On the Nature of Equilibrium

- Carlos E. Puente

**Abstract:** Searching for equilibrium and its implied contentment is one of the most instinctive and fundamental tasks we human beings perform in our lives. This is particularly difficult in this day and age when the “turbulent forces” of modernity induce a fast pace of life that hinders our ability to be fully attentive to one another and to ourselves. During the past few decades a host of ideas have been established in order to study natural complexity, and in particular the one produced by turbulence. This work explains how such modern notions help us visualize the essential options we all face regarding equilibrium and shows how such ideas point us to one and only one serene state in which we all may achieve real peace. It is argued, citing a variety of Biblical passages, that such a desired condition may be approached via the dynamic practice of humility, repentance and love, in a manner that is epitomized by the teachings of Jesus Christ.

**Key Words:** Equilibrium, Wholeness, Fragmentation, Turbulence, Unity.

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Introduction

According to the Webster’s New Collegiate Dictionary, equilibrium is “a state of adjustment between opposing or divergent influences or elements” and the closely related concept of balance is an “equality between the totals of the two sides of an account.” Based on such notions, it then appears natural to define “peace” as a balanced condition in which opposites cancel, as described graphically via the famous symbol of the Yin and Yang expressing an equilibrium between “day” and “night,” and more generally between “good” and “evil.” But, is such a general condition one that truly defines peace?

This article explores matters of equilibrium and peace by considering the opposite concepts of wholeness and fragmentation, as they arise in the study of natural turbulence. Arguing that science provides a suitable and impartial framework to reflect on our internal peace and on the nature of our interpersonal relations, including the political systems that have governed the world, this work shows that there is indeed one and only one state, defined by an improbable absence of fragmentation, that qualifies as truly balanced and which corresponds to a serene and universal condition that begets peace. It is also explained how achieving such an ultimate and desired state of wholeness is intimately related to fully heeding the teachings of Jesus Christ, including, in particular, his repeated calls for our continuous and dynamic growth to humility and repentance and our universal fulfillment of the defining maxim “love one another.”

The Breaking of Wholeness

To set the tone, it is relevant to start with a simple game that may be easily understood by molding modeling clay, as follows. Start the game with a uniform bar of clay, as it comes out of the box, and cut it by its \( p = 70\% \):

![Image of dividing clay into 70% and 30% pieces](image)

Then, mold the two pieces, piling up the one on the left and stretching the other on the right, both towards the middle, so that they make up, at the end, two contiguous bars of equal horizontal length:

![Image of two bars of equal length](image)

The height of the piece on the left is higher than the original, and the height of the one to the right is smaller. If the original bar has a height of one vertical unit, then the “rectangle” on the left has \( 2 \cdot p = 1.4 \) vertical units, for its area, i.e., the mass of such a piece, computed multiplying its base \( 1/2 \) times its height, is \( p = 70\% \) of the original. Similarly, the height of the rectangle on the right is \( 2 \cdot q = 0.6 \) vertical units.

With this process fully understood, repeat it on each of the two uniform bars, with exactly the same proportions, to obtain four pieces of horizontal size \( 1/4 \):
Now the tallest rectangle has grown to a height of $(2 \cdot p)^2 = 1.4^2 = 1.96$ vertical units and the masses (areas) of the four pieces are, in order, 70% of 70%, 30% of 70%, 70% of 30%, and 30% of 30%, which gives, multiplying, 49, 21, 21, and 9% of the total mass. As may be verified, such values correspond to the familiar expansion of $(p + q)^2$: $p^2$, twice $p \cdot q$ and $q^2$.

Clearly, as the process is repeated again and again, additional fragmentation is produced:

When the breaking continues for a total of $n$ levels, the process defines $2^n$ rectangles all having horizontal sizes $1/2^n$, whose masses, adding always to 100%, turn out to correspond to the expansion of $(p + q)^n$.

This process, properly known as a multiplicative cascade, defines $(n + 1)$ layers for the masses of the rectangles: $p^n$, $p^{n-1} \cdot q$, $\cdots$, $p \cdot q^{n-1}$, and $q^n$. As may be observed, such layers possess distinct densities as follows: there is one rectangle having $p^n$ of the mass and one having $q^n$, there are exactly $n$ rectangles with $p^{n-1} \cdot q$ and $p \cdot q^{n-1}$, $n \cdot (n - 1)/2$ pieces with $p^{n-2} \cdot q^2$ and $p^2 \cdot q^{n-2}$, and so on, according to the well known Pascal’s triangle.

