CHAPTER I
Lessons from Turbulence

The techniques I developed for studying turbulence, like weather, also apply to the stock market.

Benoit Mandelbrot,
author of The Fractal Geometry of Nature

Since childhood we all have noticed that when sufficient energy is supplied to a fluid, such as water or air, an orderly and calm condition is transformed into a disorderly and erratic state that we denote as turbulence. As more and more energy is added, the faster and less cohesively the fluid moves, and the more violent and intermittent it becomes. Natural turbulence is certainly very common, and when it happens with enough strength, we all instinctively run for cover. When we are caught by it, say while riding an airplane or traveling by boat, its unrest commonly inspires deep and conscious breaths and also sudden prayers.
Mankind has been fascinated by the varied and unpredictable nature of turbulence and has steadily uncovered and tamed many of its mysteries. Although we have accomplished clear advances, which have resulted, among other achievements, in improvements in transportation unforeseen not too long ago, the awesome destructive power of hurricanes, forest fires, floods, tsunamis, etc. reminds us that we remain, even in the twenty-first century, at the mercy of natural turbulence. These recurrent disasters certainly pierce the core of modern man whose pride of technological advances is painfully dampened by our limited abilities, and this is so despite our improving skill to foresee and track the overall evolution of some turbulent events.

This chapter explores matters of equilibrium and peace by considering the opposite concepts of fragmentation and violence, as they arise in the remarkably simple way by which energy is divided and dissipated in fully-developed turbulent flows. Arguing that science provides a suitable and impartial framework to think about our own divisiveness, both as individuals and as members of society, and also as reflected by the political systems that have governed the world, this chapter shows that there is one and only one state that qualifies as truly balanced and corresponds to a serene condition that begets peace. Quoting several passages from Holy Scripture, it is explained how achieving such an ultimate state of wholeness is intimately related to heeding the teachings of Jesus Christ, including, in particular, his repeated calls for our continuous and dynamic growth to humility and repentance and our universal fulfillment of the defining maxims “cut the mountains and fill the valleys” and “love one another.”

A Game for Kids

As shall be noted, this book contains various constructs that describe how natural complexity happens. The first one is a simple game that we kids may understand by molding clay, as follows.

Start with a uniform bar of clay, as it comes out of the box, and cut it by its $p = 70\%$, from left to right:
Then, mold the two pieces, piling up the one on the left and stretching the other on the right, both towards the middle, so that they make up, at the end, two contiguous bars of equal horizontal length:

\[
\begin{array}{c}
0 \quad \frac{1}{2} \quad 1 \\
\hline
p=70\% \quad \eta=30\% \\
\end{array}
\]

The piece on the left is higher than the original, and the one on the right is lower. If the original bar measures one vertical unit, then the bar on the left has \(2 \cdot p = 1.4\) vertical units and the one on the right \(2 \cdot q = 0.6\) vertical units.¹

With this simple process fully understood, now repeat it on each of the two uniform bars, following exactly the same proportions, to obtain four pieces having equal horizontal sizes:

Now the masses (areas) of the four pieces are, in order, 70% of 70%, 30% of 70%, 70% of 30%, and 30% of 30%, which gives, just multiplying, 49, 21, 21, and 9% of the total mass.

As may be easily verified, the values just encountered correspond to the familiar expansion of \((p + q)^2\), that is, \(p^2\), twice \(p \cdot q\), and \(q^2\), and the heights of the four rectangles are, from left to right, 1.96, 0.84, 0.84, and 0.36 vertical units.

Clearly, as the breaking is repeated, additional fragmentation is produced and the tallest triangle continues to increase:

Now the maximum height reaches \(1.4^3 = 2.74\) vertical units, and the mass, always adding to 100% due to the “principle of conser-
vation of modeling clay,” arranges into four layers according to the expansion of \((p + q)^3\).

That this is the case, not only for this step but all along, may be seen realizing that the number of “rectangles” corresponding to the layers formed are associated with the possible products of the partitions \(p\) and \(q\):

\[
(p + q)^0 = 1
\]
\[
(p + q)^1 = 1 \cdot p + 1 \cdot q
\]
\[
(p + q)^2 = 1 \cdot p^2 + 2 \cdot pq + 1 \cdot q^2
\]
\[
(p + q)^3 = 1 \cdot p^3 + 3 \cdot p^2q + 3 \cdot pq^2 + 1 \cdot q^3
\]

As may be seen, there is one layer on the original bar, \(n = 0\), two layers as the partitions start, \(n = 1\), three layers, \(p^2\), \(pq\), and \(q^2\), for \(n = 2\), and four layers for the previous figure, \(n = 3\), as there are, as prescribed by Pascal’s triangle, one rectangle for the largest and smallest layers, and three for each of the intermediate masses.

To further illustrate what is found via this simple game, one that is suitably named a multiplicative cascade, we need to implement it on a computer. For \(n = 12\), we encounter \(2^{12} = 4,096\) thin rectangles arranged in 13 layers:

As is seen, this is a rather “thorny” object for if we were to touch it from above, we would get pricked by its many spikes, all sharing a rather small base size of \(1/2^{12}\). As the height of the largest rectangle now grows to be \(1.4^{12} = 56.69\) vertical units, the object,
which is vastly different from the original bar, cannot be shown as it is, but only compressed so that it may fit on the page.

Here there are indeed thirteen layers of mass, even if only the top five or six may be seen. There is one spike, to the left, having $p^{12}$ of the mass and one small one, to the right, having $q^{12}$, there are 12 rectangles having $p^{11} \cdot q$ and also $p \cdot q^{11}$, 66 pieces with $p^{n-2} \cdot q^{2}$ and also 66 with $p^{2} \cdot q^{n-2}$, 220 for $p^{3}q^{3}$ and $p^{3}q^{3}$, 495 for $p^{8}q^{8}$ and $p^{4}q^{8}$, 792 for $p^{7}q^{5}$ and $p^{5}q^{7}$, and 924 for $p^{6}q^{6}$, with the density of thorns increasing from side to side as we enter into the middle layers, or as we enter into Pascal’s triangle from both sides.

As is seen, this simple game utterly destroys the wholeness of the initial bar and produces instead dispersed thorns that exhibit a remarkably intertwined structure. As is noticed, the progressive cuttings, pile-ups, and stretches create division everywhere and such destroys the contiguity of thorns of equal size for all layers, leading to sets that are increasingly difficult to tread. As is observed, between any two spikes of a given layer there are typically many other spikes from other layers, and hence the arduous walk.

As this silly game is repeated yet more times, the thorns and their interconnectedness multiply and the resulting object becomes truly impossible to traverse. In such a limit, containing infinitely many layers, the mass on each thorn vanishes, but altogether the then-infinite-sized spikes conserve mass in a conspicuously woven way. The structure of such a curious object, made of infinitely many thorns, may be further appreciated via another game.

**Another Game for Kids**

This game may also be understood molding modeling clay, as follows. Again start with a uniform bar, but now cut by the middle:

![Diagram](image)

Then, separate the two pieces piling them up left and right, leaving a gap of size one-third in between:
Clearly, these rectangles, having an equal base of $1/3$ and an area of $50\%$, end up being taller than the original bar. In fact, their common height may be easily shown to be $1.5$ vertical units.

With the game fully understood, this process continues by repeating the operation on each piece, cutting and separating according to the same proportions, to yield four pieces of equal height:

This construct yields another multiplicative cascade that produces, after $n$ levels of the construction, $2^n$ dispersed rectangles having equal masses of $1/2^n$, horizontal lengths $1/3^n$, and diverging heights that grow according to the formula $(3/2)^n$:

As the process is repeated many times, this second game clearly generates long spaghetti strings and ultimately spikes that never touch. This happens due to the insertion of holes everywhere, which forces the limiting thorns to emanate from a rather “empty” array of points having the structure of dust and known as the triadic Cantor set, as introduced by German mathematician Georg Cantor in 1883.³ As for the first game for kids, the sparse spikes of this game prick us when we touch from the top (and also from the bottom), as they are rather thin needles that grow to infinity, altogether conserving mass, but containing no mass individually.

It happens that the structure of dispersed thorns of equal size generated by this second game calls to mind the layers of thorns produced by the first game. In fact, varying the size of the original hole, from $1/3$ to a generic value $h$, turns out to capture not only the
non-contiguous topology of the layers on the first cascade, but also their various densities, with sparser layers, towards the periphery of Pascal’s triangle, corresponding to larger holes, and with denser layers, going inside Pascal’s triangle, requiring smaller gaps.

The moral of the story is that although the two divisive games are based on distinct constructions, none is the lesser of two evils, as they are intimately related to one another. Both cascades yield thorns and dust, and the game with holes lives inside the layers generated by the first game, but in order to more fully comprehend the detailed structure of these fractured sets, it is relevant to review some key concepts about the geometry of nature.

