

Faith Lessons from Chaotic Fig Trees

Carlos E. Puenté

OMEGA

INDIAN JOURNAL OF SCIENCE AND RELIGION

Vol.8 No.2 December 2009



Institute of Science and Religion

Little Flower Seminary, Aluva - 683 101, Kerala, India.

Phone : 0484 2623437, 2626204

E-mail : israluva@gmail.com

URL: www.lfseminary.org/htm/omega.htm

Contents

The Editorial	3
Articles	
Evolution: Science and Meta-science - An Introduction <i>K. Babu Joseph</i>	7
Can a Christian Believe in Darwin's Evolutionary Theory? <i>Sarojini Henry</i>	18
The Sacramental Character of an Evolving Creation <i>Peter M. J. Hess</i>	34
Human Origins and Original Sin in an Evolutionary Context <i>Patrick McDonald</i>	58
Science, Religion and Darwin: A Contemporary Appraisal <i>Kuruvilla Pandikattu</i>	90
Creation in Evolution <i>Francis P. Xavier</i>	110
Creation Narratives and Modern Cosmology: Are they at Loggerheads? <i>C. D. Sebastian</i>	129
Faith Lessons from Chaotic Fig Trees <i>Carlos E. Puente</i>	157



The Editorial

At the beginning of the year 2009 which marked the bicentennial of Darwin's birthday and sesquicentennial of the publication of his epoch-making work *The Origin of Species by Natural Selection*, *Omega* had published a couple of articles attempting a contextual discussion on the evolutionary theory from philosophical and theological points of view. Now, at the closure of this unique year of evolution, *Omega* is pleased to dedicate a series of articles exclusively dealing with a positive and explorative dialogue between evolution and religious belief. Apart from the constant subservience by the scientific community, what has sustained the evolution theory as a hot academic topic is its overarching existential outreach, or its imposing appeal to the horizons of humanity's meaning-world. The philosophical and religious ramifications of this theory have been so foundational that the proponents and opponents of this theory were destined to wrestle with it in one way or the other. The 18th and 19th century intellectual and religious world had difficulty in accepting Darwin. And even now many thinkers are devising ways to domesticate or ignore it. Andrew Dickson White had stated that theory of evolution was something like a plough into an anthill, annihilating the very delicate fabrics of religion and faith. As for Richard Dawkins, the pitiless indifference of the natural selection enables one to be an "Intellectually Fulfilled Atheist." Daniel Dennet found the impersonal, algorithmic, blind, purposeless processes of the universe of the evolution to be too dangerous an idea for believers to defend their faith.

Owing to the plethora of theological and philosophical accommodation that the theory of evolution has enjoyed over the decades, the voice of these ultra Darwinists is now almost a cry in the wilderness. Atheism on Darwinian grounds, now, rather than yielding to intellectual fulfillment, testifies to the intellectual bankruptcy, philosophical mediocrity, epistemic fixation, and the sabotage of almost every logical axioms and

rational standards. These claims will be substantiated by the six articles of this volume on evolution.

The opening article by K. Babu Joseph, having presented the most up-to-date features of the Darwinian formulation of the theory in terms of natural selection and its validation in the wake of developments in genetics, challenges the stereotyped image of the science-religion conflict on evolution. Evolution theory has the limitation of not being able to explain the advent of life, justifying indulgence in meta-scientific reflection. Making use of a picture of God as Energy and Information or Probability, he opens up a novel way of reconciling evolutionary and religious perspectives on creation.

In spite of the many controversies that Christian faith has to face, some eminent theologians now argue persuasively that Christians can indeed, with integrity, connect their faith in God with a scientific understanding of evolution. They point out that evolutionary science can actually be taken as an invitation to enlarge our sense of the divine. Sarojini Henry examines how theologians like Teilhard de Chardin, Wentzel van Huyssteen and John F. Haught maintain that the theory of evolution exposes a fresh view of the reality of the sacred and an astoundingly meaningful universe.

Peter M. J. Hess highlights the sacramental value of a dynamic and evolutionary world. Post-modern Christians need to absorb the reality that we inhabit a very ancient, dynamic, and evolving universe. The sacramental value of the world can be rediscovered in an evolutionary matrix. He argues that the incarnational perspective serves both as the zenith point of the inherently sacramental character of the universe and of the unity of knowledge.

Patrick McDonald's paper is a theological reflection on original sin in the context of evolution. Although there is good reason to believe humans have evolved, and that this may raise very serious questions about one account of original sin, it does not impact the central dynamic of the emergence of sin in human history and the need for healing and redemption. Accepting an evolutionary account of human origins does not undermine belief in God, belief in Christ, belief in the Bible, belief in a soul, belief in the salvation of that soul, nor belief that the means for

such salvation have been offered to us. However, holding all of these together consistently and plausibly might require some rethinking of what these beliefs mean and how we understand the sources of such beliefs.

Kuruvillea Pandikattu focuses on the impact Darwin's theory had on contemporary society, including religion. After exposing the grand idea and the response it has generated in terms of admiration and awe, the author looks at the relation between science and religion specially among American scientists to study the impact of the theory of evolution on religion. Finally the plea is made for a reasoned and creative response by believers and scientists alike, which will respect the autonomy of both science and religion and at the same time plead for a creative interaction in shaping human understanding and the evolving society at the moment.

After considering the various theories of creation and evolution Francis Xavier reaches at a constructive hermeneutical synthesis where evolution-theory and creation-faith converge into 'creation in evolution.' Both evolution and creation are processes; the former, on the physical level explaining how one being becomes a higher being, whereas the latter stating where the dynamic force for becoming comes from. God is the initial dynamic force that set the evolutionary process in motion, as he created time, and He still continues the work in the universe - and this process we understand as evolution.

In addition, we have two other articles on regular themes dealing with the constructive interlacement between science and religion. C. D. Sebastian considers the creation narratives of the religions and the inflationary cosmology. In a way, they both propose the same pledge. Creation narratives inform us that this was the action of the first cause or creator who is called Supreme Being or God. There is a promising analogical convergence between the assumptions of modern cosmology and religious beliefs as far as creation narrative is concerned. In this sense, they are not at loggerheads.

Carlos Puente does a commendable hermeneutical job in suggesting directions for finding harmony amidst chaos. The search for harmony is one of the most pressing tasks we humans attempt and it is particularly difficult when the "evil" of "chaotic forces" propels us into restless and

often helpless states whose intrinsic disorder hampers our ability to find our way to peace. During the past few decades a host of ideas have been established in order to study natural complexity, including the identification of pathways that progressively degrade “order” into the specific disorder of “chaos” and that define a host of chaotic trees, as epitomized by the iconic Feigenbaum tree, or “fig tree” in German. This work explains how such notions help us visualize the essential options we all face regarding order and disorder and shows how the ideas point us to the straight roots of such trees as the only common ground where we all may achieve true order and peace.

The thrust of this entire volume, in general, is that the clichéd notion of science-religion conflict is now no news to any informed person. And, in particular, we emphasize that rather than tampering evolution with fixated and parochial soliloquies, an open and explorative approach can open up deep depositories of truth and knowledge at stake in it. The papers here argue that evolution should be sustained on the horizons of our religious and existential meanings for there is much more that has to be harvested from the hidden potentials of this theory.

-Augustine Pamplany
Guest Editor

Faith Lessons from Chaotic Fig Trees¹

- Carlos E. Puente²

Abstract: Searching for order and its implied harmony is one of the most pressing tasks we humans attempt during our lives. This quest is particularly difficult when the “evil” of “chaotic forces” propels us into restless and often helpless states whose intrinsic disorder hampers our ability to find our way to peace. During the past few decades a host of ideas have been established in order to study natural complexity, including the identification of pathways that progressively degrade “order” into the specific disorder of “chaos” and that define a host of chaotic trees, as epitomized by the iconic Feigenbaum tree, or “fig tree” in German. This work explains how such notions help us visualize the essential options we all face regarding order and disorder and shows how the ideas point us to the straight roots of such trees as the only common ground (i.e., “under the fig tree”) where we all may achieve true order and peace. It is argued, citing a host of Biblical passages, that the modern concepts provide a rich symbolism consistent with ancient Scripture that, in particular, allows us to further appreciate, in a strikingly coincidental fashion, why Jesus may have, seemingly out of character, cursed and withered a fruitless fig tree as he rebuked the wind (evil in of itself in both instances) and why He may have asked us to learn a lesson from a fig tree and other trees (even from those chaotic ones budding in science twenty centuries later) as a mysterious and yet urgent precursor to His second coming. The implications of the notions regarding our need to be always watchful, including our prescribed conversion by coming down our own “fig trees,” are emphasized.

Keywords: Wholeness, Fragmentation, Turbulence, Chaos, Peace, Love, Eschatology.