When $n = 12$, the following set of 4,096 rather thin and hence spiky rectangles is found:

This set is not drawn to scale, for its largest rectangle has $1.4^{12} = 56.69$ vertical units, a quantity that keeps on growing as the cascade continues.

As may be seen, the divisive and rather simple process utterly destroys wholeness into dispersed thorns that exhibit an increasingly intertwined structure due to the progressive pile ups and stretches. Although the mass is always conserved in the game, “moving” from thorn to thorn becomes increasingly difficult, for, as the cascade proceeds, rectangles of equal size seldom remain positioned in a contiguous fashion.

To further appreciate the rather “empty” structure generated within each layer by this simple cascade, it is convenient to introduce yet another simple game, as follows. Start as before with a uniform bar of modeling clay but this time cut by the middle:
Now, separate the two pieces piling them up left and right, in such a way that their horizontal 
lengths equal 1/3 and such that each piece contains 50% of the original mass:

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Following arguments as before, the two rectangles, having a gap between them, end up 
having a common height of 1.5 vertical units.

As with the first game, this process continues on repeating the fragmentation on each 
piece, cutting and separating according to the same proportions:

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This game clearly yields another divisive *multiplicative cascade* that produces, after \( n \) 
levels of the construction, \( 2^n \) dispersed rectangles having equal masses of \( 1/2^n \), horizontal 
lengths \( 1/3^n \), and diverging heights that grow according to the formula \((3/2)^n\):

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As the process is repeated, this second game clearly generates spikes that never touch. 
Such are sparse “thorns” that grow to infinity and that emanate from an infinite but quite 
meager set of points, which by lacking any cohesion whatsoever may be properly described 
as *dust*, a prototypical *fractal* set as introduced by George Cantor in 1883. Remarkably, in 
the ultimate limit all the infinitely many infinite spikes turn out to contain no mass at all 
and the same happens to be true for the first game, but altogether all the mass is there. 
Although by construction both cascades conserve the total mass of the original bar, they 
both have the final effect of powerfully *scattering* the mass into a multitude of *thorns*.

It happens that varying the size of the hole on the second cascade, i.e., from \( 1/3 \) to a 
generic value \( h \), captures the non-contiguous topology, i.e., empty and fractured, of the layers 
that the first cascade generates. While less dense layers, towards the periphery of Pascal’s 
triangle, correspond to larger gaps, the more dense layers happen to require smaller and 
smaller holes.

The moral of the story is that the two divisive games are intimately related to one another, 
as the second one lives inside the layers generated by the first one. While the second cascade 
divides the bar of clay into an infinite set of equal *spikes* having ultimately infinite sizes 
and supported by one set of *dust*, the first cascade yields intertwined “*multi-thorns*” over a 
multitude of disjoint *dusts* (one per layer). As such dusts reflect fragmentation and as all of 
them are fractal sets, the multi-spiky object generated by the first cascade is properly known 
in the physics literature as a *multi-fractal* measure.\(^5\)
The Devil’s Staircase

To further appreciate the thorny objects generated by both games and as all spikes ultimately grow without bound, it is convenient to portray their accumulated masses, as such are traversed from the beginning, i.e., 0, up to a point $x$, and as a function of $x$:

As illustrated first for the cascade with gaps, the “wealth” up to point $x$, $W(x)$, simply gives (following the dynamics of the game): $W(0) = 0$, $W(1) = 1$, $W(1/3) = 1/2$, $W(2/3) = 1/2$, and also for any value of $x$ in the main gap, $W(1/9) = 1/4$, $W(2/9) = 1/4$, and so on. At the end, $x$ in the horizontal vs. $W(x)$ in the vertical yields plateaus wherever the original spiky object had holes or gaps:

It happens that such a profile is quite peculiar, for if we were to parachute on it from the top, upon landing it would appear to us that such is flat:

