Forum

Lessons from complexity: The hypotenuse -
The pathway of peace

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The last few decades have witnessed the development of a host of ideas aimed at understanding and predicting nature's ever present complexity[1]. It is shown that such a work provides, through its detailed study of order and disorder, a suitable framework for visualizing the dynamics and consequences of mankind's ever present divisive traits. Specifically, this work explains how recent universal results pertaining to multiplicative cascades and fully developed turbulence entice all of us, in a logical way, to seek peace in a condition typified by the hypotenuse of a right-angled triangle.

Games for kids

Playing with modeling clay, recall the construction of generic multiplicative cascades. Start with a uniform bar cutting it into two pieces containing $p = 70\%$ and $q = 30\%$ of the mass, respectively. Then, mold the pieces uniformly, piling up the largest one and stretching the other, so that they form two contiguous rectangles having equal size. Repeat the process *ad infinitum*, as shown in Figure 1.

After *n* levels of the construction, the bar is split into 2^n pieces of size $1/2^n$ whose masses correspond to the expansion of $(p + q)^n$, yielding $(n + 1)$ clay layers arranged according to Pascal's triangle. This divisive game produces a skewed *multifractal* object made of finely intercalated *thorns* supported by intertwined Cantor *dusts*[2]. The clay concentrates on a few isolated spikes, yielding the proverbial 80% of the mass in the tallest 20% thorns, just after 16 levels.

Recall now the construction of a uniform measure over a Cantor set, cutting a bar by the middle and piling up left and right, while leaving holes of proportion 1/3 as seen in Figure 2.

After *n* levels of this construction, the bar is split into 2^n disjoint pieces of horizontal size $1/3^n$ and mass $1/2^n$, yielding thorns that grow to infinity as $(3/2)^n$, while supported by triadic Cantor dust.

This second cascade turns out to be intimately related to the first one, for if the hole size is
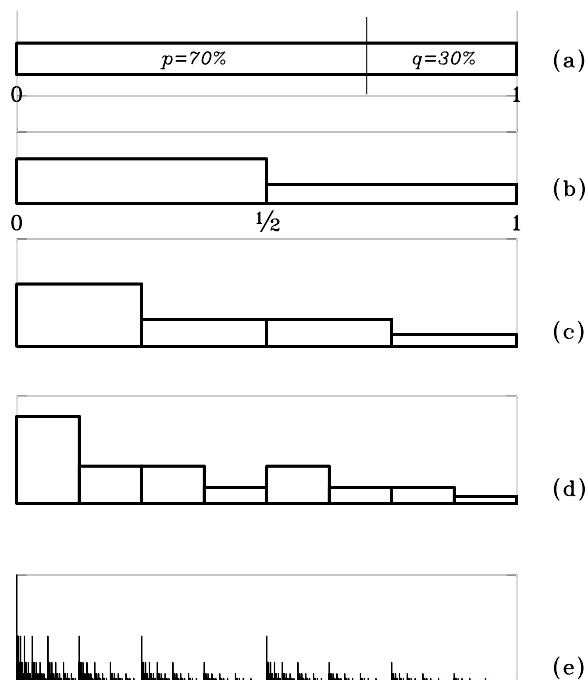

Figure 1 *A generic multiplicative cascade propagating imbalances. From a bar, (a), to 2^{12} pieces arranged into 13 clay layers, (e).*
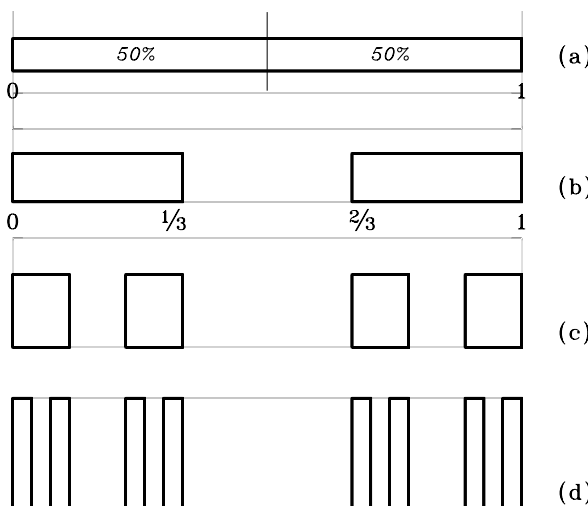

Figure 2 *A generic multiplicative cascade spreading disunity. From a bar, (a), to eight dispersed pieces, (d).*

changed from 1/3 to a general size h, then the resulting dusts match the non-contiguous organization of equally-sized thorns on the first cascade, with denser layers requiring smaller hole sizes h and vice versa (Feder, 1988).