Introduction

According to the Webster’s New Collegiate Dictionary, *chaos* is “a state of utter confusion,” one in which “chance reigns supreme.”³ In the past few decades, however, modern science has established that there is *order* in the way some of such “randomness” arises and has revealed that *simple deterministic* mechanisms, that is, not dependent on the concept of chance, are unexpectedly sufficient to comprehend such a common form of disorder.^{4,6}

Among the pathways to chaos, the most celebrated has been the one defined via a *chain of bifurcations*, which reflects the progressive splitting of a system’s ultimate dynamics, in *powers of two*, before it reaches a totally disorganized state. Defining a new paradigm for studying complexity, such ideas resulted in applications in a variety of disciplines ranging from ecology to engineering and from chemistry to physics, which included notably, in the latter, the elusive description of the dynamics of *convection*, that is, the eventual turbulent boiling of a fluid as its temperature is increased. Central to the discoveries is also the springing of a host of diagrams, reflecting all possible scenarios from *order* to *chaos* and shaped as *trees*, that share surprising *universal* properties, as first proven in the iconic *Feigenbaum tree* (“fig tree” in German) by physicist Mitchell Feigenbaum.^{4,6}

This article explores matters of *order* and *disorder* by studying the opposite concepts of *wholeness* and *fragmentation*, *unity* and *dust*, and *rest* and *wandering*, as they arise in the modern study of chaos. As previously argued via multiplicative cascades arising in the study of turbulence,⁷ this work’s premise is that we as humans may learn from the ways of nature so that, by using our common sense, we choose to avoid the prescribed pathways that take us away from *peace* and into *chaos* and its related *turbulence*. Arguing that science indeed provides a suitable and impartial framework to reflect on our internal *peace* and the *peace* of our world, this work shows that there is *one* and only one state, defined by our dynamic *abandonment* to the *origin*,⁷ that qualifies as truly centered in *love* and that corresponds to a *serene* and *universal* condition situated at the *roots* of all chaotic (*fig*) *trees*, where we all may find *peace*. It is shown how achieving such an ultimate state is intimately related to pertinent matters of *faith*, as it concerns fully heeding

the teachings of Jesus Christ,⁷ including, in particular, His celebrated teachings via (*fig*) trees.

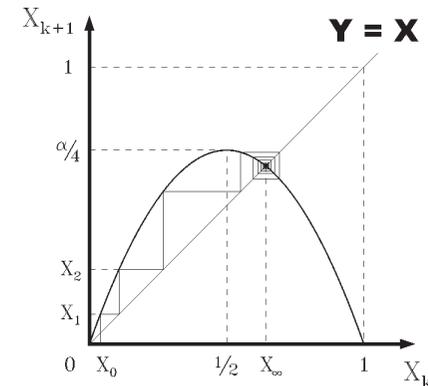
The Exquisite Dynamics of the Logistic Map

To fully appreciate the faith lessons herein, it is necessary to review first the fundamentals of chaos theory and its rather precise, and often allegorical, terminology. In order to do so, it is convenient to start first with the prototypical *logistic map* used in such studies,

$$X_{k+1} = \alpha X_k (1 - X_k),$$

where *X* is the size of a population, say “rabbits” (normalized from 0 to 1), *k* and *k + 1* are subsequent generations, and α is a *parameter* that may be any number between 0 and 4.

This rather simple *quadratic*, and hence *nonlinear*, equation, defines the *dynamics* of the population from one generation to the next. If such a map is plotted, from a generic generation X_k (in the horizontal) to the next generation X_{k+1} (in the vertical), the resulting graph gives a symmetric *parabola* whose peak value of $\alpha/4$ happens by the middle, that is, when $X_k = 1/2$:



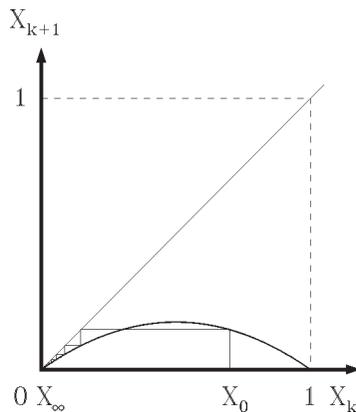
The graph above shows the evolution of a population ruled by such an expression, when the parameter α equals 2.8. As is seen, from a small initial population $X_0 = 0.04$, and following the vertical and

horizontal lines to the *one-to-one line* $Y = X$ to aid in the progressive calculations, the population grows from X_0 to X_1 to X_2 to X_3 , and to X_4 , and then spirals, after several repetitions, into a “*fixed point attractor*” $X_\infty = 0.64$, that corresponds to the non-zero intersection of the parabola and the straight line.

As from generation to generation the parabola provides an expected increase in population if there are few rabbits, but a logical decrease if there are too many of them, the repeated *iterations* of the *logistic map* gives rise to a *logical organization* of the population, and hence its name. Notice how the map is particularly severe regarding too many rabbits, for when their number equals its maximum value, that is, $X_k = 1$, the entire population extinguishes in the next generation, that is, $X_{k+1} = 0$, as one may imagine would happen due to extreme competition for limited resources.

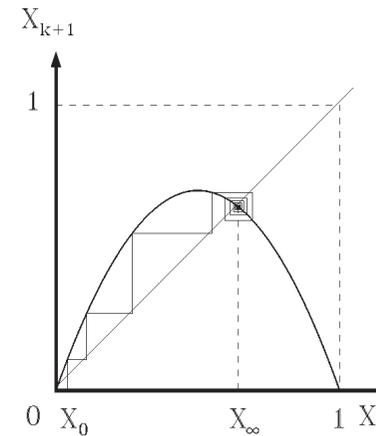
It happens that the population’s ultimate destination, X_∞ depends dramatically on the value of the parameter α , as is reviewed next.

When α is not above 1, the parabola is located *below* the one-to-one line:



and such a placement leads to a progressive decrease in population size, from an arbitrary value X_0 to $X_\infty = 0$, that is, the *origin*, as shown for $\alpha = 0.7$. As may be appreciated, irrespective of the size of the initial population, the iteration of the logistic map leads in this case to *extinction*, and hence such an ultimate fixed point attractor, that is, *zero*, is reached from everywhere and is termed *stable*.

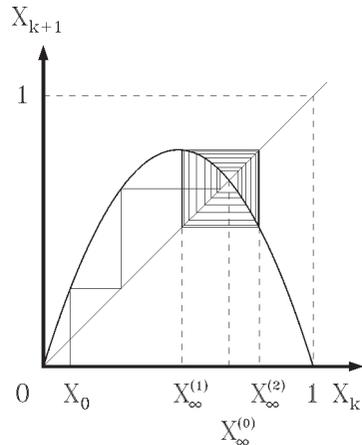
When α exceeds 1, the parabola crosses the *threshold* $Y = X$ and, as a consequence, the population no longer converges to zero, but rather *diverges* from the origin:



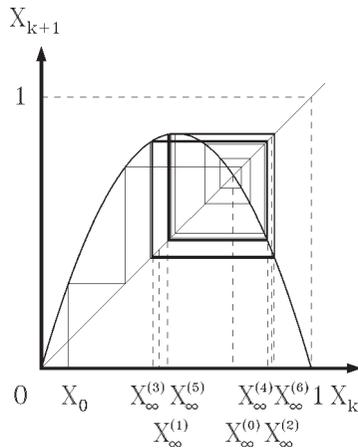
As is seen, once again for $\alpha = 2.8$, the *slope* of the parabola at *the origin* exceeds that of the *straight line*, and, as such, a small value X_0 yields larger values X_1 , X_2 , and so on, that move away from the origin and never return. As *the line* is crossed by the parabola, *zero* becomes a *repeller* and such a destination is termed *unstable*, as an arbitrarily small *initial condition* X_0 close to *zero*, would no longer go back to the origin.

When α takes any value between 1 and 3, the dynamics of the logistic map do converge to the non-zero intersection between the parabola and the straight line, which, from rather simple algebra, give $X_\infty = (\alpha - 1)/\alpha$. Such an attractor happens to be a *stable fixed point* as the slope of the curve at such an intersection is *mild*, that is, one that has magnitude less than one, and hence pulls in all non-zero and non-zero populations.

When α is greater than 3, the slope at the non-zero intersection between the parabola and the line also becomes too *steep* and, as such, what happened to the origin also happens to this intersection, that is, it repels. As shown for $\alpha = 3.2$, the dynamics now settle, quite surprisingly, into *stable oscillations* that repeat every two generations and that make a limiting square in the figure:



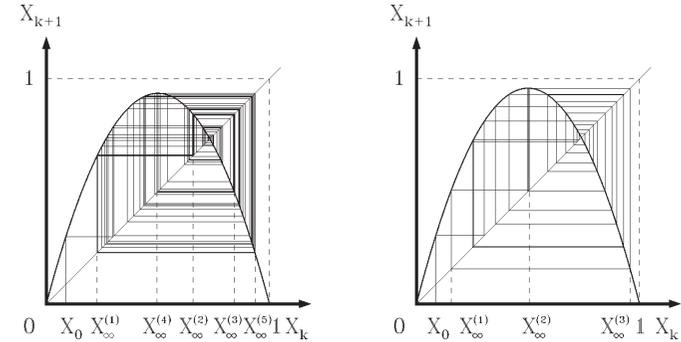
These *periodic* attractors, every *two* generations, happen for a range of values of α , but, as before, such behavior becomes unstable and repels if α is further increased. As an example, when $\alpha = 3.46$, oscillations every *four* generations appear:



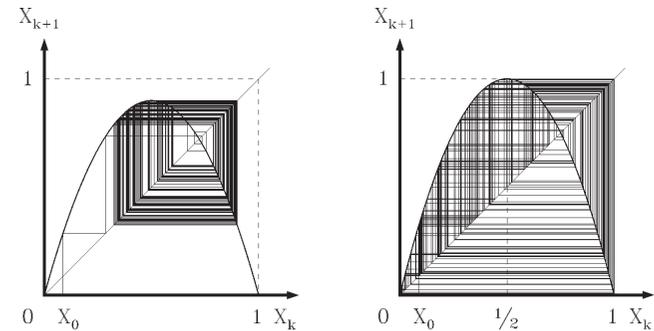
It so happens that as α is increased, a chain of *bifurcations* sets in, and the population remarkably oscillates in increasing powers of 2, first every two generations, then every four, then every eight, then every sixteen, and so on. It also turns out that the windows of stability for such oscillations decrease rather quickly, in such a way that periodic behavior, for all powers of 2, happen in the limiting value $\alpha_{\infty} \approx 3.5699$. Strikingly, the logistic map contains infinitely many *thresholds* transforming

attractors into repellers, and all of these happen via a rather simple quadratic equation just by varying the parameter α .