**Dimensions and Fractals**

The notion of dimension is fundamental in describing our world. We know that we live in three-dimensional space and that time makes up another dimension. We have been taught that points have no dimension, that straight lines are one-dimensional, that planar sets are two-dimensional, and that solid volumes are three-dimensional. In recent years, however, with the advent of the seminal work of Benoit Mandelbrot, we have learned that nature’s irregular and fragmented geometries give rise to intermediate, and commonly non-integer, dimensions and to objects we now call fractals.4

As lucidly explained by Mandelbrot, nature’s intricacies do not conform to the familiar objects of Euclidean geometry: “clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.” Yet the concept of dimension may still be used to arrive at a new language aimed at describing nature’s complexity.

Even though several technical definitions exist,5 the concepts of dimension and of a fractal may be explained via the so-called box-counting procedure. The idea turns out to be rather simple, as in yet another game, and stems from counting the minimal number of “boxes” (intervals, squares, cubes) of different sizes needed to cover a given geometric set and studying what happens as such boxes become smaller and smaller.
To illustrate the ideas, shown next is the covering by closed intervals of sizes $1/3^n$ of a straight line segment, on the left, and the triadic Cantor set on the right, as it appears while playing the second game for kids:

As is seen, when the box size, $\delta$, equals 1, both the line segment and the Cantor set may be covered with one box. But as the size is reduced to $\delta = 1/3$, the line segment requires three boxes while the Cantor set only needs two. As may be inferred, when $\delta = 1/3^n$, the line segment needs $N(\delta) = 3^n$ boxes to cover it, but the Cantor set requires, due to its holes, only $N(\delta) = 2^n$ intervals.\(^6\)

The concept of dimension arises noticing that $N(\delta)$ is related to $\delta$ in a simple manner. For the line segment, such a relation is indeed straightforward. As the box size decreases in powers of three and the number of boxes increases in powers of three, $N(\delta) = \delta^{-1}$. For the Cantor set, the calculation is a bit more involved. As the box size decreases in powers of three and the number of boxes increases in powers of two, this yields $N(\delta) = \delta^{-\ln 2/\ln 3}$, where $\ln$ is the natural logarithm.\(^7\)

In both instances, the number of boxes and their respective sizes are connected via a power-law relationship:

$$N(\delta) = \delta^{-D},$$

where $D = 1$ for the segment, and $D \approx 0.631$ for the dust.

The exponent $D$ is the box-counting dimension of the geometric object and serves to distinguish between fractal and non-fractal sets, as follows. As a point has zero box-counting dimension, for
$N(\delta) = 1 = \delta^0$ for an interval of any size covering a point, the tridec Cantor set, being a collection of sparse points, is geometrically speaking “nothing.” This is translated into saying that the topological dimension of the Cantor dust is zero. This set is termed a fractal because its box-counting dimension of $\ln 2/\ln 3 \approx 0.631$ exceeds its topological dimension. A line segment, on the other hand, is not a fractal object. This happens because such an interval, also made of great many points but without holes, has topological and box-counting dimensions that match. The line segment is a one-dimensional Euclidean object that is not a fractal as it is not fragmented or fractured. In general, an object is called a fractal if its box-counting dimension exceeds its topological dimension.

It turns out that the box-counting dimension, hereon referred to as the fractal dimension, reflects the ability of a given set to fill up the space in which it lies. The construction of dusts of varying densities, as required to describe the layers on the first game for kids, serves as an example of this general rule. If, instead of removing the middle-third subintervals, we take out $l\%$ equidistant segments, then such a construction generates other dusts, whose fractal dimensions may be shown to be any number between 0 and 1. Clearly, distinct densities are possible and such encompass the limits, for if the hole is as large as the interval, then all points would be wiped out and $D = 0$, but if there is no hole, then the interval remains and hence $D = 1$.

In regards to the games for kids, these observations help us appreciate that while the second cascade produces equal thorns over a single fractal dust, the first game yields multiple layers of thorns each defined over their own fractal dusts. The first game ultimately gives exquisitely-woven disjoint dusts, which hence correspond to a multitude of intertwined fractals having various fractal dimensions, as reflected by the location of a layer on Pascal’s triangle. For this reason, the spiky object given by the first game is properly known as a multifractal.

Other fractal objects defined over the plane or the volume, and having fractal dimensions ranging from 0 to 3, shall appear later on in this book. But for now, we continue studying how the games for kids are related to natural turbulence and our own fragmentation.
To further appreciate the thorny objects generated by the multiplicative cascades and as spikes on both games grow without bound, it is convenient to portray their accumulated masses, traversing such objects from the beginning, that is, zero, up to a point $x$, and as a function of $x$:

As may be understood following the dynamics of the games, the objects given by the cascades and shown on the left, give rise to jagged boundaries $C(x)$, shown on the right, that contain many places where tangents may not be defined.

For the first game, a frontier shaped as a “cloud of dust” is found, one that, while passing by the points $(0,0)$ and $(1,1)$, contains a multitude of horizontal-vertical notches. The most notorious notch happens at the point $(1/2, 0.7)$, for, by construction, the multifractal contains up to the middle 70% of the mass. Then, there is a notch at $1/4$ with height 0.49 (70% of 70%), another at $3/4$ with height 0.91 (100% minus 9%), and so on.

For the second game, a jagged frontier is found that includes great many plateaus that correspond to the holes in the cascade. The longest one, from $1/3$ to $2/3$, has a height of 0.5, for the object made of equal thorns contains up to $1/3$, and also up to $2/3$, 50% of the mass. Then, there are plateaus with lengths $1/9$ having heights $1/4$ and $3/4$, and so on.
As is seen, the accumulated clay boundaries are mathematical “monsters” that contain numerous kinks. While the continuous dust cloud for the first game does not have derivatives anywhere, the second one does not have them at the corners of all plateaus. This happens because in the limit all thorns contain no mass and therefore there is “nothing” to add everywhere. As ultimately there are notches and plateaus on the accumulated frontiers everywhere, both objects are locally flat, and their distances from bottom to top satisfy the odd equation:

\[ d\{(0, 0), (1, 1)\} = 2. \]

Strikingly, if we were to crisscross these boundaries, and as there is no way for us to go off on inclined tangents, even if there is an optical illusion that such lines exist due to the resolution of the graphs, we would need to walk all the horizontals and also climb all the verticals, and hence the total distance ought to be one horizontal unit plus one vertical unit, or 2 units.

This property, ascribing a maximal length to the rugged boundaries, turns out to be universal as it applies for any other imbalances \( p \) and holes \( h \). In fact, whenever there is an imbalance on the first game, no matter how small, or a hole on the second game, again no matter how small, the games do generate thorns over dust, whose individual masses tend to zero, and, as such, their locally flat and serrate frontiers give rise to maximal distances equal to 2 units.

Not only there is no advantage in having a small imbalance or a small hole, but also the same happens when the games are combined to define other cascades that contain them both:

and also when chance is used to assign the partitions from level to level, to specify yet more sophisticated divisive processes. In all such cases, their accumulated frontiers also measure 2 units.
These observations about the curious accumulated boundaries given by multiplicative cascades imply that if we were to allegorically parachute on them:

Upon landing, we would believe to have arrived to flat terrain:

Because of this clear deception, Georg Cantor himself, who thought of God as the absolute infinity, named the falsely appealing frontier for the second game the devil’s staircase, a definition that, given their universal local flatness and the divisive nature of their generating games, properly qualifies all cascades.

As a summary, and to set key symbols for further use, the two games for kids and their variants, that is, combining them and allowing for chance to dictate the divisions from level to level, give rise to thorns, dust, and devil’s staircases, and the latter, by not containing any inclined tangents, are as inefficient as possible.

**Turbulence in the Air**

As we have all seen, say while boiling water, when enough energy is supplied to a fluid, it becomes increasingly erratic and turbulent, and it rearranges into rotating elements called eddies, which swirl to transport energy from places of high pressure to regions of low pressure, that is, from plus to minus. These eddies are very
common and happen, for instance, in the whirling of high mountain streams, in the breaking of ocean waves into progressively smaller ones, in the dissipation of smoke in air, and noticeably in the scary vortex structures of tornados and hurricanes. They also occur within sets of much larger sizes such as in the galaxies and clusters of galaxies making up the universe.

As expressed by the famous Reynolds number,

\[ Re = \frac{v \cdot L}{\nu}, \]

when the internal cohesion of the fluid, that is, its viscosity \( \nu \), is overpowered by excessive inertia, as defined by the flow’s velocity \( v \) times a characteristic length \( L \), the fluid can no longer flow as a whole and in a laminar and calmed way, but rather splits into eddies carrying distinct energies, and this happens in a disorganized and turbulent fashion that is often accompanied by intermittent outbursts of violence.\(^{12}\)

While sporadic eddies form under weakly turbulent conditions, fully-developed turbulence, a situation without constraints and associated with high enough Reynolds numbers,\(^ {13} \) vastly breaks the cohesion of a fluid and generates an active array of rotating elements of a variety of sizes that conform to the notion of a cascade, as first anticipated by scientist and pacifist Lewis Fry Richardson in 1922.\(^ {14} \)

What happens to the air, as surmised via modern technology, is that observations of energy along a line are indeed consistent with the presence of a cascade:

![Diagram of energy cascade]

Remarkably, and as guided by the symbolic and pictorial ratio \( 2/3 = 0.666\ldots \),\(^ {15} \) eddies spawn baby eddies as in the first game for
kids, and yet more remarkably, the energies that such eddies carry with them correspond to the values given by the 70-30 splitting.