If our “scale” is small enough, we would touch down on a plateau with all likelihood and we would falsely believe to have landed in the original bar. Due to such a deceit and given the divisive nature of the generating cascade, such a continuous and rough boundary (i.e., one without derivatives in great many places) was properly named by George Cantor in 1883 as the devil’s staircase.
As such a stair contains only horizontal or vertical steps (seeing any inclined lines above is just an illusion due to the resolution of the graph) the length from top to bottom following such a jagged curve equals 2 units: one horizontal unit, for there are eventually gaps everywhere, plus one vertical unit, as the cascade conserves all the mass. This property happens to be \textit{universal}, for the second game always defines a devil’s staircase with maximal length of two units irrespective of the size and position of the initial hole.

For the first cascade the accumulated wealth may also be calculated following the dynamics of such a divisive game:

![Diagram](image)

For instance, after two levels: $W(0) = 0$, $W(1/4) = 0.49$, $W(1/2) = 0.7$, $W(3/4) = 0.91$, $W(1) = 1$, and so on later on.

This process turns out to yield another devil’s staircase of maximal length and shaped as a “cloud of dust.”

![Diagram](image)

As may be seen, this is a rather jagged boundary which, by being locally horizontal, results in a maximal length of two from top to bottom.

As with the second game, this length property turns out to be also \textit{universal}, for the first cascade produces a continuous and ever rough boundary made of horizontal-vertical notches everywhere (and hence without derivatives anywhere), irrespective of how the division is carried out. If there is any imbalance $p \neq 1/2$, no matter how small, such a cascade invariably grows thorns separated in layers over dust, and such give wealth stairs whose lengths are always equal to 2 units.

\textbf{Turbulence in the Air}

Recent technological advances have allowed identifying the progressive breaking given by the first cascade in the way \textit{turbulence} happens in the air:
When the air’s inertia exceeds the internal cohesion of the fluid, i.e., when its Reynolds number, \( R = v \cdot L/\nu \), is large, the air flows in an irregular and intermittent fashion. As it happens with other fluids, when the viscosity \( \nu \) is overpowered by excessive energy, as measured by the product of the flow’s velocity \( v \) and a characteristic length \( L \), the air no longer can flow as a whole and in a “laminar” and calmed way, but rather splits into inwardly rotating elements called eddies that themselves split into other eddies and carry distinct energies in a “turbulent” and often violent fashion.\(^6\)

As first suggested by pacifist Lewis Fry Richardson in 1922, a cascade indeed forms and the spiraling sets carry with them distinct amounts of kinetic energy. What is observed, via experiments along one dimension in space, turns out to be consistent with the first multiplicative cascade, for the eddies of successively smaller sizes arrange into energy layers that correspond precisely to the 70-30 splitting used in the explanations before. As the process starts and evolves, energies rearrange in layers, but the ultimate fate of the breaking is always the eventual dissipation of the kinetic energy in the form of heat, when the eddies reach a small enough scale.\(^7\)

Remarkably and as first reported by Charles Meneveau and Katepalli Sreenivasan in 1987, the crossing of the Reynolds’ threshold in the air produces just a rearrangement of the generic cascade, with the more energetic eddies happening not always to the left but either to the left or to the right, as guided by chance.\(^8\) For a variety of flows, including atmospheric turbulence and boundary layer turbulence, there is a truly excellent agreement between observations and the first cascade, hence establishing the presence of a quantized (layered) distribution of energies in natural turbulence. The expected duration of the process, as defined by the number of levels in the cascade, increases with the Reynolds number and the strength of the associated violence of turbulence also increases as the energies concentrate into thorns having smaller and smaller (horizontal) sizes.

**Speaking about Peace**

As one ponders the surprising results herein regarding natural turbulence, their striking simplicity and universality hint that it is sensible to employ the cascade notions to model how we humans create our own “turbulence” and to study how such a condition may be avoided in our lives. After all, we all, from Afghanistan to Zimbabwe, are faced with “inertial forces” that often break our “internal cohesion” and, when such happens, “crossing our Reynolds’ threshold” leads to “intermittent behavior” and to an inherent “lack of peace” that often results in the appalling expression of violence. As our distress (and certainly mine) many times is associated with the relentless repetition of a divisive trait, it also appears reasonable
to employ the two general cascades in order to symbolize the pathways that lead us (me) to “bite the dust.”

