangent on a devil’s staircase, which, as we saw, is locally flat everywhere.

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**Page 58.** To emphasize the uniqueness of true equilibrium, here is the improbable point in the midst of a sea of possibilities expressing all cascades that combine imbalances \(p\) and holes \(h\).\(^9\)

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**Page 59.** As there are devil’s staircases everywhere, we may note that it is indeed hypocritical to judge others when away from the point,\(^10\) as there are negative logs everywhere. And since the point is merely a speck within space, it is indeed easier for a camel to pass by the eye of a (sufficiently large) needle than to find the essential point.\(^11\)

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**Page 60.** At the end, however, there is an appointed faithful algorithm –a veritable sacrament– guaranteeing our arrival to the point. Confronting our selfishness, such corresponds to repentance and rectification, which may be further appreciated noting that if the cascade is performed for only a finite number of levels, it produces a convex-up surface from which we can slide to the point of true balance just by recognizing the gravity of our faults.\(^12\)

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**Page 61.** For there is a marked difference between the selfish spiral of number 6 and the always loving, unitive, and positive of number 9, as there was darkness between such precise hours when Jesus

\(^{8}\)Jn 14:6  
\(^{9}\)Mt 19:24  
\(^{10}\)Mt 7:4–5, Mt 7:1  
\(^{11}\)Mk 10:25  
\(^{12}\)1 Jn 1:9, Mt 6:9–15, Mt 11:28–30
died for us, crowned by our multiple thorns.\footnote{Mk 15:33-37, Mk 15:17}

_page 62._ These ideas remind us of our personal and collective options: equilibrium or turbulence, calmness or violence, rectitude or wickedness, 50-50 or inequities,

_page 63._ the shortest or the longest, reconciliation or separation, integration and its slender S or division and its negating symbol (for “the love of money is the root of all evil”),\footnote{1 Tm 6:10} wholeness or emptiness,

_page 64._ unity and its outward spirals or dust and the diabolical fraction, and, quite geometrically, a positive outlook to the future or a negative attitude trapped in the past, as may be inferred placing the spirals on a clock.

_page 65._ I would like to share now a song called \textit{Pathways}:

\textbf{Pages 66-68.}

Two options before us  
\vspace{0.5em}  
\textit{two pathways ahead, the one is the longest the other straight.}  
\vspace{0.5em}  
We journey directly or go by the legs, we follow intently or end up in pain.

By walking the flatness or hiking the spikes, we travel in lightness or take serious frights.

\footnote{Mk 15:33-37, Mk 15:17} \footnote{1 Tm 6:10}
The incentive is unity
and the call proportion,
the key is forgiveness
and the goal true notion.
In wandering wickedness
there is never fruit,
but in ample humbleness
one encounters the root.

There is no excuse,
o listen my friend:
it’s by the hypotenuse
or else by the legs.

There is no solution
but walking straight:
the spikes of disorder
insinuate the way.

It is true!

There is a best pathway,
the palpably smooth.

It’s by the hypotenuse
and walking in truth.

There is one solution,
I tell you the truth.

It’s by the hypotenuse
and walking in truth.

For any other pathway
will lead us astray.

It’s by the hypotenuse,
there is no other way.
O listen, you brother,
let’s brighten the day.

It’s by the hypotenuse,
there is no other way.

Otherwise, the devil
shall pull by the legs.

It’s by the hypotenuse
or else by the legs.

For such road is fractal:
as long as it gets.

It’s by the hypotenuse
or else by the legs.

O let’s mend the broken,
growing to the root.

It’s by the hypotenuse,
the one that yields fruit.

Let’s keep equilibrium,
avoiding dark soot.

It’s by the hypotenuse,
the one that yields fruit.

O listen, you brother,
a counsel from science.

It’s by the hypotenuse:
the simplest design.

I tell you integrating,
dont leave it to chance.

It’s by the hypotenuse
the simplest design.
Page 69. Now, very close to the end, I would like to remind you of other coincidences of faith you may visualize based on the concepts of science in this talk.