In a rather striking manner, energies rearrange into layers as if playing a simple game and the ultimate fate of the progressive breaking is the eventual dissipation of the kinetic energy in the form of heat, when the eddies reach a small enough scale.\textsuperscript{16}

As first reported by Charles Meneveau and Katepalli Sreenivasan in 1987, the multifractal outcome of the first cascade yields a proper encyclopedia of energy layers when the Reynolds’ threshold is high enough for the air.\textsuperscript{17} While diverse measurements do not look identical, as the larger baby eddies do not always happen to the left as herein, such sets do share similar densities and fractal dimensions and also exhibit similar separations between layers. As such, they are just suitable, albeit unpredictable, “rearrangements” of the generic encyclopedia.

It so happens that these surprising results are also remarkably universal, as they are valid for a variety of natural flows, including atmospheric and boundary layer turbulence, and also for laboratory flows such as the turbulence generated by the wake of a cylinder. At the end, there is a truly excellent agreement between observations and the first cascade when $p = 0.7$, for the parabola below, corresponding to the dimensions, $f(\alpha)$, of layers, $\alpha$, as obtained entering Pascal’s triangle from both ends, encompasses the various observations as marked by squares:\textsuperscript{9,17}

\begin{center}
\begin{tikzpicture}
\begin{axis}[
    width=6cm,
    height=6cm,
    xlabel=$\alpha$,
    ylabel=$f(\alpha)$,
    xmin=0,
    xmax=2,
    ymin=0,
    ymax=1.5,
]
\addplot coordinates {
(0,1.5)
(0.25,1.3)
(0.5,1.1)
(0.75,0.9)
(1,0.7)
(1.25,0.9)
(1.5,1.1)
(1.75,1.3)
(2,1.5)
};
\end{axis}
\end{tikzpicture}
\end{center}

It is worth emphasizing that while the physics of turbulence follow the mathematical cascade, the ending in dissipation at little eddies having a non-zero scale means that the natural cascade does not go on forever. Certainly, the duration of the game, and
its associated violence, increases with the Reynolds number, but their accumulated energy profiles do not measure 2 units, but less.

With these notions in place, that is, *eddies, turbulence, dissipation, thorns, dust, fractals, and cascades*, we may turn our attention now to our own turbulence.

**OUR TURBULENT TIMES**

As we ponder these surprising results regarding turbulence in the air, and as indeed “turbulence is in the air,” their striking simplicity and universality hint that it is sensible to employ the cascade notions to study how we kids create our own *turbulence* and to ponder how such a condition may be avoided in our lives.

After all, all of us, from Afghanistan to Zimbabwe, are faced with “inertial forces” that break our “internal cohesions” and, when such happens, “for high enough Reynolds numbers,” it leads us to “intermittent behaviors” and an inherent “lack of peace” that often results in the appalling expression of *violence*. As our distress, and certainly mine, is typically associated with the relentless repetition of a divisive trait, it also appears reasonable to employ the two general cascades to symbolize the pathways that lead us, and certainly me, to *bite the dust* when guided by our selfish “eddies.”

The general validity of the notions and the intrinsic reiteration of the breakings at various resolutions also suggest that the simple geometric ideas may be used, at the very least figuratively, to capture our “distress” at a variety of scales: within ourselves, in our relationships, in our societies, and in the world at large. For, as evidenced by the approving smiles I have witnessed whenever I share these ideas, the two cascades allow us to contemplate the most common “games” we kids employ to propagate division, that is, the establishment of *imbalance*: 

![Diagram](attachment:image.png)
and the practice of *discriminations*:

![Diagram](image)

Certainly, “turbulence are us,” and while the first game may be employed to describe the inequities generated by our preferential and competitive instincts, including the cynicism experienced by people living under such a condition, the second cascade may be used to portray the awful effects of prejudice and the related distrust and fear associated with organizations established by force. These games, no doubt, describe our *selfish* postures and actions, or the *pride* we exercise when traveling from *plus* to *minus*. For although the figurines above may bring up a condescending smile, what they display is, at the end, not a laughing matter.

As hinted and perhaps surprisingly, these simple diagrams also allow us to visualize the dynamics of the political systems that have governed the world: on the one game, the unexpected but rather predictable *dissipation of equality by force*, a disjoint system that predictably collapsed as friendship and love were curtailed by the real fear of “ending up in a gap,” and on the other game, the present rise of global inequalities via the apparent triumph of the false dogma of *the survival of the fittest*, in a system that, despite multiple claims to the contrary, also grows *thorns* and *dust* via the implementation of dehumanized *competition* inspired by the implacable quest to be number one.
For, in agreement with the quotation that starts this chapter and extending it a bit to encompass us, the general cascade ideas have an unexpected validity beyond the merely metaphorical, as the *multifractal* object generated by the first cascade, for \( n = 20 \) levels and for \( p = 0.7 \), that is, with exactly the same energy partitions as chosen by nature, closely matches the rather skewed distribution of wealth of the most powerful nation on Earth. As may be readily verified via Pascal’s triangle, and bearing in mind that for such a number of levels there are many layers that may not even be seen, there is a remarkably close fit of the wealth of the richest 5, 10, 20, and 40\% in the *United States* as reported in 1998, that is, in order, 59 (57), 71 (70), 84 (84), and 95\% (95) of the resources, with the cascade values given in parenthesis.\(^{19}\)

Although man does not live by bread alone (Mt 4:4) and the wealth of the richest 1\% is underestimated by the simple cascade, 38\% (30), these remarkably simple results help us visualize the dreadful consequences of establishing and further propagating imbalances and vividly invite us to universally reverse the trends so that we all may restore due dignities and avert the dissipation prescribed by the laws of physics and in particular by common sense.

For even if experts and others, who typically employ finite numbers to talk about us, assure us that there are better wealth distributions than others,\(^{20}\) in truth all devil’s staircases,\(^{21}\) and that the practice of *globalization* shall one day bring *justice* and *well-being* to all, and especially to the 2/3 of humanity living under extreme poverty,\(^{22}\) the very coincidence of the same allegorical ratio here, as found pictorially in the generic turbulent cascade,\(^{23}\) potently calls attention to our historical *evil* and *greed*.

These notions clearly point us to find a better landscape not guided by “competing against one another” or “dismissing one another,” but rather to one established on *Love*.

**The Faithful Solution**

Based on the games and also on our common sense—yes, our common sense—we may realize that there is indeed one solution. Such
is defined by inverting the direction of the cascade to mend the broken, living at low Reynolds numbers to avoid the anxieties and violence of our modern world, and, to say it using the language of the prophets Isaiah and John the Baptist, “cutting the mountains and filling the valleys” (Is 40:4, Lk 3:5–6) to arrive at equilibrium:

Clearly, the best action is to return to the original bar. For in a graphical and mathematical manner, the sought unity, for all of us, is made of infinitely many outwardly rotating and hence positive spirals, $1 = 0.999\ldots^{24}$ that are opposed to the negative spirals induced by diabolic division, $2/3 = 0.666\ldots$, that is, the ones ruled by “the power of the air,” in which we may appreciate, according to Saint Paul, the devil himself (Eph 2:2), the one who represents a false promise, even if he rules the world (Jn 12:31) and confused Adam and Eve inciting them to their original sin of breaking the original bar until dust, which resulted in their dissipation and due death (Gn 3:1–19). For even if he whispers in our distress that division is unavoidable and that brotherhood is impossible, there is no other solution but to go back to our essence.

Based on these geometric observations, we may see, for ourselves, the emergence of the ultimate solution in the leveled field:

Such consists of not playing any divisive games but instead maintaining dynamically the original bar, that is, the unitive level zero,
by practicing the always proverbial 50-50 with everybody, without exceptions—that is, without holes.

As is seen graphically, such a practice entails what Adam and Eve and many others did not do—repentance, conversion, and rectification—for only via such actions we may arrive at the only straight and solid condition we all can safely walk and even play hopscotch at, for it is the only circumstance that does not contain any thorns or dust. This is our best and most efficient way, a straight path that, by not falling into the lies of division, maintains the truth, and by not dissipating, stays alive.

As may be verified, the “broad valley” expressed by prophet Isaiah (Is 40:4) and the “salvation of God” alluded to by John the Baptist (Lk 3:6) are consistent with these reflections and with Jesus’ own claim, just explained, that He is the solution: “the way, the truth, and the life” (Jn 14:6), for He fulfilled the bar of perfect equilibrium, as He kept God’s law and never sinned (Mt 5:17).