When α exceeds α_{∞} , there are sometimes *periodic* repetitions, as for values of α of 3.74 and 3.83 resulting in oscillations every five and three generations:



and, more commonly, dynamics that exhibit *non-repetitive* behaviors:



for values of α of 3.6 and 4, yielding attractors that contain *infinitely* many points, one set on the left bounded by two subintervals and the other apparently filling up the whole interval from 0 to 1.

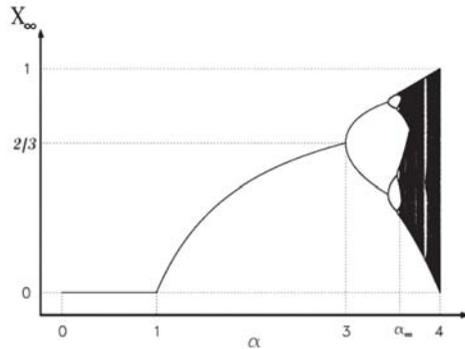
It occurs that there exist values of the parameter α greater than α_{∞} for which the population eventually repeats exactly every n generations, for any number n that is not a power of 2.⁸ As a consequence and quite remarkably, the logistic map includes periodic behavior for any natural number n . However, intertwined with this startling gamut of repetitive behavior, there are great many parameter values greater than

α_∞ for which the population does not repeat at all, but rather wanders forever following an unending dance inside an infinite set suitably known as a “strange attractor.”

These infinite but stable cases, due to their lack of finite convergence, define fittingly the mathematical notion of *chaos*. Such a name also turns out to be semantically accurate as two nearby populations, arbitrarily close, quickly *diverge* from one another when using the *nonlinear* logistic map, as if guided by chance. This extreme *sensitivity to initial conditions*, also known as the “*butterfly effect*,” explains why a small error, no matter how small and as proverbial as the flap of a butterfly’s wing, ultimately prevents us from knowing the precise evolution of a chaotic population.

The Feigenbaum Tree

A graph of the stable attractor X_∞ as a function of the parameter α is known as the *bifurcations diagram*:

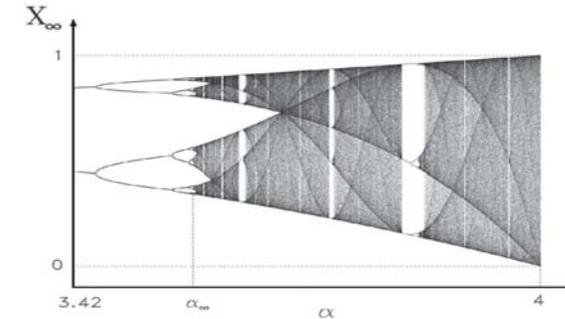


This remarkable figure is the most celebrated icon of chaos theory and shows how chaotic behavior is ultimately obtained via a chain of successive bifurcations. This diagram, when rotated counterclockwise ninety degrees, is the *Feigenbaum tree*, named after physicist Mitchell Feigenbaum who first proved some of its remarkable universal properties, as shall be reviewed later on.

As is seen, the straight *root* of the tree corresponds to the extinction of the population, $X_\infty = 0$, and there appears a positive *branch* or “*trunk*,” $X_\infty = (\alpha - 1) / \alpha$, when the parameter α is greater than 1 and

the parabola crosses the *threshold* $Y = X$. From $\alpha = 3$ onward, the chain of bifurcations gets established and such grows additional *branches* in increasing powers of two, but with progressively smaller lengths, so that all powers happen up to a value $\alpha_\infty \approx 3.5699$.

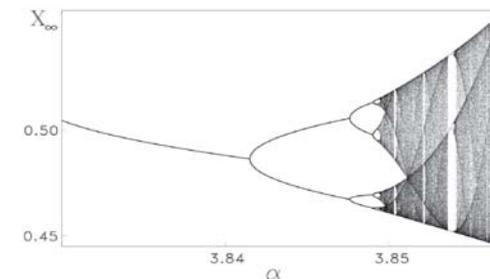
As is better seen on a magnified tail of the diagram:



from α_∞ onward, the tree contains additional periodic *branches* finely intertwined with chaotic behavior, whose infinite sequences of points in the associated strange attractors (plotted in the vertical for a given value of α) represent then the “*foliage*” of the tree.

As is seen, the tail of the diagram also exhibits several “white bands” that turn out to correspond to all periods not included in the bifurcations, with the most prominent being, from left to right, the ones related to repetitions every 6, 5 and 3 generations. As seen prominently in the band corresponding to period 3, the tree contains noticeable “*buds*” that link the tree upwards and that merit further consideration.

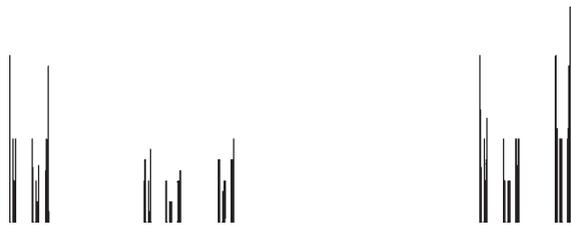
As is seen magnifying the middle bud within the white band related to period 3:



as white bands are traversed from left to right, the diagram grows yet more small *branches* and additional chaotic behavior. Strikingly, the operation results in a reduced copy of the entire tree (except the root), one that includes, in a rather remarkable fashion, bifurcation branches that *sprout* additional chaotic attractors unseen at the scale of the original graph, and yet more white bands associated with higher order periods.

As the magnification may be continued with no limit, it turns out that the Feigenbaum tree contains infinitely many *buds* that emanate from the infinitely many white bands associated with each one of the possible periodic behaviors found in the tail. Amazingly, the tree contains smaller copies of itself ad infinitum and such an extraordinary *self-similarity*, not discovered earlier than a generation ago due to intrinsic technological limitations, implies a rather deep form of *fragmentation* in the object.⁹

In order to further appreciate the degree of brokenness in the *chaotic fig tree*, it is sensible to study in some detail its first strange attractor, that is, the first infinite attractor that happens when $\alpha = \alpha_\infty$, and known as the *Feigenbaum attractor*. Iterating the logistic map for such a value of α and making a histogram over the eventual points visited yields:



This attractor certainly contains infinitely many *disperse* points, springing from the scattering of the preceding chain of bifurcations, but, as is seen, those values are not evenly visited and the iteration process defines instead a complex *multifractal* object,^{7,10} made up of multiple disjoint *thorns* of different sizes.

As may be appreciated, the Feigenbaum tree contains from α_∞ onward a multitude of *uneven thorns* at many places. If one were to pass a finger through the diagram at the value of α_∞ , one would get

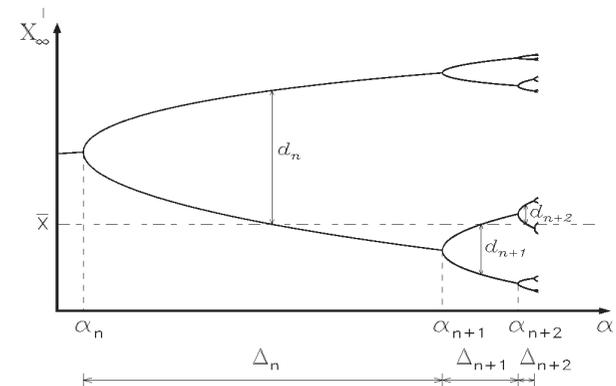
pricked by the present spikes, which literally rise to infinity as the scale of the graph is reduced. Similarly, and due to the aforementioned self-similarity of the diagram, the same would also happen by the end of any chain of bifurcations within a white band region corresponding to any period. As hinted by the uneven darkness within the tail of the tree, there are also many other regions that correspond to uneven histograms that further reflect the *rugged* nature of the object.⁸

As shall be explained later on in detail, an infinite strange attractor, that turn out to be the most common behavior within the tail of the diagram, excludes great many periodic points and hence have a *disperse* and *non cohesive* topological structure that resembles the one of “*dust*.” As a consequence, the foliage of the Feigenbaum tree is essentially *dust*.

Universality in the Fig Tree and Other Trees

It happens that all bifurcations within the Feigenbaum tree, including the ones that occur within all of the infinitely many sub-trees, take place in an orderly manner both in their *openings* and in their *durations*.

As depicted for bifurcations happening at parameter values α_n , α_{n+1} and α_{n+2} :



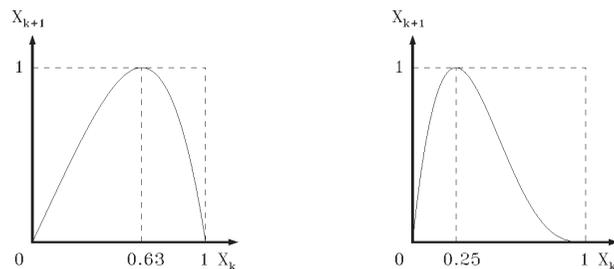
such a sequence leads to decreasing bifurcation durations Δ_n , Δ_{n+1} and Δ_{n+2} , and to alternating decreasing distances d_n , d_{n+1} , d_{n+2} , measured

from the horizontal line X (equal to 1/2 for the beginning portion of the Feigenbaum tree) to the closest branch not intersected by such a line. What Feigenbaum proved in 1978 is the remarkable fact that the ratios d_n/d_{n+1} and Δ_n/Δ_{n+1} , that is, from bifurcation to bifurcation, approach the same constants everywhere within the Feigenbaum tree:

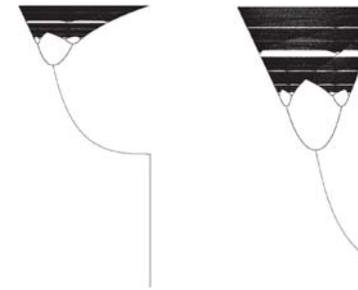
$$\frac{d_n}{d_{n+1}} \rightarrow F_1 = -2.5029... \quad \frac{\Delta_n}{\Delta_{n+1}} \rightarrow F_2 = 4.6692...$$

The limits F_1 and F_2 are *Feigenbaum's universal constants* and such are so important that they are to bifurcations in powers of two as pi is to circles.¹¹ The existence of such constants explains why there is indeed *order in the path to chaos* via successive bifurcations, but such does not imply, as it is often affirmed erroneously, that chaos itself is ordered.