Accumulated resources

As the objects generated by both cascades grow without bound, it is convenient to portray their accumulated masses, from the beginning up to an arbitrary point x.

The obtained boundaries below (Figure 3) reflect the dynamics of the games, as follows. The *notches* on the *cloud* associated with the first game correspond to the sequence of pile ups and stretches: 70% of the mass happens from 0 to $x = 1/2$, 49% up to 1/4, and so on. The *plateaus* found for the second game are related to the holes of the successive pile ups: 50% of the mass happens from the beginning to $x = 1/3$, 25% up to the ninth part, and so on. For comparison purposes, the graph also includes the *straight* cumulative distribution corresponding to the original bar, that is, the common level *zero* on both games.
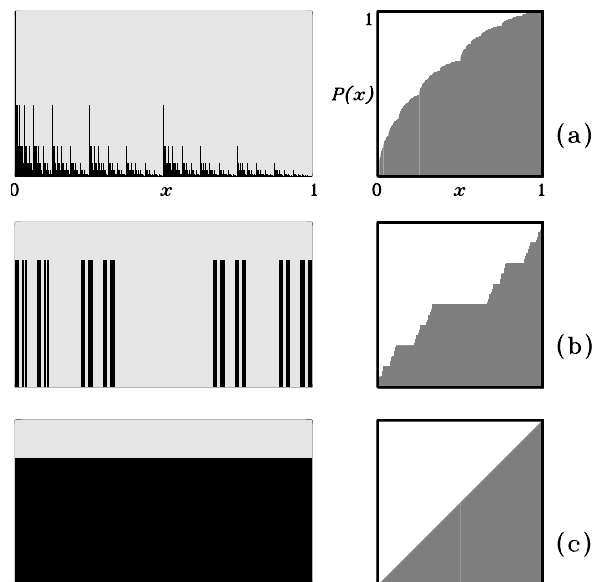

Figure 3 *Accumulated clay (right) corresponding to objects (left) generated by: (a) propagating imbalances, (b) spreading disunity, and (c) maintaining the initial bar.*

The accumulated sets for the two games turn out to be rather peculiar, for if one were to parachute on them from above, upon landing one would falsely believe that they are flat, that is, one would think to have landed on top of the uniform bar of clay. Irrespective of their *imbalances*, $p \neq 1/2$, or *holes*, $h \neq 0$, such rugged boundaries define *devil's staircases* whose lengths, from their initial and final points, equal *two* units, that is, one unit horizontally plus one vertically (Feder, 1988).

These accumulated sets and also those defined by more general multiplicative cascades, for example: combining arbitrarily the generic games (i.e., having constructs that use both imbalances and holes), splitting the bar of clay in more than two pieces, and using chance to define the associated partitions from level to level, all ultimately yield *maximal* lengths of two, as they 'travel' by the *legs* of the right-angled triangle associated with level zero.