His radical call to Love, even if it appears “irrational” to us, is certainly seen in the smallest distance of the straight ramp:

\[ d\{(0, 0), (1, 1)\} = \sqrt{2}, \]

and in the fact that if we parachute on the hypotenuse:

we end up sliding into the origin:
Notice how this commitment to ultimate faith, one expressed consistently by landing in the steep one-to-one line, is consonant with the geometric statement that “the hypotenuse is the pathway of peace” and, more importantly, with Jesus’ own words when He said that no one goes to the Father, the Origin, except through Him (Jn 14:6).

These reflections turn out to be symbolically accurate, for the one-to-one line on the efficient ramp reminds us that it is always in such a way, that is, “one on one,” that we meet the divine, and for the very equation of the simplest of lines, namely \( Y = X \), reflects, rather geometrically, the silhouette of the crucified on the cross—the one who died for us and left us His special peace (Jn 14:27), even if crowned by the thorns of our own cascades (Jn 19:2).

There is indeed a big difference between what was previously found for the crooked devil’s staircases and what is found for the smooth hypotenuse. While cascades and combinations thereof give rise to maximal lengths equal to 2 units, abstaining from the games results in the minimal length of \( \sqrt{2} \) that also reflects the attainment of the essential in the precise symbol of the root.

How beautiful it is to witness married couples in love, people exercising their outward spiral and yielding to the divine by faith and without fear. How beautiful when such a love grows as a true commitment and radiates to the world without conditions or premises. Such is definitely the miracle of the united level zero, where we all may become number one:

\[
\begin{align*}
\end{align*}
\]

A Point not to Miss

Miracles, like somebody becoming holy, even if for a while, are certainly improbable, but that does not make them impossible. The real question is whether, based on the reflections herein, there are any better choices.
As previously mentioned, when the two divisive games are combined to produce additional cascades containing both imbalances \( p \) and holes \( h \) at the same level, other more exotic sets of thorns over dust and subsequent devil’s staircases are produced. It happens then that equilibrium, in the absence of the divisive games, is only a single point within a square of possibilities:

Certainly, at such a precise location the distance on the accumulated frontier is \( \sqrt{2} \approx 1.4142 \) from top to bottom, while at any other place, no matter how close, such a distance equals the maximum value of 2 when the cascade goes all the way, or takes a value that increases towards 2 as our Reynolds number becomes large.

These observations clearly lead us to several questions: Is it worth to judge one another as Jesus tells us not to do? (Mt 7:1), and for that matter, to compare nations when we are away from the point? Do we really appreciate that going to the point implies traveling from minus to plus? Is there anything better than recognizing our faults and making amends? Is there a better solution but to share our riches with the poor and follow Jesus, as He prescribed to an unnamed rich man (Mt 19:16–24)? For by admitting the “gravity” of our sins we find the best solution in the effective restoration of communication, with God and one another, via the beautiful practice of reconciliation, but only at the point.

I realize that these notions may appear to some to be either too simplistic or too radical, since we may choose a variety of cascades to break the original bar. However, the propagation of division always yields devil’s staircases that are “clouds of dust,” as seen in controlled or uncontrolled demolitions, that properly reflect our
fragmentation and separation. It is certainly true that the original bar may be split into more than two pieces and, as mentioned before, the cascade games may be played aided by chance, that is, using variable imbalances and holes as the process is carried, but such general mechanisms, no doubt capturing the generalities of our own follies, would also result in thorns over dust and in devil’s staircases that remind us of the futility and hypocrisy in thinking that we are any better than anyone else when we are away from the just and merciful point.

As there is no way to slide to the origin via a devil’s staircase, for it lacks inclined tangents, these reflections ultimately call attention to the inherent choices we face in life and their consequences. For even if it may be argued that a bit of turbulence may be good when it shapes our characters, the choices are clear and suggest peace as our most desirable destination.

### A First Set of Choices

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Turbulence</th>
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<td>Calmness</td>
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$1 = 0.999 \ldots$

$2/3 = 0.666 \ldots$

Positive, $+$

Negative, $-$

To the Future

To the Past

As has been argued, we may indeed select between equilibrium or turbulence, calmness or the unrest of violence, rectitude and
its inherent *peace of mind* or the wickedness that comes when we decide, even if unintentionally, to live “stressed out” at high enough Reynolds numbers where we lose our internal cohesion.

The cascade notions clearly reaffirm the essential nature of our full commitment for the flourishing of true *friendships*, and in particular for the success of *married* couples who are summoned to find their happiness in clinging to the common and improbable *root*. For we may see that the shortest path is indeed far better than any form of inequities and holes that take us, politically or not, to truly longest separations, pushed by the devil bit by bit, to end in the real pain of failed relationships.

These simple ideas also remind us that we may learn from the ways of nature to choose to reverse the *natural* processes of our “troubled waters” to end up *reconciled* with one another, that is, to unite one another, even our enemies, via the always proactive notion of integration and its symbol $\int$ (the slender $S$), instead of continuing living in separation and division, as properly reflected by the symbol $\$, a humorous but truthful association that prevents us from recognizing each other as brothers and sisters, a relation that is not so funny when we realize that Jesus reminds us that it is impossible to love God and money at the same time (Mt 6:24).

For we may indeed select wholeness over emptiness, unity over the brokenness that yields thorns and dust, and, quite vividly, an infinite sequence of unnatural eddies rotating outwards denoting our quest for unity and saintly *Love*, $1 = 0.999 \ldots$, over an incomplete fraction, $2/3 = 0.666 \ldots$, that while being read from the divisive natural cascade also properly symbolizes the idiocy and emptiness of our foolish actions, that is, even if we dislike the notation, our *sinfulness*, as defined by our missing of the point.\textsuperscript{26}

For as may be seen on a clock, the ever- *positive* spiral-building unity, that is, $r = e^{+\theta}$ in polar coordinates, always travels ahead mercifully into the *future*, while the *negative* spiral, $r = e^{-\theta}$, is always rooted in the *past*, and, as such, always seeks revenge. Isn’t this fact at the core of the problems that mankind faces? Isn’t the exchange of the “natural,” from plus to minus, into the consciously human, from minus to plus, the solution that we need to implement? The following poem expresses a bit more our choices.
PATHWAYS

Two options before us
two pathways ahead,
the one is the longest
the other straight.

We journey directly
or go by the legs,
we follow intently
or end up in pain.

By walking the flatness
or hiking the spikes,
we travel in lightness
or take serious frights.

The incentive is unity
and the call proportion,
the key is forgiveness
and the goal true notion.

In wandering wickedness
there is never a fruit,
but in ample humbleness
one encounters the root.

CALLS TO CONVERSION

With the key notions explained from modern science, we may turn fully to Holy Scripture to realize that there is a consistent typology in the symbols discussed thus far, that is, that what has been said based on common sense is also expressed in the Bible and using a terminology consistent with the one of modern science.

To begin, we recall next the complete pleas mentioned earlier by the prophets Isaiah and John the Baptist. While Isaiah said “Every valley shall be filled in, every mountain and hill shall be made low; the rugged land shall be made a plain, the rough country, a broad valley. Then the glory of the Lord shall be revealed, and
all mankind shall see it together; for the mouth of the Lord has spoken” (Is 40:4–5). John the Baptist repeated eight centuries later, “Every valley shall be filled and every mountain and hill shall be made low,” (Lk 3:5) and added, “The winding roads shall be made straight, and the rough ways made smooth, and all flesh shall see the salvation of God” (Lk 3:5–6).

Then, in the decisively geometric language of these appeals, we may see that our improvement leads us away from our fractal roads and into the 50-50 condition with no holes, that is, the original bar, as the only possible destination:

These previous “earth moving” citations in the filling of valleys and cutting of mountains, also present in the book of the prophet Baruch (Bar 5:7), represent clear invitations for us to avoid the tortuous paths traveled when we follow any kind of division that, if unchecked, lead us to devil’s staircases. As the glory of God and the salvation of God, both the same concept in the ultimate destination above, appear only in the uniform condition of the original bar of clay, those pleas represent powerful reminders of the key importance of repentance and of the inherent goodness of reconciliation, as prerequisites of living with due integrity and joy.

Clearly, these images help us visualize the universal significance of “Repent, for the kingdom of heaven is at hand,” as preached both by John the Baptist and later on by Jesus when He started his ministry (Mt 3:2; Mt 4:17) and also of “Return and be converted from your idols,” as proclaimed by the prophet Ezekiel (Ez 14:6). These same notions also assist us, in their lack of dissipation, to appreciate the goodness of God’s law, above in the diagram, as God’s ordinances bring life to those who keep them (Ez 20:13).
As the condition of *wholeness* may be used to represent our commitment to rectitude in the absence of sin, the cascade symbolism reminds us that conversion, as sealed by “fellow hydrologist” John the Baptist in the baptism with water (Mt 3:6,11), is absolutely necessary. These images also help us appreciate, as explained in the letter to the Hebrews, why, to guarantee its effectiveness, the process must be “approached with a sincere heart and in absolute trust, with our hearts sprinkled clean from an evil conscience and our bodies washed in *pure water*” (Heb 10:22). Notice in these words the symbolism of cleansing via *water* and observe how the condition of absolute trust in God brings us to a state of *calmness* in which equilibrium may take place, at low Reynolds numbers.