Feigenbaum's numbers turn out to be remarkably applicable as they are also valid for countless nonlinear equations that give rise to *other* chaotic trees, beyond the Feigenbaum tree. The iteration of arbitrary *unimodal* functions, that is, those having one peak, yields similar trees and all the *budding* within these impressive objects happens precisely at the prescribed universal rates F_1 and F_2 . As an example of this extraordinary fact, the simple equations $f(X) = \alpha X (1 - X^3)$ and $f(X) = \alpha X (1 - X)^3$, representing alternatives to the logistic map with graphs that are not symmetric and not fully convex:



generate, by varying their respective parameter α , other chaotic trees also guided by the same numbers F_1 and F_2 :



that contain, as seen for the Feigenbaum tree, a straight root, a “tender branch,” bifurcation branches, and, in an intertwined fashion, periodic branches and the dusty foliage of chaos.

Amongst the most important practical discoveries of chaos theory stands the surprising work of Albert Libchaber and Jens Maurer in regards to the heating of liquid helium.¹² Such researchers found, also in 1978, that there is a discernible order in the way *convection* takes such a fluid from a state of rest to its ultimate chaotic and turbulent state. Remarkably, as the *heat* given to the fluid is increased, the helium experiences noticeable transitions that, up to the scale of the measurements, include four bifurcations that happen precisely at temperature increments given by F_2 .

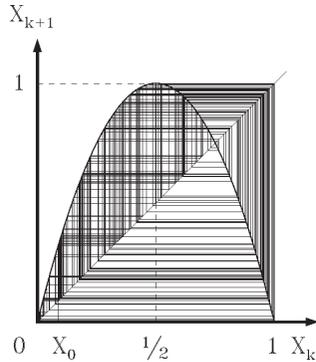
These results illustrate the surprising fact that there are various physical systems for which their relevant dynamics may be understood via iterations of simple maps and in terms of a single parameter that plays the role of α in the logistic map.⁴ As the overall notions of chaos theory became relevant in a variety of scientific fields including ecology, biology, chemistry, physics, engineering and economics, they established a new paradigm for the origins of disorder, for the ideas clearly implied the possibility of finding hidden simplicity in complexity and without employing the prescribed sets of differential equations or invoking the notion of chance.⁴⁻⁶

In the Plenitude of Chaos

Although chaotic behavior is not the most disorganized state seen in nature, it is pertinent to study the most disorganized chaotic state in order to fully appreciate its *empty* structure and some of its subtle details.

As such *plenitude* of chaos happens when the strange attractors are widest and at the top of the chaotic trees, such a special condition is hence associated with the archetypical selection of “*maximum heat*” that leads to maximum turbulence.

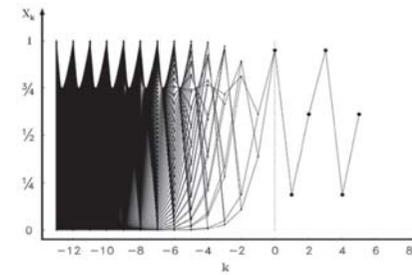
As seen before, for the logistic map such an extreme condition corresponds to the largest possible parabola defined when $\alpha = 4$, and judging by the dense horizontal-vertical lines:



it would appear that the dynamics travel everywhere within the logistic parabola, suggesting that the widest strange attractor encompasses *all* the defining interval from 0 to 1.

This, however, is not the case, as there are great many points in the interval that ought to be excluded from the strange attractor, as follows. First, the value 0 is not in the wandering attractor, for if $X_0 = 0$, then the population remains there forever. Second, the nonzero intersection of the parabola and the straight line is not in the strange attractor, for if $X_0 = (\alpha - 1)/\alpha = 3/4$, the population also remains there forever. Similarly, all the extensions to $\alpha = 4$ of all *unstable* periodic attractors, as defined by the many thresholds within the tree, need to be excluded from wandering forever, for starting in any of those precise locations leads instead to periodic oscillations forever.

These observations imply that the ultimate attractor excludes the infinitely many points corresponding to all periods. There are, however, even more points that need to be omitted. As illustrated below for an example repeating every three generations:

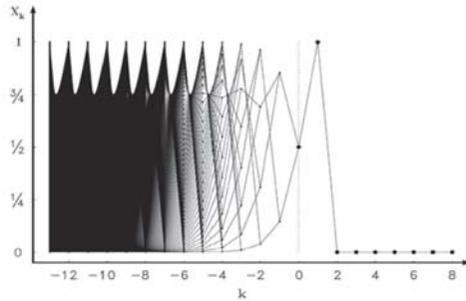


There are great many points located in the *past* of the highest repeating value, which, by not wandering forever, are not members of the final attractor either. As the parabola is read backwards again and again, such points make up a *binary tree* that, as is seen, covers the interval from 0 to 1 in a *dense* manner. In a similar fashion, there are two other similar diagrams ending in the two other repeating points that define other locations that are not included in the strange attractor.

Based on these observations, the structure of the strange attractor finally emerges. Such an *infinite* but *uncountable* set is equal to the interval from 0 to 1 minus all the *infinite* binary trees associated with all periodic points and for every period, which correspond to extensions to $\alpha = 4$ of the *infinitely* many periodic attractors that the tree contains. As such, the strange attractor for $\alpha = 4$, and similarly for other chaotic parameters $\hat{\alpha}$, by having infinitely many *countable* “holes,” is a rather *disperse* set that has the topological structure of *dust*.

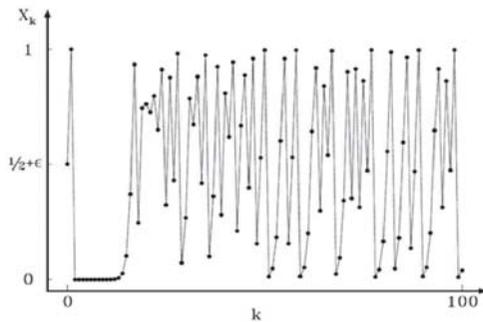
Strikingly, all periodic behaviors, and for every period, turn out to define dense subsets of the interval from 0 to 1 that end up *oscillating* forever. But as such sets are *unstable*, a minute deviation from any of such precise “jumps in high heat” would lead instead to *wandering* forever, that is, without repetition, in the *stable* infinite set that attracts it all with all probability and that is also a dense subset of the interval from 0 to 1.

These subtleties are particularly noteworthy for the logistic map, for when $\alpha = 4$, and only in such a case, there are great many countable initial values X_0 that altogether avoid wandering or oscillating forever in high heat, as such states define dynamics that eventually return to the allegorical *root* of the tree:



These are the *pre-images* of zero that find their way to the *origin* despite the most implacable *jumping* of neighboring chaotic or periodic points. As is seen, such precise locations lead to iterations of the logical map that eventually pass exactly by the *middle*, that is, $X_0 = 1/2$ in the tip of the parabola, and then arrive at *one* to finally *rest* at zero.

Chaos is, no doubt, a rather jumping process. This is illustrated by studying what happens when starting quite close to the middle but not quite, at say $X_0 = 1/2 + \epsilon$:



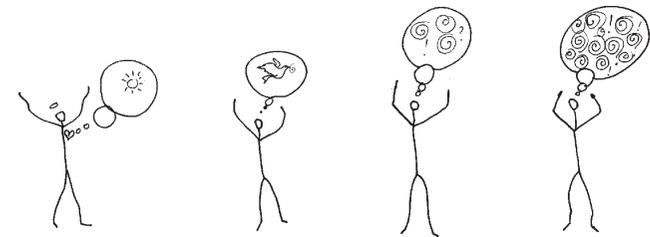
As is seen for $\epsilon = 10^{-5}$, the dynamics undershoot 1 slightly, resulting in a succession of almost zero but positive population values that eventually result in extreme divergence and the seemingly random dynamics of chaos.

Here it is, once again, the *butterfly effect*, a reflection of the unforgiving nature of *non linear chaotic* systems that prevent us from knowing their dynamics in the long run. This is one of the most important new notions of the theory, for when chaos rules, the unavoidable

propagation of errors, due to the finite precision in our calculations, prevents us from predicting the dynamics, even if we know that it happens somewhere within the strange attractor.

Chaos or no Chaos? A Sensible Question for Humans?