There is *only* one case that does not destroy the *unity* and true *equilibrium* of the original bar, and that corresponds to the 'fifty-fifty' adage (i.e., $p = 1/2$ and $h = 0$) that results in the aligned *hypotenuse* and the *minimal* distance of $\sqrt{2} \approx 1.4142...$
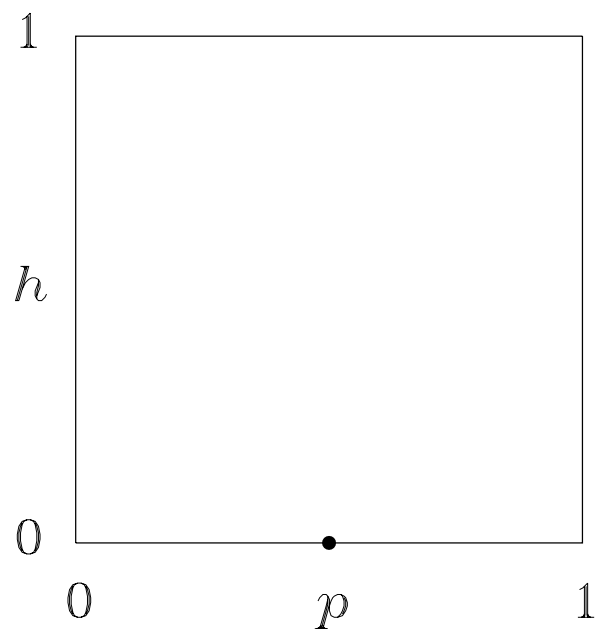

Figure 4 *The point of equilibrium within a sea of probabilities.*

The power of the air

When the Reynolds number is high, the air's internal cohesion is shattered and it flows in an intermittent and irregular fashion that is nicely captured by the progressive breaking (precisely by 70%) implied by the first game for kids (Frisch, 1995).

As depicted below, one-dimensional observations of natural turbulence, and also for other turbulent flows gathered in the laboratory, are *universally* consistent with a multiplicative cascade of *eddies*, that divide precisely according to the proportions shown: the 'offspring' eddies have half the size of their 'parent' and they transport respectively 70 and 30% of their parent's energy (Screenivasan, 1991)[2].
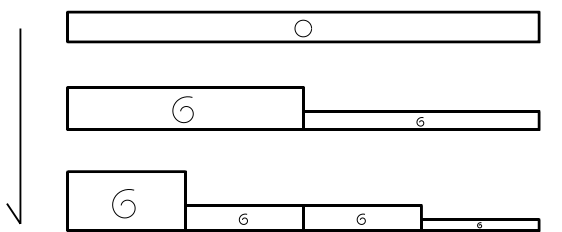

Figure 5 *A multiplicative cascade of eddies in fully developed turbulence.*

Although these results do not allow making predictions of this common condition, for the power of the air does not always use the 70% to the left but rather does it to both sides as guided by chance (Screenivasan, 1991), the process describes the formation, everywhere on earth of *violent* thorns and explains the ultimate fate of division in the eventual *dissipation* of all energy in the form of heat.

The games and the world of politics

As a simple repetitive cascade captures the complexity of natural turbulence and as various generalizations of the games herein have been found useful to represent other intricate geophysical processes, it is reasonable to employ such notions, certainly combining the games and adding randomness, in order to illustrate the possible pathways that give rise to the ample *turbulence* we experience in our modern world.

In this spirit, the simple cascade games may be used to describe the inherent dynamics of the systems that have governed the world, as follows. While the first generic game helps us visualize the proliferation of economic imbalances and the resulting distrust of competition in capitalistic societies, the second game allows us to sense the futility of totalitarian regimes whose stubborn trapping of dissenting voices into 'holes' utterly destroys unity.

Although these observations may appear to be 'too simplistic', the cascades also allow us to view the ultimate and historic consequences of the world's

political and economic systems. On the one hand, the unexpected but predictable dissipation of communism due to its rather empty and false 'equality' and, on the other, the misery experienced by many in the world today and the further dangers posed by the *thorny* game of globalization.
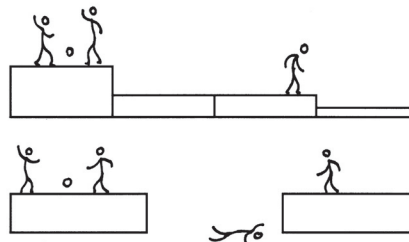

Figure 6 *Generic propagation of inequities and discriminations.*

In this regard, the first game offers a rather poignant warning against current policies by the 'civilized world', as the very object produced by such a divisive cascade after $n = 20$ levels closely matches the distribution of wealth of the most powerful nation on earth, precisely when $p = 70\%$. In fact, simple calculations, using Pascal's triangle, reveal a close match for the wealth of the richest 5, 10, 20, and 40 percentiles in the *United States*, that is, in order, 59 (57), 71 (70), 84 (84), and 95% (95) of the resources, with the cascade values given in parenthesis[3].