This trust, no doubt, is particularly present in allowing oneself to be submerged into water, and it is essential in the transformation from turbulence to serenity that follows a conversion. For as the prophet Isaiah said, “thus said the Lord God, the Holy One of Israel: by waiting and by *calm* you shall be saved, in *quiet* and in *trust* your strength lies” (Is 30:15), or as the apostle Paul would symbolically add, in a condition not being “tossed by *waves* and swept along by every wind of teaching arising from human trickery” (Eph 4:14), that is, the calmness captured by level zero.

As further explained in Scripture, the cleansing process we are all invited to follow requires us acknowledging our faults, for, as mentioned by the apostle John, “If we acknowledge our sins, he is faithful and just and will forgive our sins and cleanse us from every wrongdoing” (1 Jn 1:9). As we are exhorted to confess our sins to one another and pray for one another so that we may be healed (Jas 5:16), we realize that in the wholeness of the uniform bar we also end up liberated from our sickness. As nicely expressed in Jesus’ words, “There will be more joy in heaven over one sinner who repents than over ninety-nine righteous people who have no need of repentance” (Lk 15:7), the change of heart we are all called to experience is in complete agreement with God’s will. But, as suitably captured by the cascades and the uniform condition, the process indeed ought to be carried out because, as the Proverb explains, “He who conceals his sins prospers not, but he who confesses and forsakes them obtains mercy” (Prv 28:13).
Scripture indeed abounds with many other passages related to “the original bar,” as the fully reconciled and sinless state, a condition that we may indeed term, in its absence of sin, as *equilibrium*. Among the many references related to its *rectitude* one finds the following. “I am ‘the voice of one crying out in the desert,’ make *straight* the way of the Lord” (Jn 1:23), as proclaimed by John the Baptist; “In the desert prepare the way of the Lord. Make *straight* in the wasteland a highway for our God” (Is 40:3), as said by the prophet Isaiah; “In all your ways be mindful of the Lord, and he will make *straight* your paths” (Prv 3:6), as a Proverb teaches; “On the way of wisdom I direct you, I lead you on *straightforward* paths, when you walk, your step will not be impeded, and should you run, you will not stumble” (Prv 4:11–12), as said in another Proverb; “Teach me to do your will, for you are my God, May your kind Spirit guide me on ground that is level” (Ps 143:10), as expressed by the Psalmist; and, quite eloquently, “Let him who is wise understand these things; let him who is prudent know them, *Straight* are the paths of the Lord, in them the *just* walk, but sinners stumble in them” (Hos 14:10), as declared by the prophet Hosea.

These calls to straightness and the “dying to sin,” as explained by the apostle Paul (Rom 6:11–12), clearly remind us of the ease in walking a leveled uniform bar and playing hopscotch on it, as opposed to the outcomes of spiky cascades that are ultimately impossible to walk, hence hinting at the goodness and vitality of the *sacrament of reconciliation*.

The following passages show that the symbolism of water, already found in baptism, is also encountered in related circumstances connected to the *balanced* state. For instance, “Happy those who do not go the way of sinners, they are like a tree planted near streams of *water*, that yields its *fruit* in season; its leaves never wither; whatever they do prosper” (Ps 1:1,3), as expressed by the Psalmist and which guarantees our well being as long as we engage in continuous repentance; “I will lead them to *brooks of water*, on a *level road*, so that none shall stumble” (Jer 31:9), as explained by the Prophet Jeremiah and that gives us hope in our journey; “In green pastures you let me graze; to *safe waters* you lead me; you restore my strength, You guide me along a *right path* for the sake of
your name” (Ps 23:2–3), as famously proclaimed by the Psalmist and that tells us that it is God’s will for us to turn away from turbulence to be restored into uniformity; and “Everyone who drinks this water will be thirsty again” (Jn 4:13), as explained by Jesus to the Samaritan woman and that assures us that the cleaning process needs indeed to be done permanently.

Finally, it is worth including a rather curious quote related to the famous conversion of the apostle Paul in the road to Damascus. Quite concordantly with these notions, Paul, then Saul, was baptized by Ananias in the street called Straight (Acts 9:10–19). Rather peculiarly indeed, St. Paul was appropriately straightened up at Straight Street.

**OUR EVIL WORLD**

The adequacy of the cascade framework may also be verified while speaking about the devil and those who follow him, properly named “wicked.” As according to the apostle John, “the devil has sinned from the beginning” (1 Jn 3:8), and as Jesus added that he is a murderer and a liar not having in him any truth at all (Jn 8:44), it is reasonable to employ the negative and dissipative cascade and its associated devil’s staircase, for that matter any cascade that accumulates division and leads to death, to talk about the devil’s actions and to reflect on his maximal separation from God:

For as the apostle Paul explains, the devil’s behavior is based on “Immorality, impurity, licentiousness, idolatry, sorcery, hatreds, rivalries, jealousies, outbursts of fury, acts of selfishness, dissensions,
factions, occasions of envy, drinking bouts, orgies, and the like” (Gal 5:19–21), and hence it is indeed sensible to assume that the his dynamics ultimately happen over an empty and broken set which ought to be, very fittingly, dust.

Notably, but not surprisingly, dust, ashes and dirt are often employed in the Bible to describe the ultimate fate of the devil and his followers. As explained in the Book of Genesis, after the serpent tempted Eve in paradise, God said to him “On the belly shall you crawl, and dirt shall you eat all the days of your life” (Gn 3:14). Similarly, when God spoke against a sinful Adam, hence signaling the beginning of death for humanity, He said, “You are dirt, and to dirt you shall return” (Gn 3:19). Also recall the words of the prophet Isaiah when describing a renewed future world, “The wolf and the lamb shall graze alike, and the lion shall eat like the ox, but the serpent’s food shall be dust” (Is 65:25), and the rather categorical pronouncement by prophet Micah against those who by following their wicked ways become God’s enemies, for “They shall lick the dust like the serpent” (Mi 7:17).

As it is apparent from several passages, God does despise the wicked and if needed He Himself carries a cascade to punish them. As expressed by King David and prophet Ezekiel, the Lord says, “I ground them fine as the dust of the earth; like the mud in the streets I trampled them down” (2 Sm 22:43), and “In my fury I will let loose stormwinds; because of my anger there shall be flooding rain, and hailstones shall fall with destructive wrath” (Ez 13:13). Notice the incisive references to the repetitiveness present in the multiplicative cascade and the powerful allusions to turbulence, which furnishes us, once again and now realizing the awful consequences of our own cascades, a vivid invitation to the gentleness symbolized by the original bar.

Further connections regarding turbulence and those who choose to be wicked, like the devil, may be made realizing that the apostle Paul gave to the devil the pertinent title mentioned before “the ruler of the power of the air” (Eph 2:2), but also the name “the evil spirits in the heavens” (Eph 6:12), which compounded by Jesus’ own description “the ruler of the world” (Jn 12:31), let us appreciate the source of natural deadly turbulence in the earth and
beyond, even if called “beautiful” in exploding stars and colliding galaxies, and also the origin of our economic devil’s staircases.

For, as we may appreciate, the ancient serpent (Rv 12:9) is certainly around, but the symbolic and real exhortations for us not to follow him abound: for the wicked “are like chaff driven by the wind” (Ps 1:4), as the Psalmist said; “Yes, the hope of the wicked is like thistledown borne on the wind, and like fine, tempest-driven foam; like smoke scattered by the wind” (Wis 5:14), as the Book of Wisdom explains; “When they sow the wind, they shall reap the whirlwind” (Hos 8:7), as symbolically explained by the prophet Hosea; “The horde of your arrogant shall be like fine dust, the horde of the tyrants like flying chaff” (Is 29:5), as explained by the prophet Isaiah; and “Therefore they shall be like a morning cloud or like the dew that early passes away, like chaff storm-driven by the threshing floor or like smoke out of a window” (Hos 13:3), as expressed, once again, by the prophet Hosea.

Notice how the scattering that accompanies turbulence represents a common punishment by God. This may be seen, for instance, in the tower of Babel, where those hoping to make a name for themselves and without God ended up “scattered from there all over the earth” (Gn 11:8), for, as said by the prophet Habakkuk, “He veers like the wind and is gone—this culprit who makes his own strength his god!” (Hab 1:11). This dispersion is also encountered as a response to the obstinacy of the people of Israel, for as explained by the prophets Isaiah and Zechariah, “The people has rejected the waters of Shiloah that flow gently” (Is 8:6), “They refused to listen; they stubbornly turned their backs and stopped their ears so as not to hear. And they made their hearts diamond-hard so as not to hear the teaching and the message that the Lord of hosts had sent by his Spirit through the former prophets. Then the Lord of hosts in his great anger said that, as they have not listened when he called, so he will not listen when they called, but would scatter them with a whirlwind among all the nations that they did not know” (Zec 7:11–14).