As one ponders the rather precise results herein regarding natural chaos, their striking simplicity and universality hint that it is sensible to employ the bifurcation notions and the symbolic logistic map to model how we, as humans, often end up in “*chaos*” and to study how such distress may be avoided in our lives. After all, depending on our “internal heat,” we all experience distinct states that are accurately reflected by different locations within the Feigenbaum tree, as follows: either a *gentle* state properly maintained by the proverb of us not “crossing the line;” a *proud* state that captures our fixed but not fully abandoned actions; a *confused* state that reflects the doubts of our multiple oscillations; or, surely worse, a state of great *disarray*, often accompanied by the *violence of thorns*, where, by the excessive energies, we end up “*biting the dust*.”¹³



As, with due imagination, the logistic map may be used to represent, in the product of X and its complement $(1 - X)$, the ever present stresses we all face and as the unforgiving nature of non-linearities only creeps in when we choose to *magnify* such effects, that is, when we select a value of α greater than 1, the Feigenbaum tree can also be used to emphasize the central role that *free will* has either in aiding us to create the “chaos” we experience or in helping us pursue a pathway to “wholeness,” by *diminishing* the inherent tensions while selecting a parameter α less than or equal to 1. These arguments turn out to be consistent with *common sense* as the chaotic tree reflects our own *organizational* or *logistic chart* and as α properly captures our inherent dynamics of *order* or *disorder* as reflected in our “*mild*” or “*steep*”

slopes at the *origin*, in our essential choices of the *simple* or the *complex*, and in our subsequent attracting states of *serenity* and *peace* or ultimately *chaos* and *turbulence*.¹⁴

These reflections suggest that the posed question “chaos or no chaos?” is certainly a relevant one, for they hint that there is only one way by which we all could achieve true peace, and that is to converge to the *origin*, hence remaining in the straight *roots* of chaotic trees. Certainly, climbing the Feigenbaum tree, and other trees, takes us away from the essence of the root and into states that reflect *brokenness* and *unrest*, as it happens prominently in the chaotic states at high heat in which wandering forever, and always missing *the point* of return, represents a truly *hellish* condition.

As the best solution clearly points us to *virtue*, *humility* and *abandonment*, that is, below the first *threshold* $Y = X$, we realize that chaos theory provides overarching ideas that lay an unexpected bridge from science into other realms, as previously reported from multiplicative cascades and turbulence.⁷

The remainder of the article shall try to show that the common sense lessons drawn herein from the modern theory of chaos corroborate indeed relevant faith lessons as revealed in the ancient word of the Bible. At the end, such connections shall exhibit, via a consistent typology, an unforeseen invitation for us to fully heed the teachings of Jesus Christ based on modern science, including new consistent vistas of key passages associated with the rather symbolic *fig tree*.

The Root of the Feigenbaum Tree

As hinted from the previous section, the root of the tree symbolizes those faithful ones who, by renouncing “their rabbits,” find a superior state of harmony associated with obedience to God:



That this is the case may be seen in several passages from Scripture, as follows.

As evidenced in Jesus’ words “whoever wishes to come after me *must deny himself*, take up his cross, and follow me” (Mk 8:34),¹⁵ the key abandonment to His *radical* kind of *love*, one that excludes *no one* (Mt 5:44), may be associated symbolically with the essence of number *zero*, and consequently with our convergence to the straight *root* of the Feigenbaum tree. Clearly, this “*emptying of self*” may be seen in Jesus’ invitation for our *perfection* (Mt 5:48) and in his pleas for our *repentance* (Mt 4:17), that lead us, as the apostle Paul explains, to an associated condition devoid of any *sin* (Rom 6:11-12), consistent with the root.

Jesus’ calls to our “extinction” certainly abound, and such include some rather graphical ones inviting us to be at the root, such as “whoever wishes to save his life will lose it, but whoever *loses his life* for my sake will save it” (Mk 8:35); “Amen, amen, I say to you, unless a grain of wheat falls to the ground and *dies*, it remains just a grain of wheat, but if it *dies*, it produces much fruit” (Jn 12:24); “whoever loves father and mother more than me is not worthy of me, and whoever loves son or daughter more than me is not worthy of me” (Mt 10:37); “whoever is ashamed of me and of my words, the Son of Man will be ashamed of when he comes in his glory and in the glory of the Father and of the holy angels” (Lk 9:26) and “everyone of you who does not *renounce* all his *possessions* cannot be my disciple” (Lk 14:33), that help us appreciate the relevance of sharing “our rabbits.”

Ever present in Jesus’ message are His demands for our *conversion*. Hence, such calls may be understood as transitions towards the root of the tree associated with lowering our heat parameter \acute{a} below or at *one*, yielding a symbolic “*decrease*” of us so that Jesus’ love may increase in us, as expressed by John the Baptist (Jn 3:30), and an ultimate state of communion with Him in which we keep God’s commands, as consistently expressed throughout Scripture (e.g., Ps 119:59).

“Coming down” the chaotic tree is certainly an accurate image for our repentance, as quite literally happened to *Zacchaeus* the little tax collector, who, after listening to the voice of Jesus, quickly came

down from a sycamore tree (curiously of the same family of the fig tree), and away from his own *pride*, to be fully reconciled with God, him and all his family (Lk 19:1-10):



The Bible certainly includes several exhortations for us to move away from sin and into a condition of *straightness* associated with the very shape of the root of the Feigenbaum tree and with the inherent *halo* of righteousness of the children of God (Mt 18:3-4). Such include, for instance, citations such as “in all your ways be mindful of the Lord, and He will make *straight* your paths” (Prv 3:6), “on the way of wisdom I direct you, I lead you on *straightforward* paths” (Prv 4:11), “let him who is wise understand these things, let him who is prudent know them, *straight* are the paths of the Lord, in them the *just* walk, but sinners stumble in them” (Hos 14:10), and, prominently from the prophets Isaiah and John the Baptist, “I am ‘the voice of one crying out in the desert,’ make *straight* the way of the Lord” (Jn 1:23, Is 40:3).

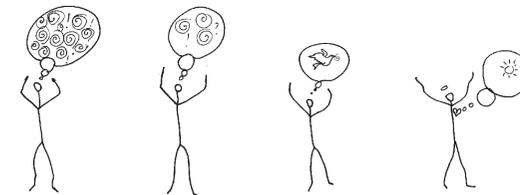
All these pleas to surrendering are consistently complemented by a host of Jesus’ sayings that invite us to *trust* in Him, such as “do not worry about your life, what you eat or drink, or about your body, what you will wear” (Mt 6:25), “do not worry about tomorrow, tomorrow will take care of itself” (Mt 6:34), “whoever wishes to be great among you shall be your servant, whoever wishes to be first among you shall be your slave” (Mt 20:26-27), that point us to *humility*, to *zero*, and hence to the attracting *Origin* of all goodness.

The Shoot of the Feigenbaum Tree

The upper portions of the Feigenbaum tree, and also of the other chaotic trees, may be used to symbolize us when our excessive heats, and associated amplifying choices, take us away from obeying God’s commands and into states that reflect our lack of peace.

Certainly, the uneasy jumping forever on strange and periodic attractors reflects the anxious and foolish frustration we often experience in our lives, and so many times not by chance, when we, by choosing to live above the “threshold” (or more precisely sometimes above many thresholds), end up living in states of disarray, traveling in proverbial turbulence without finding our “root.” Although it may appear otherwise, climbing into the main branch is not any better either, for such an impetus surely takes away our peace.

Jesus’ famous *parable of the sower* (Mk 4:1-20) may be used allegorically to illustrate what happens in the shoot of the Feigenbaum tree and also in the root, as follows. Following the four key categories coming down the tree, we may see: (a) our *chaotic* behavior in the seed that fell *in the path* and was eaten by the birds associated with Satan, for the Devil and his followers always miss *the point* while being imprisoned forever in *dust*, (Mi 7:17, Is 29:5) a condition consistent with what happens in strange attractors; (b) our *periodic* behavior in the seed that fell on *rocky ground* and whose plant withered for lack of roots representing lack of constancy under tribulation or persecution, for the inherent doubts in the oscillations clearly take us away from the proper trust (Jas 1:6); (c) our *fixed point* behavior in the seed that fell among *thorns* and giving a fruitless plant denoting us when dominated by anxiety and the lure of riches, for the main branch, by crossing the threshold, represents a stubborn state that in its quest to go higher and higher indeed encounters thorns and dust that ultimately disintegrate such a branch that becomes *unstable* as it passes $X_\infty = 2/3 = 0.666\dots$ when $\alpha = 3$, and (d) our *peaceful* behavior in the seed that fell on *rich soil* and gave a plant that produced ample fruit, for, as we argued earlier, the root is consistent with us hearing and accepting the good news:



The Feigenbaum tree reflects, no doubt, in its distinct destinations, the consequences of our actions. That, at the end, there are only two major categories that matter, either *below* or *above* the tree, may also

be seen vividly in God's decree when He said to the people of Israel and to all of us, "I set before you here, this day, a *blessing* and a *curse*: a blessing for *obeying* the commandments of the Lord, your God, which I enjoin on you today, a curse if you *do not obey* the commandments of the Lord, your God, but turn aside from the way I ordain for you today, to follow other gods, whom you have not known" (Dt 11:26-28, Ps 37:22). As a consequence, while our peaceful and fruitful behavior associated with the root of the Feigenbaum tree leads us to receive *blessings* and *life*, our sinful and fruitless conducts related to the shoot of the chaotic trees gives rise to *curses* and *death* (Dt 30:15-20).

As chaos happens in the emptiness of *dust* and always missing the best destination, such a *silly* and false state (Ps 4:3) may be used to symbolize, first in a semantic way, the "strange" dynamics to which we are "attracted" to when we choose to cross the line and try to establish our own ways without God (Rom 10:3). Such a *restless* jumping, leading to spiritual *death* if not corrected (Sir 15:17), also provides further imagery regarding the consequences of sin in God's appointed curse and punishment to sinners. For instance, strange attractors may be used to visualize how God "*disperses* the arrogant of mind and heart" (Lk 1:51) and how such ultimate sets are graphically consistent with the *nets* that trap us due to our own *pride* (Ps 31:5).

Incomplete strange attractors, made of infinitely many points always in motion, are by no means nice places to reside, especially when the underlying dynamics happen at the highest heat that reminds us of the horrific turbulence of *hell*. Certainly, the very existence of such an infernal place reflects unabated *chaos* and such is the ultimate place of punishment for the Devil. For as God Himself decreed "the serpent's food shall be *dust*" (Gn 3:14, Is 65:25) and such a dreadful state is reiterated in a future vision that affirms "the Devil who had led them astray was thrown into the pool of *fire* and sulfur, where the beast and the false prophet were; there they will be tormented day and night forever and ever" (Rv 20:10).