Although the wealth of the richest 1% in 1998 is underestimated by such a cascade, 38% (30), the simple game help us imagine what would happen if imbalances persist, either as n or p increases, or both[4]. For even if current conditions are termed 'stable' and 'prosperous' by experts, it is easy to foresee an 'unhealthy' future[5]. For as kids everywhere know, and as the laws of physics assure us, the game of imbalances ends as the game of disunity does: with 'all biting the dust'.

The games and our quest for peace

Clearly, the simple ideas herein also apply to us as individuals, as the cascades reflect and encompass our ever changing attitudes. But the notions also provide a positive image, for they allow us to contemplate the *balanced* state where we all may experience *friendship* and *peace*.

As may be appreciated, to break or to mend equilibrium is our ultimate decision. For we, as kids that we are, may choose to 'sail' the natural cascade in reverse, exchanging a selfish and turbulent spiral

($r = e^{-\theta}$), and the symbolic 2/3 = 0.666... that may be read on any divisive game[6], by an outward and loving one ($r = e^{+\theta}$) that dreams a just future in order to restore *unity*. For in a rather graphical fashion, both arithmetically and geometrically, 1 = 0.999..., as may be appreciated when mountains are cut-down and valleys are filled-in on any (multifractal) object that gives rise to a devil's staircase.

As with the arrow of time we may basically engender two distinct behaviors: either the shortest path that maintains *equilibrium* or an evil stair that through fragmentation augments *entropy*, that is, either √2 or 2, these observations remind us that it is wise to live life below the threshold[7]. For, it is thoughtful to augment our internal *cohesion* to repel the false lure of the superiority postures embedded in the pervasive cascades, which prevent us from truly standing together in solid and common high ground[8].
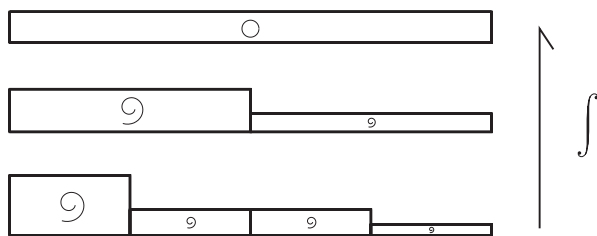

Figure 7 *Multiplicative reparation of unity via integration.*

These universal concepts reiterate that it is up to each one of us to find and to spread peace, for we may choose to break equilibrium using either cascade or their combinations, irrespective of the nation's political system in which we reside. These images also reaffirm that our most desired state is intimately related to *rectitude*, *balance*, *forgiveness*, *friendship*, and *Love*, while its antithesis is associated to indifference, vengeance, hate (even against our enemies) and with any type of 'inequality' or 'discrimination' that produces distrust, emptiness, and ample loneliness.

For the common but anomalous states defined by our divisive traits are potently symbolized by the absurdity of 'walking over thorns', a feared condition that may not be escaped via hallucinogens or other worldly distractions, but by our daring to live in solidarity helping one another.

The evils we all face are, at the end, very similar. As our seemingly distinct 'power games' eventually yield ample dust and extreme separations (infinite in size if not corrected), some based on our relentless desire for wealth leading to obvious disparities (the first game), that is, \$, which is the opposite of the desired integration, the slender letter S; and others caused by trying to impose a given life-style by force (the second game), these reflections point us to a *unique pathway*, to one condition truly worthy for all, to one improbable but feasible organization that dismisses hypocrisy and terror, to one just state that 'heals the wound' and respects all life, that is, to the one and only situation that allows us to find our common *root:* the plain and simple case symbolized by the precious and imperturbable hypotenuse, $Y = X$[9].

THE HYPOTENUSE
(Carlos E. Puente)

By the wisdom of science
simply divides the air,
to dissipate all its heat
coding a subtle cascade.