The notion of scattering as God’s punishment may also be seen through the precise words by the prophet Ezekiel, “Then shall they know that I am the Lord, when I disperse them among the nations
and scatter them over foreign lands” (Ez 12:15), for he adds, “A third of your people shall die of pestilence and perish of hunger within you; another third shall fall by the sword all around you; and a third I will scatter in every direction, and I will pursue them with the sword” (Ez 5:12), which closely evokes the construction of the Cantor set.

Notwithstanding the goodness of stirring our coffees in the morning, the relationship between God’s punishment and our turbulent ways and the implied invitation to repentance in the original bar are also seen in the words of the prophet Nahum, “The Lord is slow to anger, yet great in power, and the Lord never leaves the guilty unpunished. In hurricane and tempest is his path, and clouds are the dust at their feet” (Na 1:3), and in those of the prophet Amos, “I struck you with blight and scaring wind; your many gardens and vineyards, your fig trees and olive trees the locust devoured; yet you returned not to me, says the Lord” (Am 4:9). Notice also how God’s chastisement of the devil’s broken and worthless dust is very appropriately “blown away” when the Messiah, Jesus Christ, comes again, for as explained by the prophet Isaiah and later on by the apostle Paul, “With the breath of his lips he shall slay the wicked” (Is 11:4), as “The Lord Jesus will kill him with the breath of his mouth and will render him powerless by the manifestation of his coming” (2 Thes 2:8).

It also happens that the imagery of Cantor sets and their obvious emptiness, as opposed to the wholeness of equilibrium, also appears in Scripture, as a powerful symbol of division. As said by the prophet Isaiah in regards to sinners, “No one brings suit justly, no one pleads truthfully; they trust in emptiness and tell lies; they conceive mischief and bring forth malice” (Is 59:4), “This is a people despoiled and plundered, all of them trapped in holes, hidden away in prisons” (Is 42:22); or as said in the Book of Proverbs, “The wicked man makes empty profits, but he who sows virtue has a sure reward” (Prv 11:18). Other relevant connections between emptiness and turbulence are given throughout the Book of Ecclesiastes when it is said “This is also vanity and a chase after wind” (Eccl 6:9), and in the words of the prophet Isaiah when discussing the fatuity of an idol which opposes our almighty God, “Ah, all
of them are *nothing*, their works are *nought*, their idols are *empty wind!*” (Is 41:29), and in “The wicked are like the *tossing sea* which cannot be *calmed* and its waters cast up *mud* and *filth*” (Is 57:20).

An obvious trait of turbulent behavior, which often affects our lives, is the intense and unpredictable violence that it generates. As the Psalmist says regarding the wicked, “Pride adorns them as a necklace; *violence* clothes them as a robe. Out of their stupidity comes sin; evil thoughts *flood* their hearts” (Ps 73:6–7). This behavior is clearly against God, for as the Psalmist explains, “The Lord tests the good and the bad, *hates* those who love *violence*, and rains upon the wicked fiery coals and brimstone, a *scorching wind* their allotted cup” (Ps 11:5-6), and He assures us that “all sinners will be destroyed; and the future of the wicked will be cut off” (Ps 37:38).

Notice how these last two verses allude to the turbulent inferno that modern man prefer not to think about and that is known as *hell*. As we shall study in more detail in the next chapter, it is all there in the manual that is the Bible, and hence it is best to be reconciled with God. For as the prophet Isaiah assures us “The Lord repays his enemies their deserts” (Is 66:6), that is, we get emptiness and dust if that is what we plant.

As previously explained, following a turbulent cascade all the way, mathematically, yields an infinite sequence of progressively small eddies. Given that those elements split and rotate inwardly, it is indeed customary, as we have done, to include inside each depicted rectangle a spiral symbol in the shape of number 6. As such, the cascade reads zero-six-six-six, as in the previous citation from Isaiah, and \( \frac{2}{3} \) becomes an accurate reminder of the devil’s actions via his ever-present selfish negativity, an infinite sequence that is also consistent with the very famous three sixes assigned in the Book of Revelation to the upcoming antichrist (Rv 13:18).

Although it may appear that the presence of such a fraction is arbitrary, it is pertinent to remember that it was precisely \( \frac{2}{3} = 0.666 \ldots \) the proportion of Israelites who were killed out of God’s wrath, one third by pestilence, and one third by the sword, as mentioned earlier in the account of the prophet Ezekiel and that the apostle Peter denied Jesus Christ precisely three times before
the cock crowed twice (Mk 14:66–72). Curiously, it is worth to notice that such same fraction, which metaphorically may be used to represent the “emptiness of sinfulness,” appears frequently in the study of turbulence.\textsuperscript{23,29}

To conclude this section, it is worth mentioning other instances where the symbolic dust appears. First, it is pertinent to realize that as expressed in the first Book of Samuel, “The Lord raises the needy from the dust; from the ash heap he lifts up the poor” (1 Sm 2:8), and hence it is possible to be reconciled by repentance, reversing the flow of a cascade as we saw before. Second, these connections between sinfulness and dust may shed some new light as of why, in the Gospels, Jesus instructed His disciples to shake only the dust from their feet when not accepted during their journey (Mt 10:14), and why, in consonance with the words of the prophet Isaiah, “how beautiful are the feet of him who brings glad tidings” (Is 52:7), Jesus only washed the feet of the disciples during the last supper (Jn 13:4–11).

Even though it may appear to some that connecting modern science with ancient Scripture is unwarranted and altogether silly, especially in regards to the “obsolete” notion of the devil, we have seen in this section that there is a rather vast set of relations that end up pointing us, by default and even in this day and age, to the best and unique solution for all.

**The One and Only**

As mentioned earlier, the difference in length between the original bar and any cascade, one containing either imbalances, holes or both, provides suitable metaphors to contrast justice with sinfulness. This naturally point us to Jesus, the just, as, according to several citations, He always acted in perfect rectitude according to God’s will, and committed no sin (Is 53:9, Jn 8:46, 2 Cor 5:21).

Based on these citations, Jesus always maintained the uniform bar and its efficient linear ramp and, hence, He is potently symbolized by such straight elements, and in particular by the defining equation $Y = X$, as explained before. Notice how these geometric
images are indeed consistent with Jesus’ own statement of perfection, “Do not think that I have come to abolish the law or the prophets. I have come not to abolish but to fulfill” (Mt 5:17), as only He was able to always keep the original bar.30

As it is clearly the case, it is not possible for us, and certainly not for me, to always maintain equilibrium. As such, we invariably wander between good and evil. In this regard, the words of the apostle Paul best describe our limitations when he says, “We know that the law is spiritual; but I am carnal, sold into slavery to sin. What I do, I do not understand. For I do not what I want, but I do what I hate. For I do not do the good I want, but I do the evil I do not want” (Rom 7:14–15,19). As further explained by the apostle John, “If we say, ‘We are without sin,’ we deceive ourselves, and the truth is not in us. If we say, ‘We have not sinned,’ we make him a liar, and his word is not in us” (1 Jn 1:8,10), or as said in the Book of Ecclesiastes, “There is no man on earth so just as to do good and never sin” (Eccl 7:20), then we ought to realize that our sinful nature indeed requires of a Redeemer who may set us free. That this is the case is poignantly stressed by the apostle Paul when he explained that the price of sin is death (Rom 8:13), which in turn implies that following a devil’s staircase, even if seemingly harmonious, results in a deadly option that can only be restored in the perfection of Jesus Christ. Of course, it is Jesus’ divinity what empowered Him to always do God’s will and hence fulfill the symbolic \( \sqrt{2} \), and for that same reason He is our able redeemer.

As the uniform condition may be used to represent the ultimate state of virtue and cohesion and as any cascade eventually leads to maximal separation, this contrast allow us to better appreciate the goodness of God’s way as opposed to our own proud ways. Hence we may cherish Jesus’ invitation when He says, “Come to me, all you who labor and are burdened, and I will give you rest. Take my yoke upon you and learn from me, for I am meek and humble of heart; and you will find rest for yourselves. For my yoke is easy, and my burden light” (Mt 11:28–30), as we may see that He indeed is our best choice, even if we have to come back to him and from our sins again and again. That Jesus is “the way and the truth and the life” (Jn 14:6) and the precious cornerstone rejected by the people
of Israel (Is 28:16, Lk 20:17–18) becomes readily perceptible based on the geometric interpretations already made to these passages.

As the perfect maintenance of equilibrium neither uses lies nor is based on a condition of “hardness of heart” that dissipates, and, as seen before, the straight condition is the tried way of God, Jesus is indeed defined by the famous phrase “the way and the truth and the life,” and no one slides to the Origin, God the Father, as explained earlier, except through the precious hypotenuse (Jn 14:6).