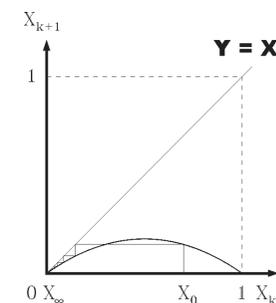
As expressed in Scripture, these categorical pronouncements regarding the symbol of dust and its related hell also apply to those following the Devil, that is, the *wicked*. This is seen, for instance, in the words of the prophet Micah when he assures us that the wicked "shall

lick the *dust* like the serpent" (Mi 7:17), in the strong statement by the Psalmist when he adds "all sinners will be destroyed, and the future of the wicked will be cut off" (Ps 37:38), and in God's response to Job when He asked "can you bring down the haughty with a glance, bury them in the *dust* together, in the hidden world imprison them?" (Jb 40:12-13). In regards to this last passage, the subtle sensitivity to initial conditions ever present on strange attractors indeed helps us visualize how God could achieve the feat of imprisoning sinners, for God may enter a multitude of them at different times and trap them inside the same dusty set, and there, all of them would be jumping forever, and without ever knowing that there are others doing just the same, as the dynamics on such grim strange attractors do not repeat.

As is seen, despite visiting infinitely many places, traveling inside strange attractors is not fun at all. This happens because the butterfly effect does not provide us with good options, as it always leaves us irremediably trapped in an *empty* strange attractor in which there is only a deadly and eternal stress that always misses *the point* at the *origin*.⁷

The Identity of the Key Threshold

As already explained, the consistent calls to *humility* included in Scripture relate *goodness* to the *root* of the Feigenbaum tree. Consequently, exhortations such as "everyone who exalts himself will be humbled, but the one who humbles himself will be exalted" (Lk 14:11) and "whoever wishes to be great among you shall be your servant, whoever wishes to be first among you shall be your slave" (Mt 20:26-27) point us to choose $\alpha \leq 1$, so that our underlying dynamics happen below the key *threshold* $Y = X$:



As our little and *obedient* parabola has a mild slope at the origin, the *one-to-one* line quite graphically shelters us from the “wind” of evil (Is 32:2) in the chain of bifurcations and beyond above the tree, and the threshold becomes a literal “refuge and fortress” (Ps 91:2) that allows us to converge to the *origin*. As the formula of *the line* portrays, also quite geometrically, the defining equality between the crucified silhouette, *Y*, and the cross, *X*, we may see that the threshold symbolizes *Jesus Christ* Himself, who, consistently said “I am the way the truth and the life, no one comes to the *Father* except through me” (Jn 14:6), that may be clearly appreciated if $X_{\infty} = 0$ represents God the Father, the *Origin*, with capital O, of course.⁷

That $Y = X$ is the proper separatrix, the anointed one (Is 45:1), the Messiah (Jn 1:41) and the *only* divine son of God (Jn 1:18), is seen in the fact that “the Father has given all judgment to His Son” (Jn 5:22), and also in a variety of assertions by Jesus himself that include: “I am the *gate* for the *sheep*” (Jn 10:7), “I am the *good shepherd*” (Jn 10,11), “do you think that I have come to establish peace on the earth? No, I tell you, but rather division” (Lk 12:51), and notably “enter through the *narrow gate*, for the gate is wide and the road broad that leads to destruction, and those who enter through it are many” (Mt 7:13-14) that hints at the chaos above. As the origin is not reachable, with all probability, from the shoot of any chaotic tree, all these citations clearly point us, consistently and geometrically, to a little parabola and under the protection of the threshold.

These notions, unexpectedly linking geometry and Scripture, are however compatible with other passages that explain how Jesus, the one who spoke in *parabolas*,¹⁶ shall change the prevailing and apparent order of matters. For, as He said, “many who are first will be last, and the last will be first” (Mt 19:30) as the humble *sheep* below shall be separated from the arrogant *goats* above during the judgment of nations (Mt 25:31-46).

As implied by these observations, the very *logistic of salvation* and hence the very reason for rejoicing while understanding the good news (Prv 2:1-2, 5-11, Mt 13:23) is intimately related to us being *below* the *one-to-one* line. For, Jesus’ words “Amen, amen, I say to you, whoever hears my word and believes in the one who sent me has eternal

life and will not come to condemnation, but has passed from death to life” (Jn 5:24), can only be fulfilled at the *Origin*, and because His defining motto “God so loved the world that he gave his only Son, so that everyone who believes in him might not perish but have eternal life” (Jn 3:16, Jn 8:24, Jn 11:25-26), further explains the protecting geometry of the “shelter of the Most High” and “shadow of the Almighty” (Ps 91:1), who shed His precious *blood* for us (Lk 22:20) while stretching His arms to end up fulfilling the simplest, and yet hardest, of equations $Y = X$ (Jn 19:18).

A Cursed Fig Tree

Few days before his crucifixion, Jesus performed an unusual miracle and surprisingly cursed a living fig tree, a living *feigenbaum*. According to St. Matthew, “when He was going back to the city in the morning, He was *hungry*; seeing a fig tree by the road, He went over to it, but found nothing on it except leaves; and he said to it, ‘*may no fruit ever come from you again,*’ and immediately the *fig tree withered*; when the disciples saw this, they were amazed and said, ‘how was it that the fig tree withered immediately?’ Jesus said to them in reply, ‘Amen, I say to you, if you have *faith* and do not waver, *not only will you do what has been done to the fig tree*, but even if you say to this mountain, ‘be lifted up and thrown into the sea,’ it will be done, whatever you ask for in prayer with *faith*, you will receive’” (Mt 21:18-22).

In the less spectacular account according to St. Mark, the event is recorded in two stages that sandwich Jesus’ overturning of the tables on the temple area in Jerusalem (Mk 11:15-19). Such a version says, “the next day as they were leaving Bethany He was *hungry*; seeing from a distance a fig tree in leaf, He went over to see if He could find anything on it; when He reached it He found nothing but leaves; *it was not the time for figs*; and he said to it in reply, ‘*may no one ever eat of your fruit again!*,’ and his disciples heard it” (Mk 11:12-14). And, “when evening came, they went out of the city; early in the morning, as they were walking along, they saw the fig tree *withered to its roots*, Peter remembered and said to him, ‘Rabbi, look! The fig tree that you cursed has withered,’ Jesus said to them in reply, ‘have *faith* in God, Amen, I say to you, whoever says to this mountain, ‘Be lifted up and thrown into

the sea,' and does not doubt in his heart but believes that what he says will happen, it shall be done for him; therefore I tell you, all that you ask for in prayer, believe that you will receive it and it shall be yours" (Mk 11:20-24).

Although the two accounts share the main cursing of the *fig tree* followed by exhortations to *faith*, they differ in two significant ways, as follows. First, the version in St. Mark does not include the assertion that the disciples could also curse and subsequently wither the fig tree. Second, the story in St. Matthew does not include the perplexing and simple explanation that the fig tree had no fruit because it was not the time for figs, as the tree was not in season.

Did Jesus metaphorically curse the unbelief of the people of Israel at His time? Certainly, such an interpretation is consistent with His overturning of the tables and the rejection He experienced during the crucifixion.^{17,18} But in the spirit of the symbols of modern science herein and with due humility given the passage of twenty centuries, we may argue a more *encompassing* curse of all our prideful roads to chaos, already and consistently cursed (Dt 30:15-20), that take us away from the root of God and into sin, pathways that if unchecked lead us indeed into chaos and death (Rom 8:13). For, after all, the modern Feigenbaum tree, like the ancient and metaphoric one, has an accursed *shoot* that does not produce any visible fruit in any season and hence it is rightfully withered to its *root*, but the *root remains*.

Scripture is certainly consistent on matters of disobedience, for those who turn their hearts away from the Lord are, once again, *cursed* (Jer 17:5) and they justly receive the same *rebuking* Jesus gave to the *wind*, leading to the same astonishment on the disciples after a great miracle (Mk 4:39-41), as it happened also with the withered fig tree. For the *Devil* is the *ruler of the power of the air* (Eph 2:2), the master of turbulence and division,⁷ the one who pulls us up the chaotic fig tree to start a chain of bifurcations precisely by $2/3 = 0.666 \dots$, and in such terms, even if unexpected coming from modern science, we may understand why we, as Jesus' disciples, may also utter such a curse ourselves against the shoot of the symbolic tree, as a manifestation of the powers that Jesus gives us to wage battle against evil (Mk 16:17-18).

As expressed before via the parable of the sower, God's desire for us to bear good fruit (Jn 15:8) is satisfied symbolically in the *root* of the Feigenbaum tree (Jn 15:8) and not above. That this is the case may also be seen directly from Jesus' own words when He said that "people do not pick *figs* from *thornbushes*" (Lk 6:44), as the structure of the shoot of Feigenbaum tree contains plenty of them by the end of its infinitely many "white bands." As John the Baptist's proclaims "even now the *axe lies at the root of the trees*, therefore every tree that does not bear good fruit will be *cut down* and thrown into the *fire*" (Mt 3:10), these reflections enhance God's invitation for all of us to *always* "produce good fruit as evidence of our repentance" (Mt 3:8), a godly sorrow at all seasons, a "*sweet fig of joy*" (Song 2:13) that, as we have seen, may only be attained via Jesus in the ever precious *root*.

A Modern Prophetic Fig Tree?

The fine symbolic structure of the Feigenbaum tree explained herein and the Biblical symbolism of the fig tree may be used, with due humility, to study the history of God's people and the delicate topic pertaining to the end of the age and the return of Jesus Christ.