Turbulence is selfish game
for it scatters the whole,
and its sequence is a frame
for the options of the soul.

Two options before us
two pathways ahead,
the one is the longest
the other straight.

We journey directly
or go by the legs,
we follow intently
or end up in pain.

By walking the flatness
or hiking the spikes,
we travel in lightness
or take serious frights.

The incentive is unity
and the call proportion,
the key is forgiveness
and the goal true notion.

In wandering wickedness
there is never a fruit,

but in ample humbleness
one encounters the root.

**There is no excuse,
let's practice fair game:
it's by the hypotenuse
or else by the legs.**

**There is no solution
but walking straight:
the spikes of disorder
insinuate the way. (2)**

There is a best pathway:
the palpably smooth.

**It's by the hypotenuse
and walking in truth.**

There is one solution,
I tell you the truth.

**It's by the hypotenuse
and walking in truth.**

For any other pathway
will lead us astray.

**It's by the hypotenuse,
there is no other way.**

Oh listen, you brother,
let's brighten the day.

**It's by the hypotenuse,
there is no other way.**

Otherwise, the devil
shall pull by the legs.

**It's by the hypotenuse
or else by the legs.**

For such road is fractal:
as long as it gets.

**It's by the hypotenuse
or else by the legs.**

Let's end all disparities,

growing to the root.

**It's by the hypotenuse,
the one that yields fruit.**

Let's keep equilibrium,
avoiding dark soot.

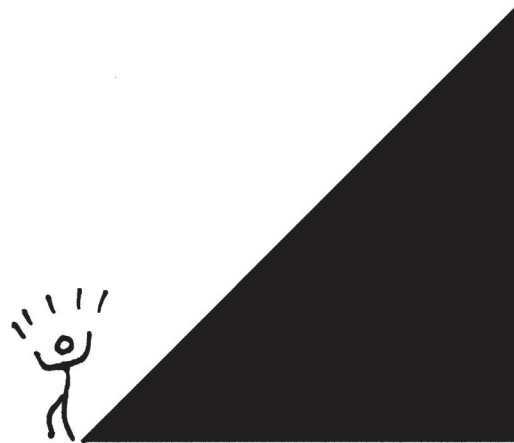**It's by the hypotenuse,
the one that yields fruit.**

Oh listen, you sister,
a counsel from science.

**It's by the hypotenuse:
the simplest design.**

I tell you integrating,
don't leave it to chance.

**It's by the hypotenuse:
the simplest design.**

**It's by the hypotenuse:
the simplest design...**

Acknowledgments

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Notes

1. For instance, Mandelbrot (1982), Bak (1996), and Wolfram (2002).

2. The cascade model is due to pacifist L. F. Richardson (1922).

3. For statistics on wealth see Keister (2000) and the web site www.globalpolicy.org. For information on globalization consider, for instance, Hertz (2002).

4. If $p = 70\%$ and $n = 30$, the richest 5, 10, and 20% on such a society would have, in order, 73, 84, and 92% of the wealth. If $p = 75\%$ and n = 30, the disparities become, in order, 90, 95, and 98%.

5. Wealth distributions of countries throughout the world may be fitted via general random multiplicative cascades leading eventually to devil's staircases, irrespective of their Gini indices.

6. The fraction 2/3 is found prominently in diverse power-laws related to natural turbulence, for details see Frisch (1995). Coincidentally, the poor in the world represent 2/3 of the population (de Soto, 2000).

7. Although a bit of turbulence may be welcomed when it shapes our characters, it is best to avoid its ultimate consequences. This means lowering the 'velocities in our lives', diminishing our 'characteristic lengths', and augmenting our 'viscosities'.

8. Only if one parachutes into the hypotenuse, one slides into the origin!

9. A more detailed exposition of the material herein and in the form of a tale is given in Puente (2006).

Choices

Equilibrium	Turbulence
Calmness	Violence
Rectitude	Wickedness
Reconciliation	Separation
$\int$	$
1 = 0.999...	2/3 = 0.666...
Wholeness	Emptiness
Life	Death