The general validity of the notions and the inherent repetition of patterns at distinct resolutions, i.e., self-similarity, also suggest that the simple geometric ideas may be used figuratively to capture our “distress” at a variety of scales: within ourselves, in our relationships, in our societies, and in the world at large. For in a vivid sense, the two cascades provide accurate depictions that allow us to contemplate the most common ways we employ to propagate division in the establishment of imbalances and discriminations:

But in matters of peace and love there is a escape however. When the first game is played always by the middle, i.e., precisely at $p = 1/2$, and the second game is performed without holes, i.e., for $h = 0$ at all levels, then there is no division whatsoever and the wholeness and unity of the original bar is maintained in a dynamic way:

In such an improbably perfect case, the wealth frontier simply gives a straight ramp having minimal length of $\sqrt{2}$ from top to bottom, for in such a case $W(x)$ is simply $x$ when $x$ is between 0 and 1. In this case, landing in the “one to one line” does not keep a parachuter stuck on a plateau or a notch as with the devil’s staircases before but rather sleds him or her to the bottom of the ramp, at the intersection of the axes defining the origin:

As may be appreciated, while the maintenance of wholeness yields a wealth function that travels through the hypotenuse of a triangle, the practice of a divisive game results in a jagged devilish stair that is eventually as long as the legs of the triangle. Based on these observations, there is only one case, only one truly balanced and whole, that may be used to properly define equilibrium and that corresponds to us not playing the divisive games at all.
The best state is simple and corresponds to the plain original bar on both games. Clearly, the serene keeping of low Reynolds numbers avoids the rush of modern life and its associated distress and, in the absence of violent thorns and deadly dust, allows us also to contemplate the sought condition of ultimate peace.

This desired and joyous condition turns out to be rather improbable, as we all know, and this fact may be fully appreciated via the cascades as follows. When the two divisive games are combined to produce additional cascades containing both imbalances $p$ and holes $h$ at the same level, other more exotic sets of thorns over dust and subsequent devil’s staircases are produced. It then happens that equilibrium is only one point within a square of possibilities, for only at such a precise location the distance on the wealth frontier is $\sqrt{2}$ from top to bottom while any place else it equals 2:

![Graphical representation of equilibrium condition]

But the uniqueness of equilibrium is yet more striking, for the original bar may be split into more than two pieces and the cascade games may be played aided by chance, i.e., using variable imbalances and holes as the process is carried, and such a general mechanism, that no doubt captures even better the intrinsic variability of our own follies, would also result in thorns over dust and in devil’s staircases.

**Talking about Politics**

In the spirit of the associations thus far, the cascade notions may also be used to vividly expose the inherent fallacies in the political systems that have governed the world, as follows. While the first game allows us to visualize the proliferation of economic imbalances present in capitalistic societies, the second game allows us to appreciate the increased fragmentation and isolation caused by totalitarian regimes:

![Graphical representation of political systems]

These associations may be corroborated not only from common sense but also from our recent history. One the one hand (even if on the right above!), the unexpected but rather predictable failure of “equality by force” due to the induced fear and mistrust citizens
experience realizing that they may “end up in a gap,” a generalized attitude that surely dissipates the loving energy of friendship that may truly sustain us as human beings. And on the other hand (even if on the left), the apparent triumph of the false dogma of “the survival of the fittest.”

which, despite multiple claims to the contrary, also grows thorns and dust via the implementation of dehumanized competition inspired by the implacable and selfish quest “to be number one.”

The ideas herein turn out to have an unexpected validity beyond the merely metaphorical, for the multi-fractal object generated by the first cascade, for twenty levels and for $p = 0.7$, i.e., with exactly the same energy partitions from level to level as found in nature, closely matches the rather skewed distribution of wealth of the most powerful nation on earth. As may be readily verified, simple calculations via Pascal’s triangle reveal a remarkably close fit of the wealth of the richest 5, 10, 20, and 40% in the United States as reported in 1998, that is, in order, 59 (57), 71 (70), 84 (84), and 95% (95) of the resources, with the cascade values given in parenthesis.¹¹

Although the wealth of the richest 1% is underestimated by the simple cascade, 38% (30), these remarkably simple results help us visualize the dreadful consequences of establishing and further propagating imbalances and vividly invite us to reverse the trends in order to find true prosperity in the serene and truly humanized condition of equilibrium.