Clearly, this presentation follows the spirit of fellow “hydrologist” John the Baptist –as he baptized with water– who with Jesus, called us to rectitude when they said “repent, for the kingdom of heaven is at hand.”\(^{15}\) This is the case as the one crying out in the desert (perhaps a little bit myself) also described, together with prophet Isaiah, the key geometric algorithm defining the broad valley in which the glory of God is revealed, by stating that “every valley shall be filled and every mountain and hill shall be made low. The winding roads shall be made \textit{straight}, and the rough ways made \textit{smooth}.”\(^{16}\)

Page 70. The ideas also remind us of some precise statements. Such include Jesus’ command that we ought to “forgive others \textit{seventy} times \textit{seven} times,”\(^{17}\) as seen allegorically in the second level of the natural cascade in the product of 0.7 times 0.7; the statement by the prophet Isaiah that “our strength lies in \textit{quiet} and \textit{trust},”\(^{18}\) that is, at low Reynolds numbers; and God’s edict that “\textit{sinners}, who love \textit{violence}, shall lick the \textit{dust} like the \textit{serpent}.”\(^{19}\)

Page 71. At the end, the notions are very old, for God prescribe to Adam not only the appointed dust to his sin, but also the decree that “the ground shall bring forth (for him) \textit{thorns} and \textit{thistles}.”\(^{20}\) For it is indeed true that one is with or against Jesus, for as He said “whoever does not \textit{gather} with me \textit{scatters},”\(^{21}\) as does the one who rules the wind and wishes our division.

\(^{15}\)Mt 3:2, Mt 4:17
\(^{16}\)Lk 3:5, Is 40:4
\(^{17}\)Mt 18:22
\(^{18}\)Is 30:15
\(^{19}\)Ps 73:6, Mi 7:17
\(^{20}\)Gn 3:18
\(^{21}\)Mt 12:30
Page 72. How wonderful it is to appreciate the precious gift of reconciliation that God offers us, for it makes a difference such “as far as the east is from the west,”\textsuperscript{22} and as far as the square root of two is from 2. How lovely it is and how powerful when two or more find the root! For \textbf{unity} may be understood \textit{geometrically}, and one plus one is not two but, by the cross of love, a larger one, as explained by Jesus regarding \textbf{marriage} –between a man and a woman, of course– and his bride the \textbf{church}.\textsuperscript{23}

Page 73. Now to end, and giving you thanks for your attention, I would like to share another song entitled \textbf{609}. I hope you like it...

Pages 74-75.

\begin{verbatim}
Six, zero, nine, a dear song
numbers unfolding daylong,
six, zero, nine, a clean gong
symbols inviting us to love.

From six to six
revolving inwards,
from six to six
I went downwards.

From six to six
dividing selfishly,
from six to six
lying endlessly.

From six to six
trying to be a rose,
from six to six
\end{verbatim}

\textsuperscript{22}Ps 103:12
\textsuperscript{23}Mt 19:6, Jn 10:16
being only a nasty thorn.

Six, zero, nine, a dear song
numbers unfolding daylong,
six, zero, nine, a clean gong
symbols inviting us to love.

From six to zero
I tapered my speed,
from six to zero
the tempest did not lead.

From six to zero
I no longer postponed,
from six to zero
I finally atoned.

From six to zero
I experienced peace,
from six to zero
my loneliness ceased.

Six, zero, nine, a dear song
numbers unfolding daylong,
six, zero, nine, a clean gong
symbols inviting us to love.

From zero to nine
the spiral turned over,
from zero to nine
I dared to love others.

From zero to nine
I attempted prayers,
from zero to nine
I became a repairer.

From zero to nine
infinity flowed free,
from zero to nine
unity grew in me.

Six, zero, nine, a dear song
numbers unfolding daylong,
six, zero, nine, a clean gong
symbols inviting us to love.

From nine to nine
weaving my reality,
from nine to nine
dreaming its totality.

From nine to nine
conquering my greed,
from nine to nine
planting a new seed.

From nine to nine
despite a clear spite,
from nine to nine
knowing there is light.

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Page 76. Thank you very much for your presence. For additional information, please refer to the two books *The Fig Tree & The Bell* and *The Hypotenuse*. 