As seen geometrically, and even if in a childish way, the easiness of Jesus’ yoke and the lightness of His burden are appreciated in the ability we gain in walking freely above the leveled bar and, if desired, also play hopscotch there, as opposed to the spiky objects generated by any cascade:

Observe how this free walking in equilibrium, in the fullness of our forgiveness and in the positive cross of Jesus Christ, in addition to being perfectly efficient, also removes from us any fear of stumbling. This is nicely captured in the words of the prophet Ezekiel when he said, “The Lord speaks to the prophet in the plain” (Ez 3:22–23), and in particular by the apostle John when he so eloquently spoke about love, “There is no fear in love, but perfect love drives out fear because fear has to do with punishment, and so one who fears is not yet perfect in love” (1 Jn 4:18).

Even though our ego may still tell us that the notion of the cascade may be too strict in describing our behavior, these observations suggest that, as expressed previously as a first set of options, there are, at the end, basically two possible pathways and only two that we may follow (Sir 39:24). There is either a sequence of steep selfish acts that eventually leads us to dust and death, or there is the preservation of the leveled bar which, as explained by the apostle Peter and by virtue of Christ’s death in the cross, becomes for us a life of righteousness (1 Pt 2:24). That this is the case is beautifully expressed in Jesus’ own words when He said, “I
came into the world as light, so that everyone who believes in me might not remain in darkness” (Jn 12:46), that is, *dissipation*, for as explained by the prophet Isaiah, light and peace are found in what is right and darkness in what is crooked (Is 59:8–9).

The explicit difference in paths and their relation to Jesus are also plainly seen in His famous and authoritarian words, “Whoever is not with me is against me, and whoever does not *gather* with me *scatters*” (Mt 12:30), which poignantly contrast integration and division and very subtly speak of the devil’s scattering, as performed, appropriately, by the wind, and, moreover, following a simple cascade in whose eddies we may read our common and essential enemy.

That Jesus, in the original bar, is our best destination may also be seen in other citations. For instance, the evident gap that exists between our two possible behaviors, that is, $\sqrt{2}$ vs. 2, is nicely captured by the words of the prophet Jeremiah when speaking about two baskets of *figs*, ones excellent but others so bad that they could not be eaten (Jer 24:1–3), and very graphically by the Psalmist when he said, “As far as the east is from the west, so far have our sins been removed from us” (Ps 103:12), and also by Jesus’ own words when He said, “It is easier for a camel to pass through the eye of a needle than for one who is rich to enter the kingdom of God” (Mk 10:25), that point us to Jesus himself, who by obeying God’s plan, including dying on the positive *cross*, fulfilled the ultimate and perfect *dot* (Phil 2:8):

Notice how the best choice is the shortest in all aspects, even if it indeed has a probability of *zero* that makes it satisfy the adage, and observe how the benefit of *forgiveness* becomes evident. This may be further observed in the words of the Psalmist when he adds,
“If you, Lord, mark our sins, Lord who could stand? But with you is forgiveness and so you are revered” (Ps 130:3–4) and “Happy the sinner whose fault is removed, whose sin is forgiven” (Ps 32:1), for these notions point us to the original state that the prophet Isaiah describes as “a highway clear of stones” (Is 62:10), the same joyous place of an everlasting covenant described in the Book of Exodus as a “spacious land flowing with milk and honey” (Ex 3:8).

As is seen, Jesus’ invitation is a call for perfection, “Be perfect, just as your heavenly Father is perfect” (Mt 5:48) and such a call was accepted by the apostle Peter via three symbolic positive spirals when Jesus asked him, after His resurrection and in contraposition to his early negative denials, if he loved Him (Jn 21:15–17):

$$0.999... = 1$$

This invitation implies, as expressed in the Lord’s prayer (Mt 6:9–15), removing our sinfulness and earnestly going to God, walking in love with Jesus, co-creating a chain of outward spirals yielding unity, and getting our “halos”—yes, halos shaped as the symbolic zero of the perfect circle that defines the holy Origin. For as the apostle Paul said, “You should put away the old self of your former way of life, corrupted through deceitful desires, and be renewed in the spirit of your minds, and put on the new self, created in God’s way in righteousness and holiness of truth” (Eph 4:22–24).

Notice how, as explained in the Gospel, the old self corresponds to the selfish spiral, to the “wooden beam” (Mt 7:3) of negativity and lies present in us, or, as expressed by the apostle James, “the tongue, a restless evil, full of deadly poison” (Jas 3:8), which if untamed so easily leads us to cascades of division that bring separation, violence, and death. Observe how this state happens to be of maximum separation because, as the same apostle James explains, “Whoever keeps the whole Law, but falls short in one particular, has become guilty in respect of all of it” (Jas 2:10). Notice how in contraposition, the new self represents the mind of Jesus who calls us to “obey and teach the commandments” (Mt 5:19).
Ultimately, the clear distinctions, between short and long, just and wicked, and straight and crooked, demand of us a conscious effort to repair the breach (Is 58:12) and to attain equilibrium. For as the apostle Paul expresses in “Rejoice with those who rejoice, weep with those who weep” (Rom 12:15), the leveled state is indeed of key relevance. The following poem expresses the very same feeling of solidarity via the related symbol of the lovely hypotenuse.

**THE HYPOTENUSE**

There is a best pathway,  
the palpably smooth,  
*it’s by the hypotenuse,*  
and walking in truth.  

There is one solution,  
I tell you the truth,  
*it’s by the hypotenuse,*  
and walking in truth.  

For any other pathway  
will lead us astray,  
*it’s by the hypotenuse,*  
there is no other way.  

Oh listen, you brother,  
let’s brighten the day,  
*it’s by the hypotenuse,*  
there is no other way.  

Otherwise, the devil  
shall pull by the legs,  
*It’s by the hypotenuse,*  
*or else by the legs.*

For such road is dusty,  
as long as it gets,  
*it’s by the hypotenuse,*  
*or else by the legs.*

Oh let’s mend the broken,  
growing to the root,  
*it’s by the hypotenuse,*  
the one that yields fruit.

Let’s keep equilibrium,  
avoiding dark soot,  
*It’s by the hypotenuse,*  
the one that yields fruit.

Oh listen, you sister,  
a counsel from science,  
*it’s by the hypotenuse,*  
the simplest design.

I tell you integrating,  
don’t leave it to chance,  
*it’s by the hypotenuse:*  
the simplest design.

**THE BEAUTY OF UNITY**

Quite vividly, the *original* bar also provides us with suitable geometric imagery to describe *unity*. As previously described, Scrip-
ture contains unmistakable calls to reconciliation that point us to the goodness of unity. For instance, and as expressed by the Psalmist, “How good it is, how pleasant, where the people dwell as one!” (Ps 133:1), and the very specific exhortation by Jesus who argues against the snowball effect of a cascade which represents the Biblical basis for the concept of purgatory, “Why do you not judge for yourselves what is right? If you are to go with your opponent before the magistrate, make an effort to settle the matter on the way; otherwise your opponent will turn you over to the judge, and the judge hand you over to the constable, and the constable throw you into prison, I say to you, you will not be released until you have paid the last penny” (Lk 12:57–59).

The relevance of the original bar is further enhanced by realizing that unity is precisely what Jesus desires for us, that is, unity in Him. This may be seen in His prayer to God the Father regarding the disciples, as recorded in the Gospel, “I pray not only for them, but also for those who will believe in me through their word, so that they may all be one, as you, Father, are in me and I in you” (Jn 17:20–21), which is consistent with other of His assertions, such as, “The Father and I are one” (Jn 10:30).

As the role of the Holy Spirit, to be fully explored in chapters to come, is to guide us to all truth in Jesus Christ (Jn 16:12-15), the Spirit also participates on such a special unity with the Father and the Son, as He leads us to the uniform stage of perfection in which we ourselves become one with Jesus. Notice here how the original bar plays a dual role, that is, unity and oneness, and observe how it symbolizes geometrically the fact that two or many could indeed “become one” in the same leveled and holy ground. For quite eloquently this image explains why true unity, no matter how else we may call it, can only be fulfilled in Jesus Christ and why He apparently said so little about marriage via His famous words that “they are no longer two, but one” (Mt 19:6).

Of course, the symbol of equilibrium does not imply that those fulfilling God’s will shall become identical to one another, but rather that only on such a stage a symphony of love may exist, in full communion with God and one another. For the role of individuality is indeed expressed, for instance, in the words of the
The apostle Paul, when he said, “For as in one body we have many parts, and all the parts do not have the same function, so we, though many, are one body in Christ and individually parts of one another” (Rom 12:4–5).