For even if experts, who typically use finite numbers to describe us, assure us that the practice of globalization shall bring justice and well-being to all, and especially to the 2/3 of humanity living under extreme poverty,¹² and that there are better uneven wealth distributions than others, the universal cascade notions and its devilish stairs remind us of our historical evil and greed that point us to the only point of peace where “competing against one another” or “dismissing one another” may truly be replaced by “loving one another.”
The Faithful Solution

Division may indeed be defeated if we humans learn from the nature of turbulence and run all cascades in reverse in order to heal our “fractal” world:

This entails heeding the voice of prophets in their explicit calls to rectification, such as the rather geometric exhortations by John the Baptist: “I am the voice of one crying out in the desert, make straight the way of the Lord” (Jn 1:23, Hos 14:10) and “every valley shall be filled and every mountain and hill shall be made low; the winding roads shall be made straight, and the rough ways made smooth” (Lk 3:5, Is 40:4), that clearly points us away from the cascades yielding scattered thorns, dust and devil’s staircases and into the 50-50 and no holes condition and its associated one to one ramp.

Running the natural cascade in reverse means to move away obediently from our transgressions and into the rectitude of God’s law. It means listening to pleas of repentance (Mt 4:12) to ultimately arrive at the “broad valley” in which the glory of God is revealed to all mankind (Is 40:5), that is, to encounter Jesus Christ himself who, by never sinning, always maintained the “level ground” of the “original bar” properly symbolizing the law (2 Cor 5:21, Jer 31:9, Mt 5:17).

As hinted from the diagram above, the turbulence associated with sin and dust (Gn 3:19) disappears in the process of conversion. This is consistent with several ancient calls for us to be “calm” and to “trust” in God (e.g., Is 30:15), which point us away from the high Reynolds numbers of the divisive cascades in our lives. For even if a bit of turbulence may be welcomed when it shapes our characters, it is best to avoid its ultimate consequences lowering the “velocity in our lives,” diminishing our “characteristic lengths” and augmenting our “viscosities.” For as described by Jesus himself in his explanation of the central parable of the sower, the daily thorns of anxiety and the lure of riches are the ones that choke the planted word of truth in us and the presence of such prevents us from being truly fruitful (Mt 13:22).

The uniqueness of equilibrium within the cascade notions help us appreciate that there is indeed only one solution that avoids distress, and that is, once again, Jesus Christ who appropriately defined himself as “the way and the truth and the life” (Jn 14:6). Notice how his words “Whoever is not with me is against me, and whoever does not gather with me scatters” (Mt 12:30) may be fully appreciated geometrically in the absence or presence of thorns and dust and how his exhortation “come to me, all you who labor and are burdened, and I will give you rest; take my yoke upon you and learn from me, for I am meek and humble of heart; and you will find rest for yourselves; for my yoke is easy, and my burden
light” (Mt 11:28-30) may only be fully grasped in the simplicity of equilibrium. For there is no fear of punishment in walking the original bar (1 Jn 4:18), unlike what happens with the thorny cascades that lead to frightful steps that make us stumble (Sir 39:24).

The universal message of Jesus Christ turns out to be vividly present in the notions of modern science herein. Quite beautifully and besides the associations thus far, Jesus is also symbolized by the radical concept of the root and by the shortest and simplest straight line in the hypotenuse, which by its graphical equation, \( X = Y \), represents him in the very geometry of the cross and him crucified on it! For only by parachuting into the straight ramp one may slide into the origin, in consonance with his words “no one comes to the Father except through me” (Jn 14:6).

Our Evil World

The divisive cascades may be used to portray graphically the dynamics of the works of the Devil, the one always divisive and selfish, for as Scripture explains: he has sinned from the beginning (1 Jn 3:8) and there is no truth in him at all (Jn 8:44). His appellatives “the ruler of the power of the air” (Eph 2:2), “the evil spirits in the heavens” (Eph 6:12), and “the ruler of the world” (Jn 12:31), further relate the “ancient serpent” (Rev 12:9) to turbulence in the air and to the spawning of eddies leading to dissipation, to the violence and dust present in the universe as discovered by modern astronomy, and to the ever present economic Devil’s staircases that plague our world in contraposition to God’s will (Lk 16:13).