The story starts, of course, with Adam and Eve, and their *fall* from obedience and nakedness (Gn 2:25) into sin and "coverage" with, precisely, *fig leaves* (Gn 3:7). As the serpent, forever eating dust (Gn 3:14), lied to the woman, a figurative "cascade of bifurcations" quickly happened, and this led to death, for God decreed "for you are dirt, and to dirt you shall return" (Gn 3:19) as He banished man from the Garden of Eden (Gn 3:23). The story reached a crescendo when God established a *covenant* with His people, the descendants of Abraham, Isaac and Jacob (Israel), through the circumcision of every male, and later via the following of *God's law* as given, in particular, to Moses. Scripture relates the struggles of the Israelites in keeping the law, and tells us of their times of peace and their times of distress.

Throughout the story, the *fig tree* and the *vine* appear as consistent symbols describing what would happen to the people of Israel. For instance, under the ruling of King Solomon, "Judah and Israel lived in security, every man under his *vine* or under his *fig tree*" (1 Kgs 5:5); but, in punishment, God "will lay waste their *vines* and *fig trees*" (Hos

2:14, J1 1:7,12). The fig tree also represents *perseverance* and *hope*, as a *fig* poultice administered by the prophet Isaiah healed a repentant King Hezekiah of Judah (Is 38:21).

As seen before in the distinct stages in Jesus' famous *parable of the sower* (Mk 4:1-20), the particular state of grace of an Israelite (and intrinsically of any human being) may be traced at the alternative stages of the Feigenbaum tree, for a fall in grace implies traveling a "road to chaos." Invariably, the reasons for people's distress are found in disobedience, for example, "why is the land ravaged, scorched like a wasteland untraversed? Because they have abandoned my law, but follow rather the hardness of their hearts" (Jer 9:11-13) or "my people go into exile because they *do not understand*" (Is 5:13). This rebellious behavior, happening despite awesome displays of power by God, for instance, the manna He sent them from heaven (Ps 78:27-32), points the Jewish people at that time away from the symbolic root of the Feigenbaum tree, as the Psalmist explains about them, "their hearts were *not constant* toward him", as "they were not faithful to God's covenant" (Ps 78:37).

These observations are particularly consistent with Israel's *stubbornness*, "the first fruits of God's harvest" (Jer 2:3), at the time when the story reached its plenitude with the coming of Christ. For "although he had performed so many signs in their presence they did not believe in him" (Jn 12:37), for "they stumbled over the stone that causes stumbling" (Rom 9:31-32), that is, for they chose to cross the line over $Y = X$. Scripture explains that this happened because "they did not acknowledge it openly in order not to be expelled from the synagogue, for they preferred human praise to the glory of God" (Jn 12:42-43). For, although, they had a zeal for God, they could not discern and hence submit to the righteousness of God in Christ (Rom 10:2-4).

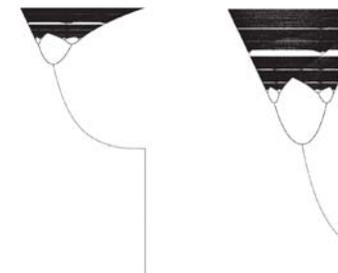
These arguments regarding the "*obstinate branch*" (in singular) having a broken faith (Ez 15:1-8) become quite relevant when considering Jesus' eschatological discourse and, in particular, the very precise wording of Jesus' *parable of the fig tree*, as recorded in the synoptic Gospels. While according to St. Matthew and St. Mark, Jesus said, "learn a *lesson* from the *fig tree*; when its *branch* becomes tender and sprouts leaves, you know that summer is near; in the same way when you see

all these things, know that he is near, at the gates; Amen, I say to you, this generation will not pass away until all these things have taken place; heaven and earth will pass away but my words will not pass away" (Mt 24:32-35, Mk 13:28-31), according to St. Luke, "He taught them a *lesson*; 'consider the *fig tree* and all the *other trees*; when their buds burst open, you see for yourselves and know that summer is *now* near; in the same way, when you see these things happening, know that the kingdom of God is near; Amen, I say to you, this generation will not pass away until all these things have taken place; heaven and earth will pass away, but my words will not pass away'" (Lk 21:29-33).

As the recently discovered Feigenbaum *tree* does have a rather thin and hence rather "*tender branch*" that loses its stability as it passes precisely by $X_{\infty} = 2/3 = 0.666 \dots$, once again a symbolic repetition of a negative spiral forever and denoting the Devil's perennial sin and deception (1 Jn 3:8, Rv 13:18),⁷ and as such a symbolic tree filled with thorns also quite literally "*sprout leaves*" of *dust* reflecting death, one wonders if Jesus' words are being surprisingly fulfilled in our times via these scientific discoveries:



For we may *see* rather clearly for ourselves this fig tree and also many *other trees* with their "buds burst open" into dust, rotten trees that capture universally our ways to disorder when we choose to disobey and go beyond our prescribed means:



Is this just a coincidence? Is this just an unwarranted association between modern science and Holy Scripture? Certainly, the overall message of this work is one of *love*, ultimate *love* associated with the *root*, and in the very same spirit we may interpret the very advent of the chaotic tree as a consistent act of *God's mercy*, an always fitting call to *repentance*, even if it originates in the unlikely confines of chaos theory.

For without lessening the fact that within the parable's context "these things" may naturally include other **chaotic** events in the *eschatological discourse*, such as persecution of believers (Lk 21:12-19), a time of great tribulation (Mt 24:15-28), and powerful signs in the sky (Mt 24:29-30), the "geometric" connections herein yield an unexpected but relevant call to *conversion* towards the *Origin*, not only for the people of Israel, but for all of us.

Clearly, an exact date for Jesus' return may *not* be drawn, for Jesus Himself said "but of that day and hour no one knows, neither the angels of heaven, nor the Son, but the Father alone" (Mt 24:36, Acts 1:6-7). However, these reflections, in consonance with the predicted modern advent of the nation of Israel (Ez 36:1-37), provide indeed relevant reminders as they reaffirm that we ought to be *watchful* (Mk 13:32-37), for in the days of God's chastisement the stars in the sky shall fall to the earth like unripe *figs* shaken loose from the tree in a strong *wind*" (Rv 6:13).

The message drawn here is consistent and is one of *fruitfulness* in the *root*, for as Jesus said, once again referring to the symbolic tree, "there once was a person who had a *fig tree* planted in his orchard, and when he came in search of *fruit* on it but found none, he said to the gardener, 'for three years now I have come in search of *fruit* on this *fig tree* but have found none, so cut it down, why should it exhaust the soil?' he said to him in reply, 'sir, leave it for this year also, and I shall cultivate around it and fertilize it; it may bear *fruit* in the future, if not you can cut it down'" (Lk 13:6-9).

The Improbable Elect and the Church

Right before the parable of the fig tree, the Gospel according to St. Matthew records Jesus' prophetic words, "and then the sign of the

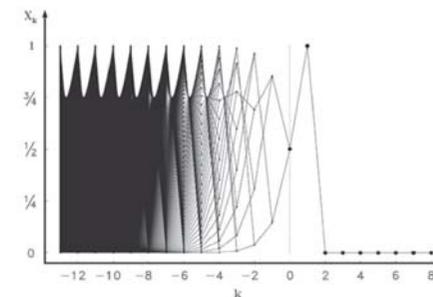
Son of Man will appear in heaven, and all the tribes of the earth will mourn, and they will see the Son of Man coming down upon the clouds of heaven with power and great glory; and He will send out His angels with a trumpet blast, and they will gather His *elect* from the four winds, from one end of the heavens to the other" (Mt 24:30-31).

These "God's chosen ones, holy and beloved" (Col 3:12) may indeed be visualized, once again, in the *root* of the Feigenbaum tree:



where the ultimate union between *one* flock and *one* shepherd (Jn 10:16) yields the geometric equation of *unity* $1 = 0.999\dots$ that symbolizes *love* in the positive spirals.⁷

But, there are some elect, not yet in the *root*, that travel towards the origin in the highest heat when $\acute{a} = 4$, following the precise dynamics of the *pre-images of zero*:



and these correspond, in consonance with the teachings of the Catholic Church, to the holy souls in *Purgatory* that eventually arrive, aided by our prayers, to the heavenly destination with God.¹⁹

The diagram above is quite remarkable as it allows us to visualize yet more citations regarding our *salvation*. Curiously, if the diagram is

rotated twice, once from left to right and once from bottom to top, it portrays, quite consistently and admirably, the symbol of the *square root*, which helps us appreciate the final destination and the true *freedom* enjoyed by the elect. For as Jesus, “the *root* and o spring of David” (Rv 22:16), explains, “whoever enters through me will be saved, and will come in and go out and find pasture” (Jn 10:9), as the elect find the *root* even if subject to the purification of heat.

As the fullness of chaos contains great many (but countable) *unstable* paths that, when precisely followed, lead to the desired state of *surrendering*, and as these roads are extraordinarily intertwined with periodic scenarios for any period and also with the most probable chaotic regime, one may vividly see why “many are invited but few are chosen” (Mt 22:14), why Jesus consistently distinguished between the faithful *wheat* and the superfluous *weeds* (Mt 13:24-30), or between the humble *sheep* and the arrogant *goats* (Mt 25:31-46), and why we must not judge (Mt 7:1-2), for all diagrams leading to distinct destinations look just the same few generations to the past.