In any event, the key equation of unity, graphically satisfied by joining two index fingers vertically to make a larger one, or Biblically in two sticks made one and representing the once divided houses of Judah and Joseph, as explained by the prophet Ezekiel (Ez 37:15-22), yields simply,

$$1 + 1 + \ldots + 1 = 1,$$

which symbolically denotes the Church, that is, the people of God, who by living according to His plan fulfill the ultimate prediction that “there will be one flock, one shepherd” (Jn 10:16), or the desired and veritable unity in Jesus Christ.

In summary, only in the uniform and upright condition and in its associated one-to-one root we may find the rectitude and the required communication that allow us to encounter unity. Only by listening to Jesus’ truth (Jn 18:33–37) and following His positive spiral of love, $r = e^{+\theta}$, that flows towards the other, even toward our enemies (Mt 5:44), so that we may achieve the lovely equation $1 = 0.999\ldots$ that truly raises the bar.

For as Scripture emphasizes, Jesus’ disciples are to be recognized by their love for one another (Jn 13:35), a truly unnatural condition that, contrary to the natural eddies, travels from minus to plus. For, quite accurately and in the fullness of the symbols herein, there is darkness, dissipation, between our selfish postures and our loving ones, as there was a prescribed eclipse of the sun (Am 8:9), precisely between the 6th and 9th hour, when Jesus was crucified for us (Mk 15:33–37) and crowned by the thorns that denote our transgressions (Mk 15:17, Is 53:5).

Even if these associations may, once again, appear unwarranted to some, there is even more, for we may unfold the intertwined 6 and 9 making up a circle in the famous symbol of the Yin and Yang, to describe not a balance between good and evil, but rather the triumph of Jesus in His positive cross. For although turbulence is in the air, the coherent light of love wins (Eph 5:8–14).
Six, zero, nine, a dear song
numbers unfolding daylong,
six, zero, nine, a clean gong
symbols inviting us to love.

From six to six from zero to nine
revolving inwards, the spiral turned over,
from six to six from zero to nine
I went downwards. I dared to love others.

From six to six From zero to nine
dividing selfishly, I attempted prayers,
from six to six from zero to nine
lying endlessly. I became a repairer.

From six to six From zero to nine
trying to be a rose, infinity flowed free,
from six to six from zero to nine
being only a nasty thorn. unity grew in me.

From six to zero From nine to nine
I tapered my speed, weaving my reality,
from six to zero from nine to nine
the tempest did not lead. dreaming its totality.

From six to zero From nine to nine
I no longer postponed, conquering my greed,
from six to zero from nine to nine
I finally atoned. planting a new seed.

From six to zero From nine to nine
I experienced peace, despite a clear spite,
from six to zero from nine to nine
my loneliness ceased. knowing there is light.

Six, zero, nine, a dear song
numbers unfolding daylong,
six, zero, nine, a clean gong
symbols inviting us to love.
To Equilibrium

We have seen that there is a consistent relationship between the teachings of Holy Scripture and what may be inferred based on how turbulence happens. No doubt, it is best for us to avoid our turbulent cascades in order to find the point and it is best for us to avoid thorns, dust, and devil’s staircases in order to rest in the perfect peace of $Y = X$, that is, in true equilibrium. For even if turbulence naturally happens to us all, these reflections remind us that it is best to heed Jesus’ symbolic words when He explained, “Everyone who listens to these words of mine and acts on them will be like a wise man who built his house on rock. The rain fell, the floods came, and the winds blew and buffeted the house. But it did not collapse; it had been set solidly on rock. And everyone who listens to these words of mine but does not act on them will be like a fool who built his house on sand. The rain fell, the floods came, and the winds blew and buffeted the house. And it collapsed and was completely ruined” (Mt 7:24–27).

For completeness, this chapter includes an additional list of Biblical precepts that help us attain or/and maintain the sought rock, the balanced condition beautifully represented by the uniformity of the original bar.

On the one hand, we may avoid holes and imbalances by: forgiving others, seventy times seven times as seen with the same numbers on the turbulent cascade (Mt 18:22), not letting anger accumulate (Eph 4:26), not grumbling or complaining (Phil 2:14, 1 Cor 10:10, 1 Pt 4:9), refraining our tongue from evil and lies (Ps 34:14), loving our enemies (Lk 6:27, Prv 25:21), blessing those who curse us (Lk 6:28, Rom 12:14, 1 Pt 3:9), loving one another without partiality (Jn 13:34, Jas 2:8–9, Dt 1:17), applying the golden rule, that is, proactively doing to others what we would like others do to us (Mt 7:12, Lk 6:31), giving more than receiving (Acts 20:35, Sir 4:31), not “missing the point” by judging others (Mt 7:1, Rom 2:1–2, 1 Cor 4:5), giving to the hungry and the thirsty (Mt 25:35), sharing our wealth with the poor (Mt 19:16–24, Lk 3:10–14), clothing the naked and visiting those in prison (Mt 25:36), and, in general, by serving one another (Mt 20:26, Mk 9:35).
On the other hand, some specific calls for our perfection in the original bar are found in us: being sincere toward God (Dt 18:13), praying without ceasing and being thankful to increase our “internal viscosities” (Mk 14:38, 1 Thes 5:17), renewing our spirit and mind (Eph 4:23, Rom 12:2), taking our thoughts captive to Christ (2 Cor 10:5), avoiding any kind of cascade so that we do not end up being hypocritical (Mt 7:5), remaining in the love of Christ (Jn 15:9–10), rejoicing in the Lord (Phil 2:18, Phil 4:4, 1 Thes 5:16–18), imitating God with due perseverance (Eph 5:1, Mt 5:48, 1 Pt 1:16, Jas 1:4, Jn 7:18), examining our conscience to avoid sin (Ps 4:5), receiving the special peace of God that takes away fear (Phil 4:7, Jn 14:27, Col 3:15), giving evidence of repentance (Acts 26:20, Mt 3:8), living without impurity and in holiness (1 Thes 4:7), not living according to our strength (Wis 2:11, Phil 2:3), living in trust one day at a time (Mt 6:34), not letting the thorns of anxiety and the lure of riches choke the Word of God (Mt 13:22), and in general, by us growing in humility (Phi 2:3, Rom 12:3) to experience the fruits of truly peaceful Reynolds numbers in our lives.

As we shall see in the next chapter, God’s call to surrendering to His divine will is closely related to our ability to grow in service and in humility, as the precious $Y = X$ did.

**Symbols and Further Reflections**

It has been shown that elements that appear via the study of turbulence turn out to be useful to visualize the paths that we often take in order to destroy equilibrium and unity. Some of the key symbols encountered in the analysis are summarized below:

- **Multiplicative cascades**, general mechanisms that represent the progressive propagation of imbalances and holes as found in natural turbulence and that we often employ in order to break equilibrium.

- **Dust and thorns**, the ultimate consequences of the disintegration produced by cascades as unity is shattered by the practice of division.
• *Eddies*, the inward rotating elements of natural turbulence that reflect the movement of our own selfishness, as in the negative spiral that corresponds to number 6.

• *devil’s staircases*, graphs showing the crooked and most inefficient placements of accumulated mass found while following dissipative cascades.

• *Equilibrium*, the even distribution of equality and serenity as found by practicing the unnatural flow of positive spirals represented by number 9.

• *Power of zero*, the condition of trust and repentance required in order to mend sins and achieve unity and peace, as “saints” are called to do.

• *The hypotenuse*, the most efficient and straight accumulated state that, by keeping God’s commandments, avoids the division of all cascades.

• *The root of two*, the shortest distance in the hypotenuse and the essence that two may achieve while being protected by perfect love.

• *Y = X*, the formula of the hypotenuse and the silhouette of Jesus crucified on the cross as the one and only solution for peace and unity.

This chapter has shown that there is a consistent typology in Holy Scripture between the images associated with turbulence and equilibrium and those related to the sinful and just persons, respectively. The reader is encouraged to further verify such a relationship by considering passages from the Bible, for instance, some of the Psalms and prophetic books cited herein, and the Gospels, including Jesus’ famous Sermon on the Mount (Mt 5–7).

As multiplicative cascades have been shown to be related to the thorny topic of wealth inequalities and also to our own daily actions, the reader is also encouraged to reflect on the particularly sad economic state of our modern world and on the applicability of the cascade notions to describe our communities and our personal
relationships, including, in particular, the one we may have with God Himself.

A Bit More on the Solution

\[ Y = X \]

is justice that illuminates,
is balance that fascinates:
\[ Y = X. \]

\[ Y = X \]
is the practical alliance,
is the precious reliance:
\[ Y = X. \]

\[ Y = X \]
is true word that matures,
is a spiral that endures:
\[ Y = X. \]

\[ Y = X \]
is the spotless resting place,
is the state of mighty grace:
\[ Y = X. \]

\[ Y = X \]
is smoothness that esteems,
is a lovely dove that gleams:
\[ Y = X. \]

\[ Y = X \]
is the short and precise root,
is the weaving of the truth:
\[ Y = X. \]

\[ Y = X \]
is a future that forgives,
is crowned science that is:
\[ Y = X. \]