In regards to the ever symbolic dust and thorns, it is pertinent to recall first the very explicit decrees of God right after the fall of Adam and Eve. To the deceiving serpent He said “dirt shall you eat all the days of your life” (Gn 3:14). And to Adam He decreed the famous death edict “you are dirt, and to dirt you shall return” (Gn 3:19), also commonly translated with dirt exchanged by dust, not before explaining that the land he would now toil outside of paradise would bring forth “thorns and thistles” (Gn 3:18).

The cascades not only may denote the Devil but also his followers, the so-called wicked. This fact is found on several citations that relate the sinner with turbulence. As an example and quite graphically, “the hope of the wicked is like thistledown borne on the wind, and like fine, tempest-driven foam, like smoke scattered by the wind” (Wis 5:14), and “the arrogant shall be like fine dust” (Is 29:5). For “all sinners will be destroyed” (Ps 37:38) and they shall lick the dust like the serpent” (Mi 7:17), for “when they sow the wind, they shall reap the whirlwind” (Hos 8:7). Other turbulent and fractal traits in the wicked are also found in, “pride adorns them as a necklace, violence clothes them as a robe. Out of their stupidity comes sin, evil thoughts flood their hearts” (Ps 73:6-7) and in “Ah, all of them are nothing, their works are nought, their idols are empty wind!” (Is 41:29).

As may be verified, turbulence is also a common punishment of the sinner by God. For instance, “then shall they know that I am the Lord, when I disperse them among the nations and scatter them over foreign lands” (Ez 12:15, Zec 7:11-14), as it happened to the people of Israel, when they disobeyed Him. This is also seen vividly in the edict, “a third of your people shall die of pestilence and perish of hunger within you; another third shall fall by the sword all around you; and a third I will scatter in every direction, and I will pursue them with the sword” (Ez 5:12), which evokes the original construction of Cantor dust.
In the progressive division of both cascades we may recognize selfishness and greed as the guiding principles. As natural eddies always run inwardly, always from more or plus to less or minus as on a destructive hurricane, in their directionality we may appreciate the famous and symbolic 666 assigned to the false antichrist to come (Rv 13:18):

This is because the shape of a natural eddy is a negative spiral given by the equation (in polar coordinates) \( r = e^{-\theta} \) that corresponds pictorially to such a number. In the spirit of these associations, we may also realize that the fraction \( 2/3 = 0.666 \cdots \) (with 0 denoting equilibrium), ever present in the divisive cascades and curiously corresponding to the number of poor people on earth today, may be associated with the infinite deceit of the devil himself.

But the devil is fully defeated as Jesus rebuked his scary winds into immediate calmness (Mk 4:39), resurrected as predicted from the dust of death (Mk 16:6) and shall ultimately kill him when He comes (2 Thes 2:8), even if, in a archetypical fashion for us all, the apostle Peter denied Jesus precisely three times before the cock crowed twice (Mk 14:66-72).

**Our Ultimate Unity**

The notorious difference between \( \sqrt{2} \) and 2 may be used to further illustrate the goodness of forgiveness. For “as far as the east is from the west, so far have our sins been removed from us” (Ps 103:12). Reconciliation with God is hence an exceedingly valuable sacrament, for “if we acknowledge our sins, He is faithful and just and will forgive our sins and cleanse us from every wrongdoing” (1 Jn 1:9), as also expressed in “The Lord’s Prayer” (Mt 6:9-15).

The difference between the shortest path and the others also permit us to appreciate the subtleties of sin and the beauty of God’s plan of salvation for us. For the inefficiencies in cascades, and especially by combining them, help us value Jesus’ words: “why do you notice the splinter in your brother’s eye, but do not perceive the wooden beam in your own eye?” (Mt 7:3), that fully corroborate his famous maxim “it is easier for a camel to pass through the eye of a needle than for one who is rich to enter the kingdom of God” (Mt 19:24):
For uniformity and wholeness are attained only by obeying God’s plan to the positive cross (Phil 2:8), forgiving those who offend us seventy times seven times as consistently allegorized in the second level of the natural cascade (Mt 18:22), and fully surrendering ourselves to obey God’s plan at low Reynolds numbers in order to find the point that leaves out the hypocrisy that prevents us from truly helping one another (Mt 7:4-5, Jas 2:10).