With due imagination, in the protected diagram we may vividly imagine Hananiah, Mishael and Azariah “walking in the fire” and without consequences, when King Nebuchadnezzar of Babylon threw them into a fiery furnace (Dn 3:1-92), and also Daniel himself when he came out unhurt from the lion’s den where he was sent after refusing to stop praying to God, as decreed by King Darius (Dn 6:2-24). In the same diagram we may also visualize yet another relevant citation pertaining to God’s faithful protection for his elect, for as the Psalmist says, “though a thousand fall at your side, ten thousand at your right hand, near you it shall not come” (Ps 91:7).

The unequivocal dynamics towards *zero* certainly allows us to cherish more fully the mystery of our *salvation*, for in the ensuing diagram we may indeed appreciate “how constricted is the road that leads to life” (Mt 7:14), “how hard is to enter the kingdom of God!” (Mk 10:24) and how *narrow* is the *gate* at $\alpha = 4$, for in such a case it is just a *single point* $X_0 = 1/2$, that turns out; however, to satisfy Jesus’ famous saying that it is easier for a camel to pass through the eye of a needle (something probable if a needle is big enough) than for a rich one to

enter the kingdom of God (Mk 10:25). It is much easier *indeed* to wander forever in high heat than to enter exactly through *the middle point*, which, borrowing from Pierre Teilhard de Chardin, is a veritable \hat{U} point, a key point of *conscious* love and mercy, “the Lord’s own gate, where the victors enter” (Ps 118:20).

These precise reflections further emphasize God’s incredible gift to us through Jesus Christ, and lead us to the *humility* and ample *joy* that are found in the *root*. For as Scripture teaches, once all is consummated, “there is neither Jew nor Greek, there is neither slave nor free person, there is no male and female, for you are all *one* in Christ Jesus” (Gal 3:28) and hence the only remaining set shall be the upright *root* of the Feigenbaum tree, the holy and united *Church*, the bride of Jesus (Rv 19:7).

That Israel shall heed to God in due time (Rom 11:25-29) is also beautifully seen in connection with the *fig tree*. This stems from the mysterious and joyful encounter of Jesus with the disciple to be Nathanael, whom Jesus describes as a *true Israelite*, after seeing him *under the fig tree* (Jn 1:47-51). The implication, based on the symbolic notions herein, is that the *root* of the Feigenbaum tree is precisely where true Israelites belong, even if they doubt, as it happened to the disciple before he emotionally acknowledged the truth (Jn 1:49). After all, there are a host of pathways that end up in the origin and the ones that reach the mid-point from above and from below may be used to denote the ultimate salvation of the chosen Jews and Gentiles.

Inside the root there is certainly plenty, for when saintly “zeroes” agree, the miraculous power of infinity rules (Mt 18:19-20), and such could be seen, once again with the aid of geometry, via the lovely equation $0 + 0 = \infty$ that reflects that nothing is impossible to God.

Summary and a Poetic Tribute to the Threshold

It has been shown how in the confines of science majestically stands a tree that includes all numerals in a dance that allows us to see how chaos emerges. Such a fig tree contains foliage of disorder trapped in symbolic dust and chaos is expressed in jumps astir forever due to a rather subtle thrust that pushes us above the key threshold. Once the

crossing of the outset happens, the faithful root repels and that yields a tender offset or branch that fails to produce any fruit. The process defines a cascade of bifurcations that increases heat within and such an escalation produces an inescapable succession of branches that are bent by the wind of evil and division. At the end, there is a sprouting of dynamics attracted to the craziness in strange attractors, a condition that turns out to remind us that it is at the origin, at the root, where the flame of God resides.

It has been explained how in the midst of chaos, when such a condition happens at its maximum heat, there is an escape from the hellish situation of wandering forever without finding the proper destination, for, up there, there is a small gate, a point, that leads to fine rest and loyal pathways that, by averting a fright, end up inviting us to an everlasting dance. There are faithful routes leading to the root that are consistent with the symbols of a sought needle's eye and wheat exquisitely surrounded by weeds.

At the end, this work exhibits an improbable rhyme between modern science and Holy Scripture, one that, by reminding us of a rotten tree that foretells the very advent of time, invites us to grow small. Realizing the consistent typology herein in the invitation of Jesus to *love* (Jn 13:34-35), and heeding the converging fact that "he who tends a fig tree shall eat its fruit" (Prv 27:18), this work ends exalting the precious threshold $Y = X$ who shall return in glory and in due time:

$Y = X$

$Y = X$
is justice that illuminates,
is balance that fascinates:
 $Y = X$.

$Y = X$
is the incarnate alliance,
is the established reliance:
 $Y = X$.

$Y = X$
is true word that matures,
is a spiral that endures:
 $Y = X$.

$Y = X$
is the precious resting place,
is the state of mighty grace:
 $Y = X$.

$Y = X$
is smoothness that esteems,
is a hummingbird that gleams:
 $Y = X$.

$Y = X$
is the short and precious root,
is the weaving of the truth:
 $Y = X$.

$Y = X$
is a future that forgives,
is crowned science that is:
 $Y = X$.

$Y = X$
is the ever tender tune,
is the impartial tribune:
 $Y = X$.

$Y = X$
is all innocence that heeds,
is a garden with no weeds:
 $Y = X$.

$Y = X$
is the simple clear sign,
is the majestic design:
 $Y = X$.

Y = X

is independence that heals,
is matrimony that shields:

Y = X.**Y = X**

is the real chaste embrace,
is the goodness of a yes:

Y = X.**Y = X**

is a smile that edifies,
is a spin that rectifies:

Y = X.**Y = X**

is all gentleness in us,
is the everlasting plus:

Y = X.**Y = X**

is inspiration that calls,
is growing to be small:

Y = X.**Y = X**

is the forgotten territory,
is the improbable story:

Y = X.**Y = X**

is revelation that nests,
is surrendering the rest:

Y = X.**Y = X**

is the dustless short incline,
is the faithful narrow line:

Y = X.**Y = X**

is renouncing all spears,
is experiencing no fears:

Y = X.**Y = X**

is the perennial giveaway,
is pure life with no decay:

Y = X.**Y = X**

is the only perfect remedy,
is loving, even the enemy:

Y = X.**Notes and References**

1. This work is warmly dedicated to Fathers Andrew Coffey, Richard Blinn, Daniel Looney, Loreto Rojas, John Kakinda, José Aguilar, William Weigand, Edwino Silvera, Camilo Bernal, Diego Jaramillo, Antonio José Sarmiento, Hernando Silva, Roberto Jaramillo, Sergio Bernal, Jorge Mario Posada, Fernando Cuenca, Daniel Ferreira, Melchor Sánchez de Toca y Alameda, Rafael Pascual, Pedro Barrajón, Lucio Florio, Rafael Martinez, Javier López, Alfonso Miranda, Luis Alfonso Ruíz, Claudio Peña, William Aparicio, Augustine Pamplany, Stuart Swetland, Peter Sanders, James Tarantino, John Campoli, John Hampsch, George Coyne, Andrew Whitman, William Stoeger, and former Catholic Priest Alberto Cutié, for their friendship and loving support.
2. Carlos E. Puente lectures at the Department of Land, Air and Water Resources, University of California, Davis.
3. Webster's Ninth Collegiate Dictionary (Merriam-Webster Inc., 1991), p. 1564.
4. P. Bergé, Y. Pomeau and C. Vidal, *Order within Chaos* (John Wiley: Interscience, 1987), p. 380.
5. F. C. Moon, *Chaotic Vibrations* (John Wiley & Sons, 1987), p. 309.
6. J. L. McCauley, *Chaos, Dynamics and Fractals* (Cambridge University Press, 1993), p. 323.
7. C. E. Puente, "On the nature of equilibrium," in *Omega*, 6.2 (2007), pp. 85-105.
8. H-O. Peitgen, H. Jürgens and D. Saupe, *Chaos and Fractals* (Springer-Verlag, 1992), p. 984.

9. B. B. Mandelbrot, *The Fractal Geometry of Nature* (Freeman, 1982), p. 468.
10. J. Feder, *Fractals* (Plenum Press, 1988), p. 283.
11. M. J. Feigenbaum, "Quantitative universality for a class of nonlinear transformations," in *Journal of Statistical Physics*, 19 (1978), pp. 25-52.
12. J. Maurer and A. Libchaber, "Rayleigh-Bénard experiment in liquid helium frequency locking and the onset of turbulence," in *Journal de Physique Letters* 40 (1979), pp. 419-423.
13. The figurines herein were made by Fernando Duarte, <http://www.duarteid.com/>.
14. C. E. Puente, "More lessons from complexity. The origin: the root of peace," in *E:CO, Emergence, Complexity and Organization*, 8.3 (2006), pp. 115-122.
15. All Biblical quotations come from *New American Bible, Saint Joseph Edition* (Catholic Book Publishing Co., 1992).
16. The words parable and parabola are the same in Greek and in other languages.
17. W. R. Telford, *The Barren Temple and the Withered Tree* (She eld, 1980), p. 319.
18. G. R. Beasley-Murray, *Jesus and the Last Days* (Hendrickson Publishers, 1993), p. 491.
19. *Catechism of the Catholic Church: Second Edition* (Doubleday Religion, 2003), p. 825.

OMEGA

INDIAN JOURNAL OF SCIENCE AND RELIGION

Printer and Publisher

Dr. Augustine Pamplany
Institute of Science and Religion
Little Flower Seminary, Aluva - 683 101
Ernakulam District, Kerala.

Editor

Dr. Job Kozhamthadam
De Nobili College, Ramwadi, Pune - 411 014

Printed at

Ayodhya Printing Press
Kaloor, Ernakulam, Kerala.

Place of Publicaiton

Institute of Science and Religion
Little Flower Seminary, Aluva-683 101
Ernakulam District, Kerala.

Owner

Dr. Augustine Pamplany
Director, Institute of Science and Religion
Little Flower Seminary, Aluva - 683 101
Ernakulam District, Kerala.