The uniform condition through its dynamic constancy allows us also to visualize and value unity, within ourselves, in our interpersonal relations, in our societies, countries, and the world at large. In this context, the original bar denotes united societies, “aligned” with God’s plan, while, as explained earlier, dusty cascades represent societies based on imperfect human systems, Devil’s staircases that generate emptiness with their maximizing of personal profit or with their forced and false equalities.

Clearly, unity is essential in the teachings of Jesus Christ, for in particular He said “I pray not only for them, but also for those who will believe in me through their word, so that they may all be one, as you, Father, are in me and I in you” (Jn 17:20-21). Such a lovely condition is found on several other passages that include welcoming one another as Christ welcomed us (Rom 15:7) and rejoicing with those who rejoice and weeping with those who weep (Rom 12:15). For in fact, as the psalmist exclaims, “how good it is, how pleasant, where the people dwell as one!” (Ps 133:1).

This desired state is also reflected in Jesus’ famous words about marriage, when He succinctly explained, “they are no longer two, but one” (Mt 19:6), surely in them being “covered” by a common root as in √2, and also in the elusive unity of the Church as explained by the apostle Paul, “for as in one body we have many parts, and all the parts do not have the same function, so we, though many, are one body in Christ and individually parts of one another” (Rom 12:4-5, 2 Cor 8:8-15). These last citations suggest a geometric rather than an arithmetic relation for both marriage and the Church: “1 = 1 + 1” and “1 = 1 + 1 + . . . + 1,” that reflect, in the latter case, Jesus’ prophetic words about the ultimate marriage with Him when He says, “there will be only one flock, one shepherd” (Jn 10:16).

As depicted graphically in the diagrams in this work, only in the uniform condition and its associated hypotenuse we may find the proper radical state that yields unity. This leads us, by repairing the breach (Is 58:12), to the positive “spiral of love,” i.e., \( r = e^{+\theta} \), (with a positive internal derivative rather than a negative one) and to the simple and eloquent equation \( 1 = 0.999\ldots \) that depicts the attaining of unity in the admission and practice of love, as epitomized, once again for us all, by the three times the apostle Peter acknowledged Jesus after his resurrection (Jn 21:15 17):
For Jesus’ disciples are to be recognized by their love for one another (Jn 13:35), and this unnatural condition always travels from minus to plus, i.e., to the cross, and grows in us by the practices of humility and of service to one another (e.g., Phi 2:3, Rom 12:3, Mt 20:26, Mk 9:35).

Quite accurately given the ultimate dissipation in natural cascades, there is darkness between our selfish postures and our loving ones, as there was, quite infallibly, a prescribed eclipse of the sun between the 6th and the 9th hour when Jesus, the Lord, was crucified for us (Mk 15:33-37) and crowned by our many thorns (Mk 15:17).

Unwinding the Yin and Yang symbol while recognizing not a false balance between good and evil but the triumph of love via Jesus’ resurrection, we may arrive at the following poetic summary for this work:}\(^\text{10}\)
Six, zero, nine, a dear song
numbers unfolding daylong,
six, zero, nine, a clean gong
symbols inviting us to love.

From six to six
revolving inwards,
from six to six
I went downwards.

From six to six
dividing selfishly,
from six to six
lying endlessly.

From six to six
trying to be a rose,
from six to six
being only a nasty thorn.

From six to zero
I tapered my speed,
from six to zero
the tempest did not lead.

From six to zero
I no longer postponed,
from six to zero
I finally atoned.

From six to zero
I experienced peace,
from six to zero
my loneliness ceased.

Six, zero, nine, a dear song
numbers unfolding daylong,
six, zero, nine, a clean gong
symbols inviting us to love.
Notes:

1. This work is warmly dedicated to María Helena Buitrago, Teresa Latorre and Aurora Solano for their unconditional example of service and love.
2. Carlos E. Puente lectures at the Department of Land, Air and Water Resources, University of California, Davis.